Transmission Lines for Short-Wave Radio Systems*

By E. J. STERBA AND C. B. FELDMAN

The requirements imposed on transmission lines by short-wave radio systems are discussed, and the difference in the requirements for transmitting and receiving purposes is emphasized. Various line types are discussed, particular attention being given to concentric tube lines and balanced two-wire lines. The concentric tube line is particularly valuable in receiving stations where great directional discrimination is involved and low noise and static pick-up is required.

Excellent agreement between calculations and measurements is found for the high-frequency resistance of concentric lines, using the asymptotic skin effect formula of Russell. Other losses in correctly designed concentric tube lines are found to be negligible. Measured losses in two-wire lines are found to be greater than losses predicted by the asymptotic skin effect formula owing, in part, to losses brought about by unbalanced currents.

Practical aspects of line construction such as joints, insulation, and provision for expansion with increasing temperature are discussed.

Some difficulties encountered in transmission line practice, such as losses due to radiation, reflections from irregularities, effects of weather, and spurious couplings between antenna and line are discussed.

I. General Requirements

THE transmission line systems employed for the purpose of transferring energy between radio units and antennas are fundamentally no different from line systems used in power or telephone work. Owing, however, to the high frequencies employed in radio transmission an operating technique differing from that found economical in low-frequency practice is necessary. An important consideration in 60-cycle power practice is that the voltage at the far end of the line be maintained constant irrespective of load variations. At radio frequencies a transmission line may be many wave-lengths long and the reflections from a load other than one equal to the characteristic impedance of the line produce standing waves. Transmission losses in radio-frequency lines are appreciably augmented when the currents and voltages on the line appear in the form of standing waves. The operation of the radio unit connected to the line is sometimes affected by the presence of standing waves.

Induction and cross-talk problems familiar to every telephone engineer are increasingly important as line operation approaches radio frequencies. Owing to the high sensitivity of radio receiving equipment as compared with that of telephone equipment the difficulties

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arising from static and noise pick-up are more marked at radio frequencies. Spurious radiations from lines operated at radio frequencies may completely destroy the directional characteristics of an antenna and in addition may cause interference to other radio stations. Under certain conditions the spurious power radiated by a transmission line may be an appreciable fraction of that radiated by the antenna connected to the line.

It follows that although the primary purpose of a transmission line in a radio station is to provide a means for transferring energy between an antenna and the radio unit, a consideration of great importance is the degree of isolation from its associated antenna, from other antennas and lines, and from extraneous sources of signals. This is particularly true in receiving stations where discrimination against undesired signals is oftentimes of greater importance than the over-all sensitivity to the desired signal. Extraneous pick-up on a line to the receiving unit may not only destroy the directional pattern of the antenna but it may also introduce a noise level into the receiver output comparable to the desired signal and so destroy the utility of the apparatus. As compared with transmitters, receivers are generally small units. It is economical to house several units in one building. The lines not only to one receiver but those to other receivers must of necessity be in close proximity. Thus the possibilities for cross-talk with ensuing increase in noise levels, loss of circuit gain, and loss of discrimination are greatly augmented.

Of course, cross-talk possibilities between lines of adjacent transmitters cannot be ignored. Transmitters, however, usually occupy sufficient space so that a desirable degree of line separation is obtained. With the exception of local lightning storms static pick-up is of no great importance. High voltage surges due to lightning may be drained by means of properly placed horn gaps and grounds.

Insulation for high voltages at radio frequencies is an important consideration for the case of lines connected to transmitters. Insulators for balanced open-wire construction may be selected from materials designed for high voltage power transmission. However, insulators for concentric tube lines capable of transmitting several kilowatts of modulated radio-frequency power require special consideration.

The lines commonly employed in radio stations may be divided into four classes: single-wire lines, balanced open-wire lines, multiple-wire lines, and concentric-tube lines.

Single-wire lines are of limited utility owing to the low efficiencies arising from the marked radiation characteristics of such wires. The power radiated by a single-wire line several wave-lengths long may be equal to that radiated by the antenna to which it is connected.[1] In fact, single-wire lines, particularly when terminated, are for certain services desirable radiating elements. Diamond-shaped arrays of such elements are employed in some of the radio facilities of the Bell System.[2]

It is generally appreciated that the power losses due to radiation may be reduced by employing two conductors in a go-and-return circuit, the wires being separated a small fraction of a wave-length. A necessary requirement is that the two wires carry equal currents exactly opposite in phase. Otherwise, there will appear current components which employ the two conductors in parallel. In the latter event the radiation losses ascribed to single-wire conductors occur.

Although there is a very great reduction in radiated power in balanced two-wire lines as compared with single-wire lines there are many practical cases where the radiation from two-wire lines produces cross-talk and loss of signal discrimination. Multiple-wire lines comprising several pairs of conductors in go-and-return circuits may be employed to reduce the undesired radiation couplings. As in the two-wire case, care must be exercised in maintaining the required current amplitudes and phases since otherwise the radiation losses ascribed to single-wire lines may destroy the utility of the multiple-wire system. Multiple-wire lines, of course, reduce static and noise interference.

From the standpoint of isolation an ideal electrical connection between antennas and radio apparatus is approached when one conductor completely encloses the other conductor. A concentric-tube line comprising an outer sheath and an inner conductor is the practical form of this construction. Long transmission lines often pick up a large amount of static and other electrical disturbances. Spurious couplings may introduce these disturbances into the radio circuit. Electrical disturbances so introduced are greatly reduced when the outer sheath of a line may be grounded at frequent intervals. In fact, concentric-tube lines may be buried in the ground.

The effect of weather is a factor which in some instances may determine the type of construction to be employed in radio-frequency lines. It is generally appreciated that rain and sleet storms may materially lower the insulation of a line. The velocity of propagation and characteristic impedance also are affected by a coating of water or sleet upon the wires. Concentric-tube lines may be constructed so as to be weather proof.

This paper will be confined entirely to concentric-tube lines, to

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[1] See calculations in the appendix.

[2] E. Bruce, *Proc. I. R. E.*, p. 1406, August, 1931.

balanced two-wire lines, and to the apparatus associated with these two line types.

II. CONCENTRIC-TUBE LINES

A shielded line comprising an inner tubular conductor and an outer concentric shield is the form most commonly employed in radio practice. Owing to the circular symmetry of the line the case is capable of rather exact mathematical analysis. At radio frequencies the results are surprisingly simple. This simplicity is very evident for the two important parameters of a transmission line, the propagation constant P and the characteristic impedance Z_0.

The propagation constant of a line is defined by:

$$P = \sqrt{R + j\omega L} \cdot \sqrt{G + j\omega C}, \tag{1}$$

in which

$(R + j\omega L)$ is the complex impedance and

$(G + j\omega C)$ is the complex admittance, both per unit length. It is well known that the propagation constant is a complex number and that at radio frequencies (1) reduces to:[3]

$$P = \alpha + j\beta = \frac{R}{2Z_0} + \frac{GZ_0}{2} + \frac{j2\pi}{\lambda}, \tag{1a}$$

in which R is the resistance and G is the leakage conductance, both per unit length and at the wave-length λ.

The characteristic impedance Z_0 is defined by the ratio:

$$Z_0 = \frac{\sqrt{R + j\omega L}}{\sqrt{G + j\omega C}}. \tag{2}$$

The characteristic impedance also is a complex quantity, but at radio frequencies it is for most practical purposes the real quantity:

$$Z_0 = \sqrt{\frac{L}{C}}. \tag{2a}$$

In the case of concentric-tube lines the expression for the capacity C per unit length is the familiar relation:

$$C = \frac{1}{2 \log_e \frac{b}{a}} \text{ e.s.u.}, \tag{2b}$$

in which a is the outer radius of the inner conductor and b is the inner radius of the outer conductor. The inductance L per unit length may

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[3] J. A. Fleming, "The Propagation of Electric Currents."

be obtained from an expression derived by Lord Rayleigh upon assuming that the two tubes comprising the line are of negligible thickness. This is permissible because at radio frequencies the conduction of currents is essentially a skin effect. Upon this basis the inductance per unit length of a concentric-tube line becomes:

$$L = 2 \log_e \frac{b}{a} \text{ e.m.u.} \tag{2c}$$

Upon substituting (2b) and (2c) into (2a) with proper regard of units a simple expression for the characteristic impedance at radio frequencies is obtained:

$$Z_0 = 138 \log_{10} \frac{b}{a} \text{ ohms.} \tag{2d}$$

The high-frequency resistance of concentric-tube lines has been treated by a number of investigators, notably by A. Russell.[4] The asymptotic formula for resistance as the frequency is increased without limit is:

$$R = \sqrt{\rho \mu f} \left(\frac{1}{a} + \frac{1}{b} \right) 10^{-9} \text{ ohms/cm,} \tag{3}$$

in which:

ρ is the resistivity in e.m.u. (for pure copper ρ is about 1730 e.m.u.),

μ is the magnetic permeability,

f is the frequency, c.p.s.,

a is the outer radius of the inner conductor,

and

b is the inner radius of the outer conductor, the two latter being in centimeters.

It is of interest to note that the wall thickness of the conductor is not involved. At radio frequencies the current is confined to a very thin layer on the outside of the inner conductor and on the inside of the outer conductor.[5] The skin effect is, of course, not so pronounced at low frequencies and more complicated formulas involving wall thickness must be employed.

Some typical experimental data are submitted to show that for frequencies higher than one megacycle and for several practical line constructions the foregoing equation (3) holds with a very useful degree of accuracy. The physical dimensions and construction details of

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[4] A. Russell, *Phil. Mag.*, April 1909; and "Alternating Currents," Vol. I, p. 222, 1914, Cambridge Press.

[5] Frequency is not the sole criterion, resistivity, wall thickness, and diameter also being involved.

the lines for which the observations were made appear in Fig. 1. With the exception of one rubber insulated line all inner conductors were supported on porcelain insulators. The latter were attached to the inner conductor by means of spring clips, extruded metal ears or by means of soldered rings. Some measurements were made on lines assembled with soldered joints and some on lines connected by means of pipe unions with miniature plug and jack connections for the inner pipe. Various line lengths were employed. Most of the observations comprised measurements of the quantity $(R/2 + GZ_0^2/2)$. The measurement procedure will be described later.

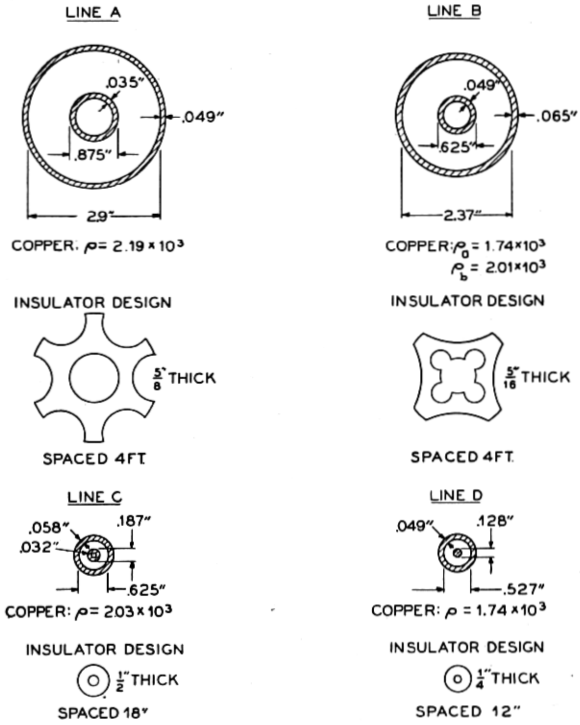

Fig. 1—Principal dimensions of the concentric tube lines upon which the experimental resistance measurements shown in Fig. 2 were made.

Line *E*—Same dimensions as "C." Brass ($\rho = 6.6 \times 10^3$) outer pipe.
Line *F*—Same dimensions as "C." All brass ($\rho = 6.6 \times 10^3$), insulators spaced 22 inches.
Line *G*—Same as "C" but filled with insulators.
Line *H*—Lead sheath cable, No. 18 B & S copper ($\rho = 1.7 \times 10^3$), rubber insulation, lead ($\rho = 17 \times 10^3$) $\frac{1}{8}$-inch inside diameter.
Line *I*—Same as "C" but insulators spaced 9 inches.
Line *J*—Same as "F" but insulators spaced 18 inches.

The results of these measurements appear on Fig. 2. The solid curves were computed by means of (3), neglecting the conductance term. The points are the experimental observations. Note that, excepting lines G and H, the small margin between measured and calculated values shows that the leakage losses are very small. It is believed that the scattering of the observed points could have been reduced appreciably if corrections for variations of resistivity with temperature had been made.

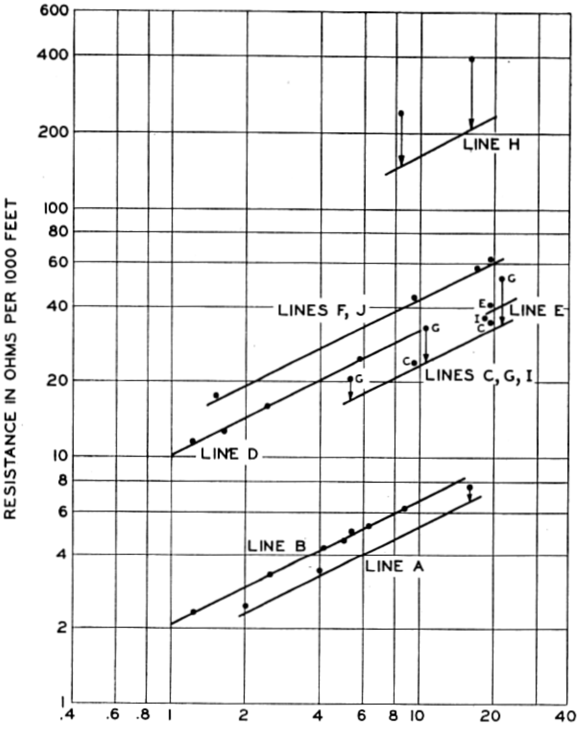

Fig. 2—Radio-frequency resistance measurements upon concentric tube lines. The curves are based upon computations and the points are experimental observations. See Fig. 1 for line details.

In lines B, E, and H the sheath and the inner conductor comprised materials of different resistivities. Calculations for these cases were made with the assistance of a modified form of (3):

$$R = \frac{1}{a}\sqrt{\rho_a f} + \frac{1}{b}\sqrt{\rho_b f}, \qquad (3a)$$

in which the subscripts denote the inner and the outer conductors.

Line *G* was completely filled with porcelain insulators and line *H* was a rubber insulated, lead sheathed, cable. The curves of Fig. 3 were derived from the difference between the observed and calculated resistance in these two cases. The results so obtained are a fair approximation of the leakage losses. Note that the curves are nearly proportional to the frequency which is to be expected if for a constant voltage the dielectric absorbs a fixed amount of energy each cycle.

As may be expected the velocity of propagation for both lines *G*

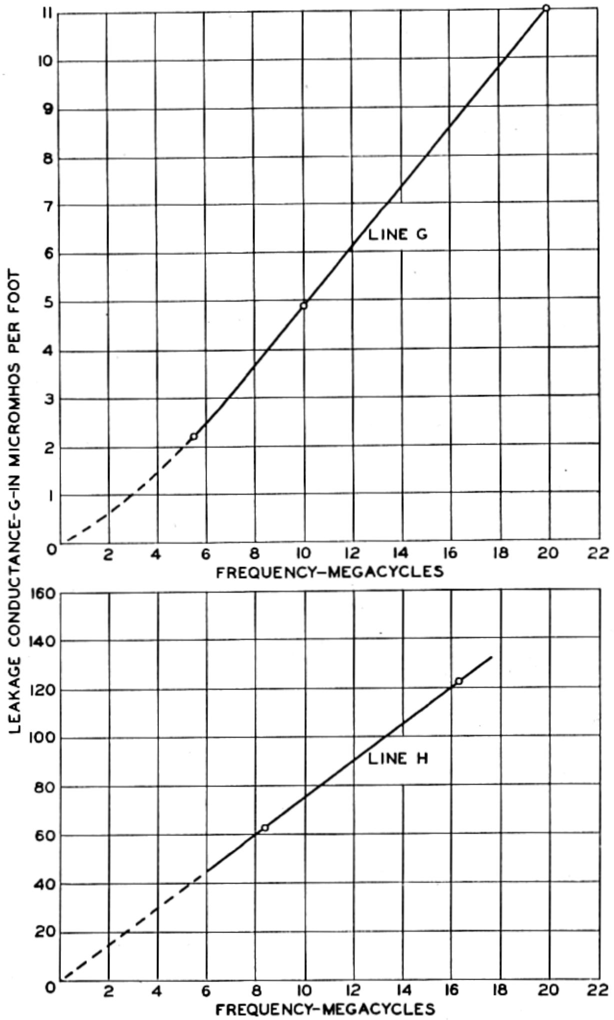

Fig. 3—Derived leakage conductance for the concentric tube lines *G* and *H* of Fig. 1. Line *G* was completely filled with porcelain insulators. Line *H* comprised a No. 18 B & S conductor with rubber insulation and lead sheath.

and H was reduced by a factor of approximately 1.8 which corresponds to a dielectric constant of about 3.2. Line I which was made with insulators spaced at 9-inch intervals was the only other line which showed a pronounced reduction in the velocity of propagation, the factor in this case being 1.18.

It is of interest to observe that if for economic reasons the diameter of the outer conductor is fixed there is an optimum inner conductor size for minimum attenuation. Employing (1a), (2d), and (3) the real part of the propagation constant may be written as:

$$\alpha = \frac{\sqrt{\rho\mu f}}{276} \frac{\left(\frac{1}{a} + \frac{1}{b}\right)}{\log_{10} \frac{b}{a}} \times 10^{-9}. \tag{4}$$

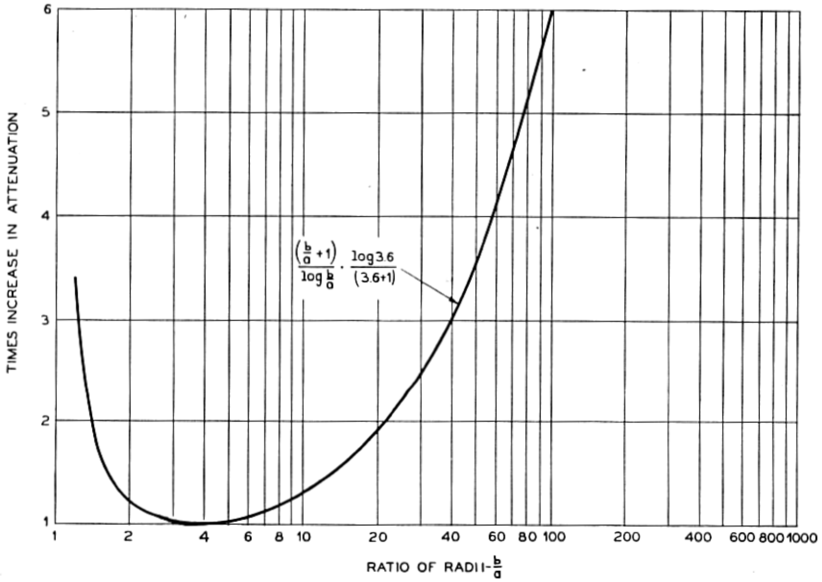

Fig. 4.—The most desirable ratio (36) of outer to inner conductor and the penalty incurred in departing from this value.

This neglects leakage loss and assumes that both conductors are made of the same material. Upon minimizing with respect to a, the optimum ratio:[6]

$$\frac{b}{a} = 3.6 \tag{4a}$$

is readily obtained. This ratio corresponds to a characteristic impedance of 77 ohms. Fig. 4 gives the manner in which the attenuation

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[6] An experimental figure for the optimum ratio was given by C. S. Franklin in a British Patent, No. 284005. The above derivation for the optimum ratio was disclosed to the writers by E. I. Green and F. A. Leibe, American Telephone and Telegraph Company, New York City.

varies as a function of b/a. Note that a moderate departure from the optimum ratio does not greatly increase the line losses.

So far it has been tacitly assumed that the conductors were exactly concentric. Eccentricity affects all of the line constants.[7] However, experience has shown that the departures from concentricity usually encountered in practice produce no appreciable increase in the attenuation constant of the line.

At commercial installations the actual power loss in the terminated line is measured directly in decibels and has invariably been found to agree with the predictions within the precision of such measurements which is about 0.5 db, in field work. Certain of the lines referred to in this paper have been similarly tested in the laboratory under more favorable conditions and yielded agreements within 0.3 db where the total loss was of the order of 4 to 6 db.

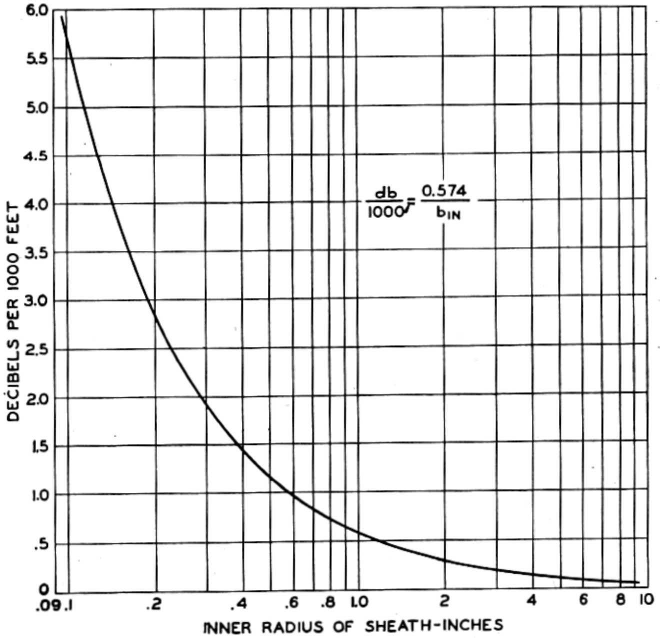

Fig. 5—Calculated losses at 20 megacycles expressed in decibels for concentric-tube lines constructed from copper and employing the optimum ratio (3.6) of outer to inner conductor.

Briefly summarizing it may be said that the attenuation in well constructed concentric lines is proportional to the square root of frequency, inversely proportional to the diameters (optimum ratio) and

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[7] A. Russell, "Alternating Currents," Vol. 1, p. 166, Cambridge Press.

proportional to the square root of resistivity. Numerically, the loss for copper lines of optimum ratio, neglecting leakage, is:

$$\frac{db}{1000\text{ ft.}} = \frac{0.128\sqrt{f_{\text{mc.}}}}{b_{\text{in.}}}\,. \tag{5}$$

A plot of (5) for one particular frequency ($f = 20$ mc.) is shown in Fig. 5.

It is important to emphasize one precaution in the use of concentric lines. The high degree of isolation afforded by concentric-tube lines may be easily destroyed. Owing to pick-up from near-by antennas or from other sources, currents of appreciable magnitude may be flowing upon the exterior of the sheath. Spurious couplings between the antenna and the line or between the equipment and the line may introduce these currents into the shielded circuit. In this manner the discrimination of a receiving circuit against undesired signals may be destroyed. Also, the currents flowing upon the exterior of the sheath may destroy the directional characteristic of the antenna to which the line is connected. Grounds placed at frequent intervals are useful in reducing these currents. Sometimes it is both desirable and convenient to bury the line in the earth. Additional improvement is obtained by constructing the circuits which transform the antenna impedance to the line impedance so as to obtain rigorous symmetry to ground.

III. Open-Wire Lines

The losses in open-wire lines may not be determined in as simple a manner or with the degree of certainty that is possible with concentric-tube lines owing to the complex nature of the electromagnetic field about open-wire lines. The high-frequency resistance of one conductor may be obtained from the foregoing equation (3) by assuming that the radius of the outer pipe is infinite. The characteristic impedance of balanced open-wire lines is obtained with sufficient accuracy from:

$$Z_0 = 276 \log_{10}\frac{2D}{d}\text{ ohms}, \tag{2e}$$

in which D is the axial spacing and d is the wire diameter. Some typical results for the resistance of a single conductor appear in Fig. 6.

At first thought it would appear that, owing to the high resistance of a single conductor, the losses in open-wire lines are higher than in concentric-tube lines. In practical constructions, however, open-wire characteristic impedances 5 to 10 times greater than those for concentric-tube lines are easily obtained. For example, the loss of a 77-ohm concentric-tube line is 6.38 times as great as that for a 770-ohm open-wire line in which the wire diameter is equal to that of the inner conductor of the concentric-tube line. Thus, the attenuation constant for a practical open-wire line may be approximately the same as that for the larger practical sizes of concentric-tube lines. Some typical computations appear in Fig. 7. A balanced two-wire line of 600-ohm characteristic impedance was chosen for the computations.

In Fig. 7 it was assumed that the proximity effect, that is, the redistribution of currents owing to the presence of the second conductor, is a correction of negligible magnitude. Only in the case of large conductors closely spaced does the proximity effect perceptibly increase the resistance. This may be seen from Fig. 8 which shows the increase in resistance due to the proximity of the conductors. There are several excellent published articles upon this subject.[8,9,10,11]

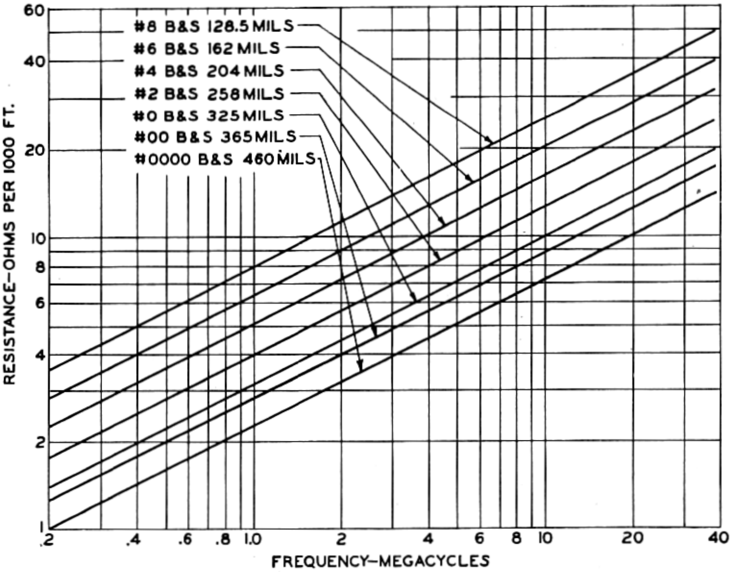

Fig. 6—Calculated radio-frequency resistance for several common sizes of solid copper conductors. Values are for one conductor only.

The foregoing results give, of course, only the power dissipated in copper losses and tell nothing about radiation losses. If the line spacing is less than 1/10 of a wave-length and if the line length is more

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[8] J. R. Carson, *Phil. Mag.*, Ser. 6, Vol. 41, p. 607, April, 1921.

[9] H. B. Dwight, *Jour. A. I. E. E.*, p. 203, March, 1922.

[10] H. B. Dwight, *Jour. A. I. E. E.*, p. 827, September, 1923.

[11] S. Pero Meade, *Bell Sys. Tech. Jour.*, Vol. 4, No. 2, April, 1925. The equations given in this reference were employed in computing Fig. 8.

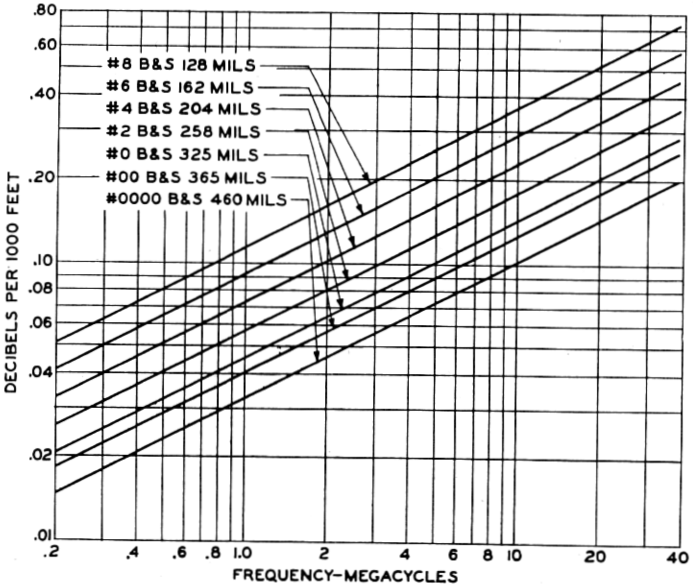

Fig. 7—Calculated attenuation expressed in decibels for copper losses in 600-ohm lines made up from common sizes of solid conductors.

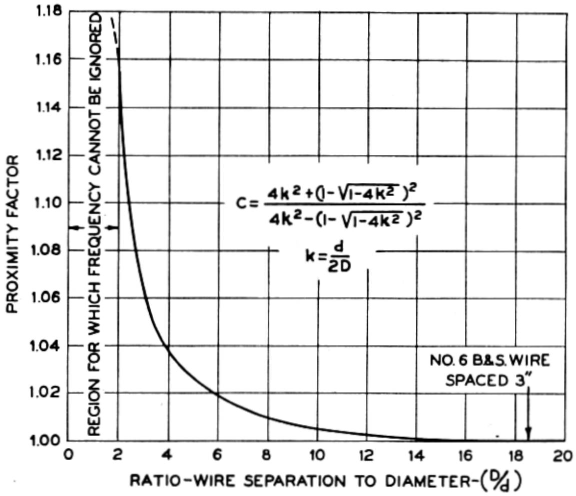

Fig. 8—Calculated values for the increase in copper losses due to the redistribution of current at close conductor spacings in balanced two-wire lines.

than 20 times the line spacing, the power radiated by a two-wire line terminated in its characteristic impedance is approximately:

$$\frac{P}{I^2} = 160\left[\frac{\pi D}{\lambda}\right]^2 \text{ watts/(amperes)}^2, \tag{6}$$

in which:

D/λ is the line spacing and
I is the r.m.s. value of the current in the line.

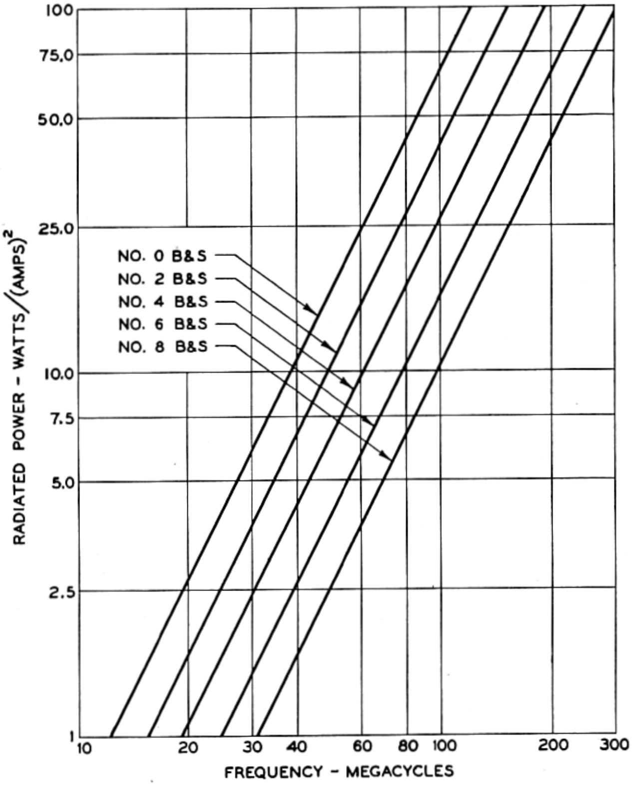

Fig. 9—Calculated power radiated by 600–ohm lines terminated in the characteristic impedance for several common conductor sizes. Note that line length is not involved.

Under these conditions the radiated power is independent of line length. More accurate equations appear in an appendix to this paper. It may be concluded from (6) that the power radiated by a terminated line is, in magnitude, approximately twice that radiated by a doublet antenna of length equal to the line spacing. Thus, the most simple circuit with which it is possible to terminate an open-wire line, a resistance of length equal to the line spacing, will radiate approximately one half as much power as the line. Therefore, considering both load and generator terminations, the total power dissipated in radiation may be approximately twice that given in (6). The equation is plotted on Fig. 9 for the cases of several 600-ohm lines constructed from practical conductor sizes. It may be seen from this figure that the power radiated by a practical terminated line is negligible as compared to the power transmitted by the line provided that operations are confined to wave-lengths other than those in the ultra-short-wave region.

If the currents in the two wires are unequal or are not exactly 180 degrees out of phase there is an appreciable amount of power radiated by a two-wire line. Unbalances of this kind become evident when the driving voltages, measured to neutral, are incorrectly balanced and phased. Such unbalances also arise if the voltages induced by the antenna set up currents in the line which employ the two conductors in parallel.

For the purpose of computation unbalanced currents may be considered as flowing in a single conductor parallel to a perfectly reflecting earth. The amplitude of the current in the single wire may be assumed to be the vector sum of the current values in the two conductors. This procedure ignores the mutual interactions of the balanced and unbalanced currents flowing in the two-wire line and hence, the results so obtained are not strictly correct. It is believed, however, that the error is small.

Based upon these assumptions the power radiated by unbalanced currents is approximately:[12]

$$\frac{P}{I^2} = 30\Bigg[0.5772 + \log_\epsilon (2L) - \sin^2 (L)\left(1 - \frac{\sin H}{H}\right) - Ci(2L) - 2Ci(H) + Ci(\sqrt{L^2+H^2} - L) + Ci(\sqrt{L^2+H^2} + L)\Bigg], \tag{7}$$

in which:

P/I^2 is expressed in watts/(amps)^2

I = r.m.s. value of current at a position along the line of maximum current,

$H = \frac{4\pi h}{\lambda}$, $\frac{h}{\lambda}$ being the height of the wires above ground in wave-lengths,

and

$L = \frac{2\pi l}{\lambda}$, $\frac{l}{\lambda}$ being the length of the line in wave-lengths.

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[12] See appendix.

The equation is plotted on Fig. 10 for two specific heights above ground and for various line lengths. Upon examining Figs. 9 and 10 it may be concluded that in practical constructions a thirty per cent unbalance in line currents radiates an amount of power roughly equal to that radiated by the balanced currents in the line.

It is our experience that losses due to current unbalances are appreciably greater and somewhat different in character from those indicated by (7). The discrepancy may reside in the assumptions employed in deriving the equation. In particular, the losses in the earth have been ignored. It may well be that the soil over which the line is erected introduces large losses in the line, particularly when the currents are unbalanced. Such losses would augment the attenuation constant of the line.[13] At least, the computations indicate the desirability of maintaining careful line current balances.

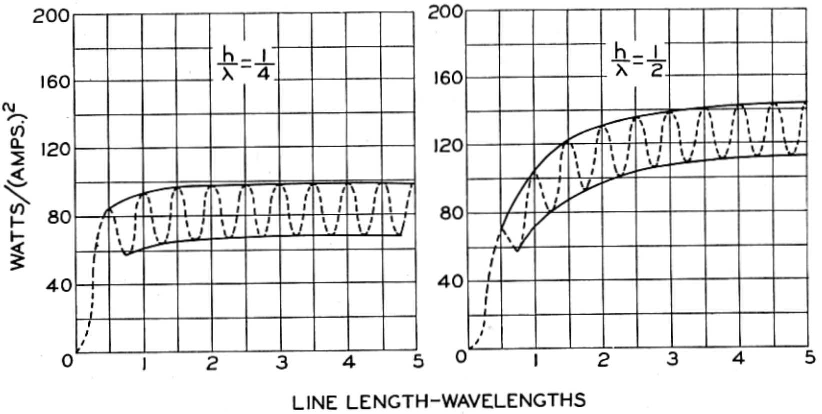

Fig. 10—Approximate power in watts radiated by an unbalanced current of 1.0 r.m.s. amperes in a long two-wire line. Also, the power radiated by a single wire parallel to the earth for 1.0 r.m.s. ampere line current. Two cases, $\frac{1}{2}$ and $\frac{1}{4}$ wavelengths above ground are illustrated.

Some remarks upon the proper procedure for inserting the power losses due to radiation into the equations for the line may be of interest. Carson [14] has shown that the conventional solution of the transmission equation for guided waves on wires is incomplete and does not explain the phenomena of radiation. He shows that a "principal wave," and hence the currents in the conductor associated with this wave, travel along the conductors without sensible attenuation due to radiation. Radiation from the line results in the attenuation of an infinite number of "complementary waves." These are highly attenuated so that the

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[13] John R. Carson, *Bell Sys. Tech. Jour.*, Vol. V, No. 4, October, 1926.

[14] John R. Carson, *Jour. A. I. E. E.*, p. 908, October, 1924.

radiation of energy is a phenomenon essentially associated with the terminals of the line or points of discontinuity which set up reflected waves.

It may be concluded from Carson's mathematical investigation that the radiation resistance is a term to be added to the impedance of the line at the terminals or points of discontinuity and that it does not appear in the propagation constant. On this basis, the power radiated by a practical balanced transmission line is negligibly small when compared to the power being transmitted by the line except perhaps for operation at the very short wave-lengths.

Experimental data for the attenuation in open-wire lines which are as complete as those already shown for concentric-tube lines are not available for this paper. Some typical observations for 600-ohm lines constructed with No. 6 B & S semi-hard drawn copper wire appear in Fig. 11. The points are experimental observations. The lower curve was computed for the case 1830 e.m.u. copper resistivity. The observed values are about 66 per cent higher than the computed values.

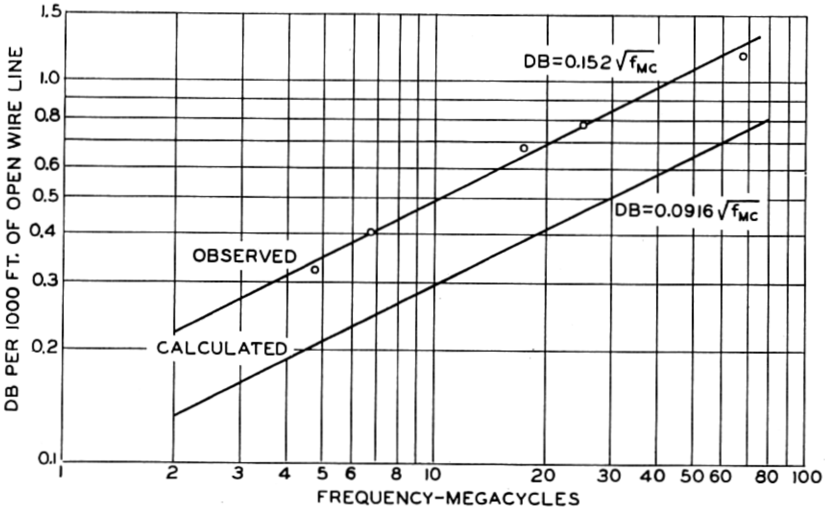

Fig. 11—Experimental observations of attenuation in a 600-ohm line comprising 0.162-inch copper conductors. The points are observed values. The lower curve is calculated only on the basis of copper losses.

The experimental procedure was as follows. A line 2000 feet long was carefully balanced and terminated by an iron wire line [15] for each of the experimental observations. By means of a portable calibrated indicating device the average currents for the one-half wave-length of

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[15] See Section VII.

line at the near end and at the far end were obtained. The attenuation in decibels was computed from average near end and far end current ratios.

It is difficult to explain the discrepancy between observed and computed values. If the resistivity employed in the computations were to be increased from 1830 to 5030 e.m.u. (a multiplication factor of 2.75) the computed curve so obtained would be in good agreement with the observed results. It is true that the wires were somewhat weathered. There is, however, little reason to believe that an appreciable amount of current flows in the oxide layer covering the wires. Effects of this kind would have been evident in the measurements upon concentric-tube lines. It already has been mentioned that small current unbalances in the line may produce losses in the earth which increase the real part of the propagation constant. Possibly, losses of this kind may explain the discrepancy.

IV. Notes on Matching Impedances

It already has been mentioned that standing waves on a transmission line augment line losses. The penalty which is imposed by improper impedance matches may be seen from Fig. 12. This figure plots line loss as a function of the degree of matching for several attenuation factors. The line loss is computed from the ratio of the power dissipated in the load to the total power obtainable from the generator. The curves were obtained from conventional transmission line theory. For the purpose of simplifying calculations the line length is assumed to be an integral number of one-quarter wave-lengths, thereby eliminating complex impedances. Otherwise the length of the line is immaterial, the product of length and attenuation per unit length being the criterion of loss.

In the diagrams of Fig. 12A and Fig. 12B the circuit M is an adjustable ideal transformer. For every value of the resistance R the transformer M is assumed to be adjusted so as to maximize the load power. This process is equivalent to matching impedances at the terminals of the line adjacent to the transformers. It is of interest to observe that where R is not equal to the characteristic impedance Z_0, this adjustment does not yield an impedance match at the line terminals remote from the transformers. It is of further interest to observe that in the case where the load impedance is variable (Fig. 12B) the optimum adjustment is a compromise between a non-reflecting termination and an impedance match at the generator end.

Conventional tuned transformers may be employed to match the line impedance to the antenna and radio equipment impedances. In the transmitting case, tuned circuits are often found to be both bulky and costly. There are a number of schemes which employ a short section of line as a transformer element. These are feasible only at frequencies for which the wave-length is short. The circuits so provided are extremely simple and cheap.

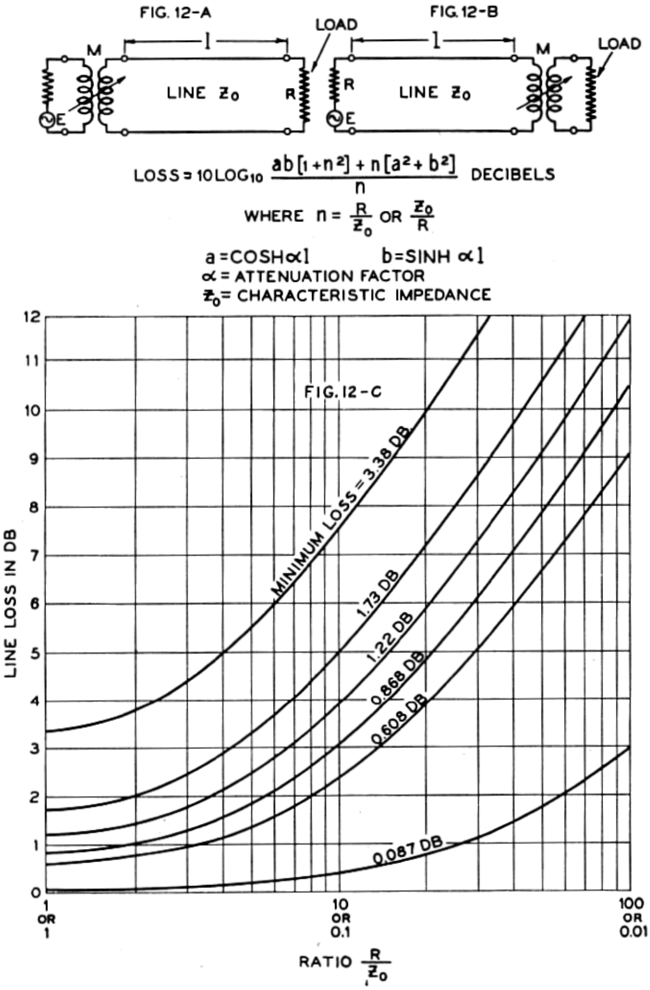

Fig. 12—Line loss as a function of the degree of matching. The several curves are designated in terms of the minimum line loss obtained for the case of a perfect match with the line characteristic impedance.

In another paper[16] a scheme for employing a one-quarter wave-

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[16] E. J. Sterba, *Proc. I. R. E.*, p. 1184, July, 1931.

length section of line as a step-up or step-down transformer was described. Briefly the principle of operation is the fact that the sending end impedance Z_s and the receiving end impedance Z_r are related to the characteristic impedance Z_0 by the simple expression:

$$Z_sZ_r = Z_0^2. \tag{8}$$

Thus, by choosing the proper characteristic impedance any two real impedances may be matched provided these do not differ too greatly.

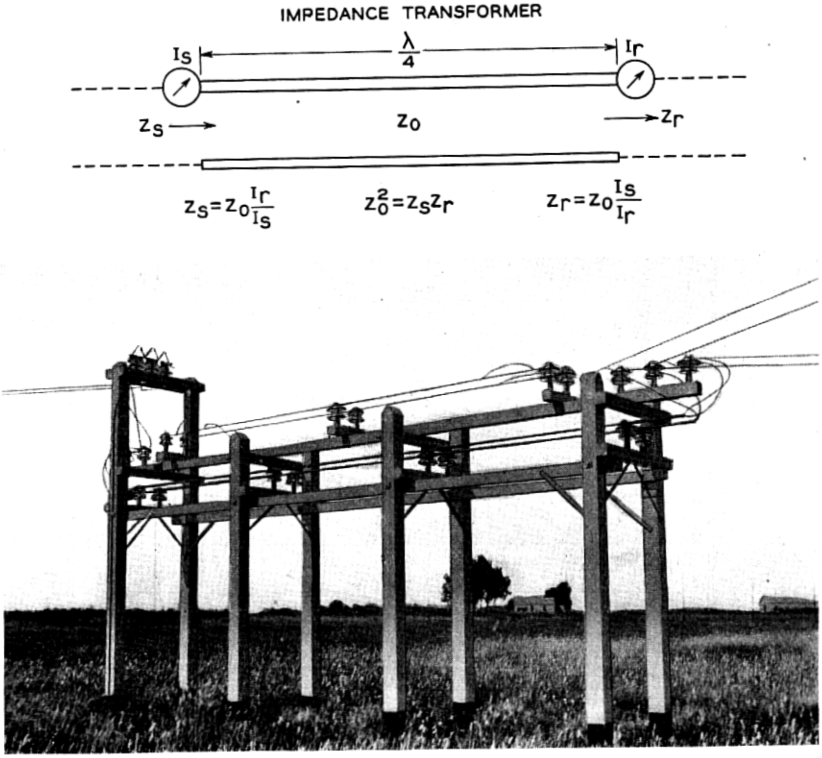

Fig. 13—The principles underlying the use of one quarter wave-length bars as a transformer are shown in the upper diagram. The lower illustration depicts a commercial installation.

Often the scheme is made workable by constricting the line spacing for a one-quarter wave-length section. Where a large difference in transformer line and transmission line spacing is undesirable the transformer line may comprise conductors of large diameter. A transformer set-up of this type is shown in Fig. 13.

There is another effective way for transforming line impedances by means of short line devices.[17] A complex impedance at the proper position along a partially terminated line is selected such that a shunt reactance at this position transforms the real part of the impedance to the surge impedance of the line at essentially unity power factor. The shunt could, of course, be a lumped reactance. It is found convenient to employ a short section of line for this reactance. A position along the line for the shunt reactance of either leading or lagging power factor may be chosen. In the former case the shunt reactance must be inductive and in the latter case capacitive. Computed shunt impedance positions for the two cases and the values of the shunt impedance in terms of line length for 600-ohm lines and various standing wave amplitudes on the unterminated section appear in Fig. 14. Actual settings correspond very well with the calculated settings.

V. RESISTANCE AND ATTENUATION MEASUREMENTS ON TRANSMISSION LINES

The following is a description of some of the measurement methods which have been found useful in the study of transmission lines. The schemes may not be applicable to every phase of the transmission line roblem. However, it is hoped that they may suggest precautions top be observed in performing transmission line studies.

One scheme, very commonly employed, is to measure the attenuation along a transmission line by actual current measurements. This method is particularly suited to measurements upon a long line terminated in its characteristic impedance. It has been found desirable to measure the current amplitudes at close intervals for at least a one-half wave-length section at the near end and the far end of the line. In this manner an average result which reduces observational errors and errors arising from standing waves of small amplitudes is obtained. From the ratio of the average sending end current I_s and the average receiving end current I_r and the average distance l between the two sections of line the attenuation per unit length is obtained from the definition:

$$\text{db} = 20 \log_{10} \frac{I_s}{I_r}, \tag{9}$$

and since

$$\frac{I_s}{I_r} = \epsilon^{Rl/2z_0}, \tag{10}$$

$$\frac{\text{db}}{l} = 4.343 \frac{R}{Z_0}, \tag{11}$$

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[17] Disclosed to the writers by P. H. Smith, Bell Telephone Laboratories, Inc., New York, N. Y.

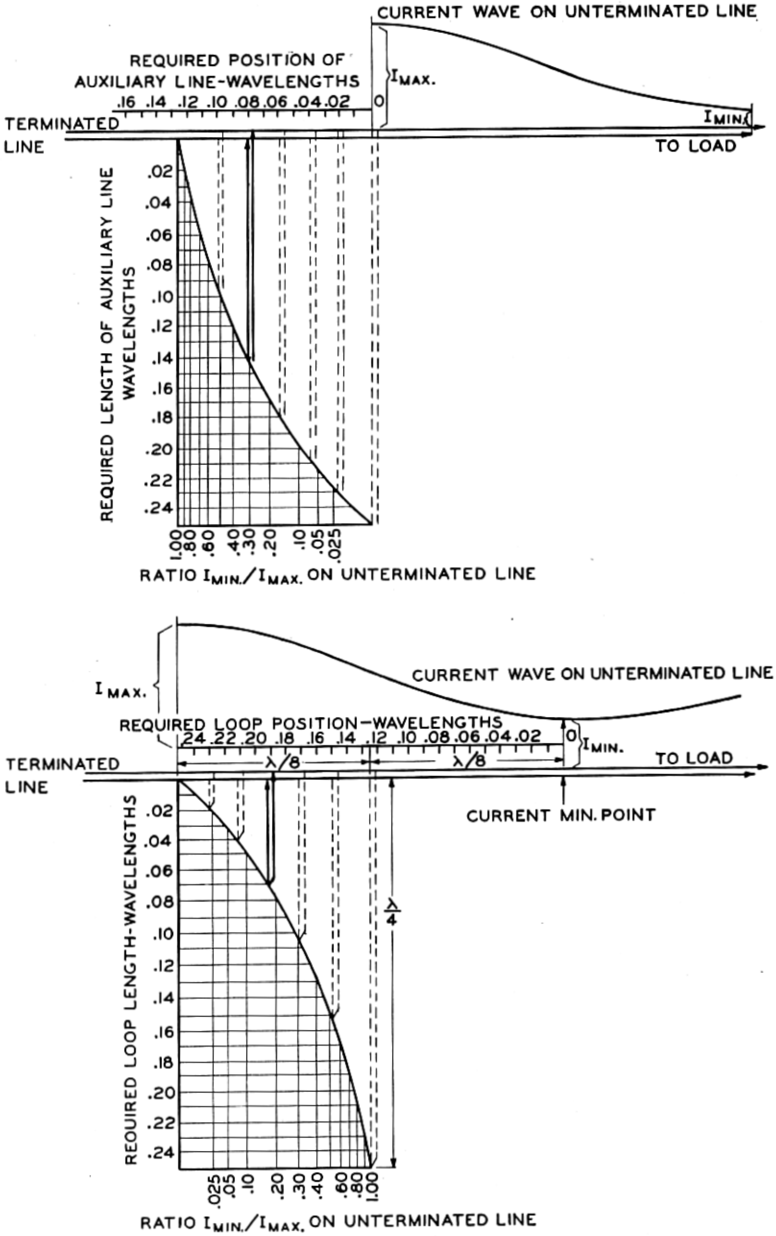

Fig. 14—The use and adjustment of an auxiliary line as a transformer element. The settings are computed for the case of 600-ohm lines. The position and length of the auxiliary line may be obtained from the curves for any given ratio of minimum to maximum currents on the unterminated portion of the line.

from which the resistance R per unit length may be obtained with a degree of accuracy depending chiefly on how accurately the characteristic impedance Z_0 is known.

The current distribution along the line is most conveniently obtained by means of a portable indicating device. Three designs which have been found useful are shown in the following figures. The indicator to the left of Fig. 15 is used for measurements upon open-wire lines. The manner in which it operates is evident from the figure. Note that except for closely spaced wires the device is sensitive only to the current in one side of the line. When this device is used the currents in both sides should be measured to assure that there are no large current unbalances and for the purpose of averaging out any small unbalances. The device in the center of Fig. 15 is coupled to both sides of the line and is not sensitive to currents which employ the two-line conductors

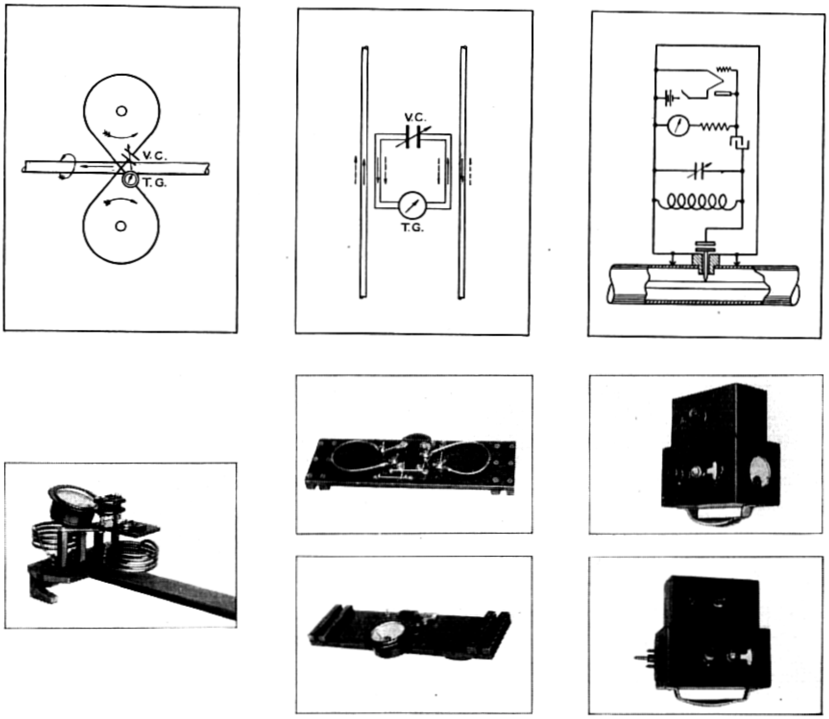

Fig. 15—Three types of portable indicating devices for observing the current distribution on transmission lines. The device to the left is sensitive chiefly to the current flowing in one conductor. The device in the center responds chiefly to balanced currents and is not sensitive to unbalanced currents. The device to the right is suitable for observing the voltage amplitudes on concentric tube lines.

in parallel. It is useful where for other reasons the unbalanced currents cannot be reduced to a desirably low value. The device shown to the right of Fig. 15 is suitable for measurements on concentric lines. In order to employ this device openings at regular intervals are required in the outer sheath. An important precaution to be observed in employing this last device is that the shielding be sufficiently thorough to assure no pick-up from stray currents flowing upon the outside of the sheath.

It is of course essential that all portable devices of this kind extract a very small proportion of the power in the line; otherwise, the device becomes a source of reflection and spurious results are obtained.

Another method of measuring the attenuation of a line which is particularly useful in studying the effects of current unbalances is to employ a small portable horizontal antenna the impedance of which matches the characteristic impedance of the line. The antenna is connected in a short section and then in a long section of the line. It is essential that the height of the antenna above ground be equal for the two positions. Also, the location for the experiment should be such that the same ground losses are present for the two positions. The ratio of the antenna currents for the two positions and for the condition of equal power input is a measure of the total line losses.

One of the most satisfactory schemes for measuring line attenuation is the direct measurement of the line sending end impedance by means of the familiar resistance substitution method. It has been used extensively in measurements of concentric lines. For this purpose it is necessary to employ lines either open- or short-circuited at the far end and to restrict the measurements to lines which contain an integral number of quarter wave-lengths.

Conventional transmission line theory indicates that under these conditions the impedance is either:

$$Z_1 = Z_0 \tanh\left(\alpha \frac{n\lambda}{4}\right), \tag{12}$$

or:

$$Z_2 = Z_0 \coth\left(\alpha \frac{n\lambda}{4}\right), \tag{13}$$

where:

Z_0 = characteristic impedance,
α = attenuation factor; i.e., the real part of the propagation constant,
λ = wave-length, and
n = an integer denoting the number of quarter wave-lengths.

If n is even and the termination is a short circuit or if n is odd and the termination is an open circuit, (12) is employed. If n is odd and the termination is a short circuit or if n is even and the termination is an open circuit, (13) is employed. The attenuation factor is given by:

$$\alpha = \frac{1}{2}\frac{R}{Z_0} + \frac{1}{2}GZ_0 \text{ napiers per foot,} \tag{14}$$

$$= 4.34\left(\frac{R}{Z_0} + GZ_0\right) \text{ decibels per foot,} \tag{14a}$$

where:

R = resistance in ohms per foot,
G = conductance in ohms per foot, and
Z_0 = characteristic impedance in ohms.

For all the lines concerned with here tanh $[\alpha(n\lambda/4)]$ may be replaced by $[\alpha(n\lambda/4)]$ without more than 1.5 per cent error. Thus, (12) and (13) reduce to

$$Z_1 = \frac{R}{2}\frac{n\lambda}{4}\left(1 + \frac{GZ_0^2}{R}\right), \tag{12a}$$

$$^{18}Z_2 = \frac{2Z_0^2}{\frac{n\lambda}{4}R}\left(1 - \frac{GZ_0^2}{R}\right). \tag{13a}$$

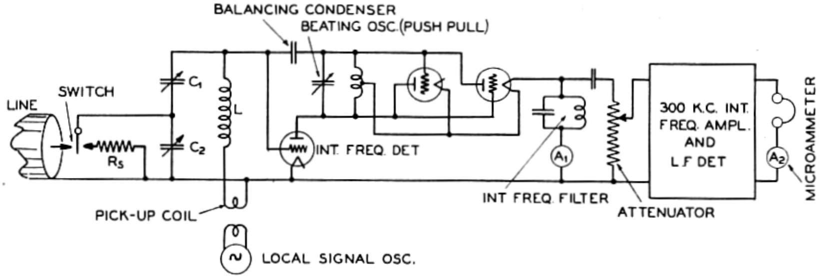

Fig. 16—Schematic diagram of apparatus for measuring line loss by means of a resistance substitution method.

Therefore, except for the small contribution of shunt conductance, Z_1 is independent of Z_0. In cases where G is negligible, the measurement of Z_1 gives directly the high-frequency resistance.

The method of measurement is shown schematically in Fig. 16 and an experimental set-up appears in Fig. 17. The modified high-fre-

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[18] It is assumed here that the conductance term in (14) is small compared with the resistance term.

quency field intensity measuring unit[19] is a convenient indicating device and source of signal. The intermediate-frequency amplifier with its adjustable gain is very useful in maintaining a desirable level in the last detector which is the indicator. Returning to Fig. 16, the local signal oscillator is the source of voltage actuating the tuned circuit LC_1C_2 in which the substitutions are made. Loose coupling is desirable between the pick-up coil and oscillator. In measuring Z_1, which is a low impedance, the condenser C_2 is set at minimum capacity thus effectively making the comparison in a series circuit. In measuring Z_2, which is of the order of hundreds of ohms, C_2 is used to transform Z_2 into an appropriate series value consistent with selectivity and sensitivity.

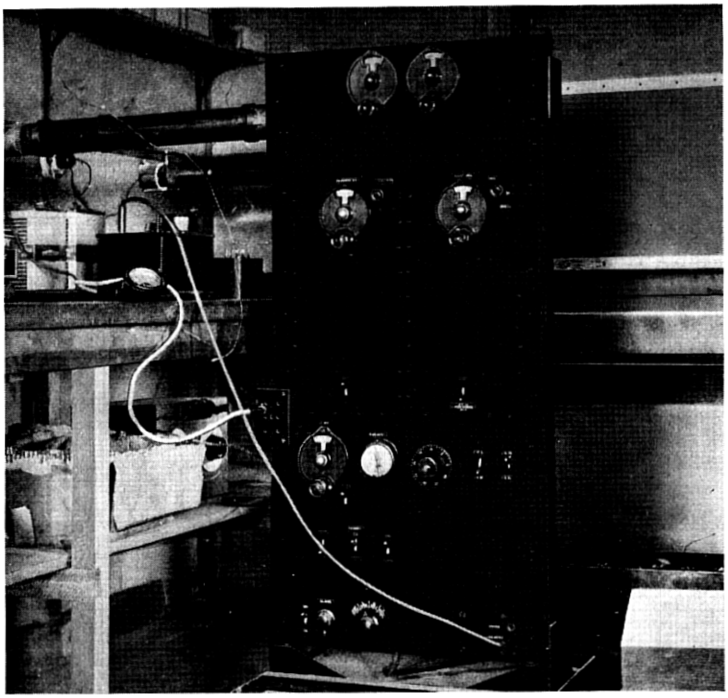

Fig. 17—Experimental set-up of apparatus for measuring line loss.

The comparison resistances used for measuring (Z_1) may be fixed units and the line impedance obtained by interpolation. These comprise straight lengths of high-resistance wire and range from a few tenths of an ohm to ten ohms. The wire size is chosen so that skin

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[19] Readers not familiar with this measuring unit may refer to a paper by Friis and Bruce, *Proc. I. R. E.*, August, 1926.

effect is negligible. Although these resistances possess appreciable inductive reactance at the higher frequencies the reactance usually may be safely tuned out.

It has been found practicable to employ continuously variable resistances made from hard drawing pencil lead equipped with spring clip contacts for measurements of high resistances such as (Z_2). The high resistivity of graphite makes it possible to obtain several thousand ohms in 5 or 6 inches, free of skin effect and with but little inductance.

The foregoing resistance substitution method has been found satisfactory for the purpose of measuring the characteristic impedance of lines. Two methods have been employed. One of these is the familiar procedure in which the sending end impedance is measured for the case of the line open and short circuited at the far end. The geometric mean of the two impedances so obtained is the characteristic impedance of the line.

Another scheme producing more precise results is also adapted to the foregoing resistance measuring method in that it requires that the line be some odd number of one-quarter wave-lengths long. Such a line transforms to a different value a terminating impedance which is other than the characteristic impedance. By comparing a variable terminating resistance directly with the value to which it is transformed by the line a setting may be found for which the line functions as a one-to-one transformer. For this condition the value of the variable terminating resistance is the characteristic impedance of the line. Here again drawing pencil leads have been found to be satisfactory termination resistances when set by direct-current measurement methods.

In practice the foregoing resistance substitution method brings to light many slight irregularities. Variations of apparent characteristic impedance with frequency as much as 5 to 10 per cent have been found for concentric-tube lines equipped with elbows, couplings, and similar fittings. It is believed that impedance variations of this order are to be expected from some such irregularities unless particular care is taken in the construction of the fittings. On the other hand, it has been found that a short straight length of carefully constructed concentric-tube line is so smooth that its characteristic impedance may be employed as a calculable standard.

VI. Practical Construction Details

Open-wire radio-frequency line construction is not very different from that employed in power practice. One outstanding difference is that line supports and insulators must be considered as individual irregularities spaced at intervals often greater than one wave-length. The effect of one such irregularity may be small. The total effect in a long line, however, is sometimes appreciable.

The body of the insulator, since it has a dielectric constant appreciably different from air and since its dimensions are comparable with the line spacing, is in itself a line irregularity. Tie wires or conductor clamps augment this effect. Cross arms and pins employed for mounting pin-type insulators also add to the effect, particularly during wet weather.

From the standpoint of line irregularities suspension-type insulators are more desirable than pin-type insulators. The latter construction, however, appears to be more practical because the lines are more rigid, sway less during wind storms, and because no intermediate spreaders are required to maintain the desired line spacing.

One other difficulty with open wire lines is the drift in velocity of propagation and surge impedance during rain and sleet storms.[16] Since a similar effect occurs in the elements of the antenna there is a decided drop in the efficiency of the combined antenna and line during rain and sleet storms. The effects of sleet may be reduced by heating the wires with sleet melting currents. The conductor size may be increased to reduce the effects of wet weather but this makes sleet melting more difficult.

There is an appreciable pick-up between balanced open wire lines on common supports. It appears desirable to separate lines to a common transmitter by at least 10 times the conductor spacing. Spacings greater than this may be required if two lines are to be operated simultaneously and in some cases it is more desirable to employ separate line supports in order to reduce the possibility of cross-talk difficulties. Of course any current unbalances in two parallel lines greatly increase the danger of cross-talk.

Concentric tube line construction is not as simple as open wire construction. Considering the transmitting case, there is a smaller safety factor for voltage overloads. Insulators are required to withstand high voltage gradients. Temperature changes with ensuing line expansions and contractions must be given consideration. It is these factors in addition to the added expenditure for copper which make concentric line construction more costly than open wire construction.

The first consideration in the design of a concentric line is the weight of the outer sheath. If the line is to be employed for high power transmitting purposes the voltage safety factor may be so low that accidental dents in the sheath may lead to breakdown. Obviously, there is a choice between a large diameter, lightweight sheath and more rugged small diameter sheath without an appreciable difference in copper expenditure. Other factors which involve the remainder of the radio plant often determine the size of the outer sheath. We have found that for outer sheaths a diameter of 2.5 inches and a radial thickness of 0.0875 to 0.10 inch provides lines which are sufficiently rugged for transmitting 15 kw of modulated power at 16 meters wave-length.

Careful consideration needs to be given to the problem of protecting concentric lines from voltage overloads which may be brought about by accidental open or short circuits or by flashovers. Voltages of the order of 30,000 to 90,000 volts may easily be built up in this manner at the shorter wave-lengths. Horn gaps are useful if located in the proper way. It is fortunate that conventional line input circuits are apt to be detuned in the event of an accidental open or short circuit on the line and that very little power may then be transmitted to the line.

Beads of high grade porcelain in diameters up to one inch are satisfactory insulators for low power and receiving lines. However, such simple insulators are not suitable for high power work. Owing to the volume of dielectric in large annular insulators sufficient heating may occur at the higher voltages to destroy the insulator. Insulators such as those described for line *B* Fig. 1 have been found suitable at the higher voltages.

The air film between the insulator and the inner conductor lies in a region of steep voltage gradient. Even under what is considered normal operating voltage there may be enough corona in this region to produce heating of the insulator. It may be of interest to mention that a line approximately as described in *B* Fig. 1 has been found satisfactory for normal operation at 16 meters for a carrier power of 15 kw. The line breaks down in the region of the insulator at 9000 r.m.s. volts.

For transmitting purposes it has been found desirable to employ glazed insulators in concentric tube lines because dirt, soldering fluxes, etc. acquired in assembly operations are more readily removed from glazed insulators.

There are a number of simple ways in which insulators may be held in place in concentric tube lines. For low power work and receiving purposes wire clips, rivets or even extruded metal ears upon the inner conductor, are satisfactory. As a rule these do not prove satisfactory at higher powers owing to high potential gradients at points and sharp edges. Small rings riveted or soldered upon either side of the insulator have proven satisfactory. Lines with soldered rings are more easily repaired. Care must be exercised, however, to prevent condensation of metal and fluxes in the pores of the insulator.

In open wire construction it is customary to accommodate line variations brought about by temperature changes by adjusting the sag of the conductors. Provisions for temperature variations in concentric tube lines are not so simple. The first obvious remedy is to employ lines buried at sufficient depth so that temperature changes are reduced to a slow seasonal variation. At the present time a buried 3/8-inch line has been in service for more than one year without developing faults. Without longer experience with concentric lines we would question the advisability of burying larger lines which are to be employed at high voltages due to the difficulty of finding faults should these occur.

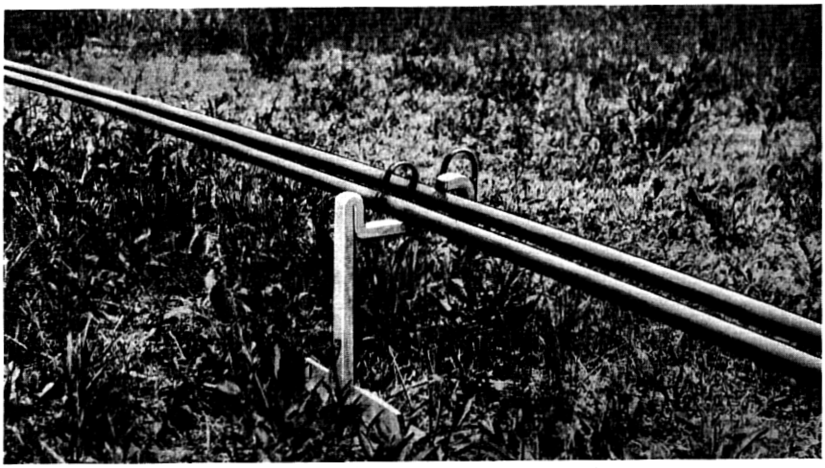

Fig. 18—A short section of three-quarter inch diameter line showing support for holding line in a sinuous form.

A very simple scheme, suitable for small lines, is to reduce the effects of temperature variations by laying the line in a sinuous path as shown in Fig. 18. This construction permits the line to buckle slightly at the curves as the length varies and cumulative changes in length do not appear at the line terminals. The inner conductor is held loosely within the sheath so that it may buckle independently of the sheath. The outer conductor changes its length both at a different rate and at a different time from the inner conductor. With increasing temperature the sheath is at a higher temperature than the inner conductor. There is an appreciable time lag in heating of the inner conductor due to the heat insulation of the air space between the conductors. Small lines laid in a sinuous manner have been found remarkably free from mechanical breakdowns brought about by temperature variations of length.

Sliding joints may be employed to accommodate variations in line length brought about by temperature changes. It is very difficult to make such joints water-tight without recourse to expensive fittings. There is also the possibility of microphonic contacts which are particularly objectionable in receiving work.

The expansion joints shown to the right of Fig. 19 have been employed with some success. Dimensions and shapes should be chosen to minimize the irregularities in line impedance caused by expansion joints. It is a step in the right direction to maintain constant the ratio of conductor diameters at the joint. Even then, it has been found that the irregularities caused by 10 such joints in a 600-foot line are observable (approximately 10 per cent standing waves).

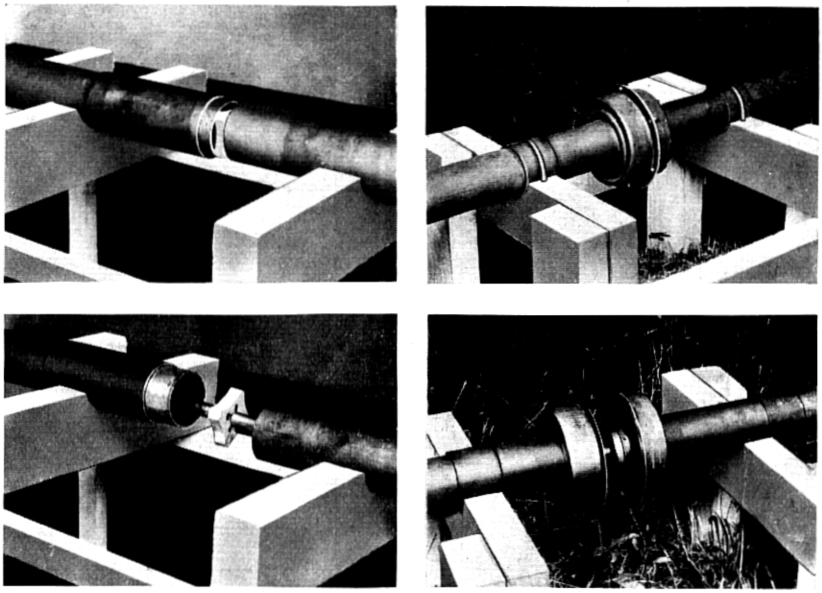

Fig. 19—Experimental expansion and lock joints for large sizes of concentric tube line.

It is necessary that expansion joints be employed in conjunction with lock joints so arranged that no one joint is required to take more than a predetermined portion of the line expansion. One lock joint with an expansion joint 25 feet in either direction has been found to be a satisfactory length within which line variations are corrected. The lock joint proper, Fig. 19, comprises an insulator of the same design as the intermediate insulators but made with an outer diameter equal to that of the outer sheath. It is held in place by a sleeve sweated to the sheath, the sleeve continuing the electric circuit. The insulator is also fixed to the inner conductor by means of rings. Since the lock joint is in a position symmetrical with respect to the two expansion joints it is required to withstand a shearing load brought about only by inequalities in the expansion of the conductors. In order to distribute the line expansion uniformly among the expansion joints it is necessary to clamp the outer sheath of the lock joint to a substantially braced support.

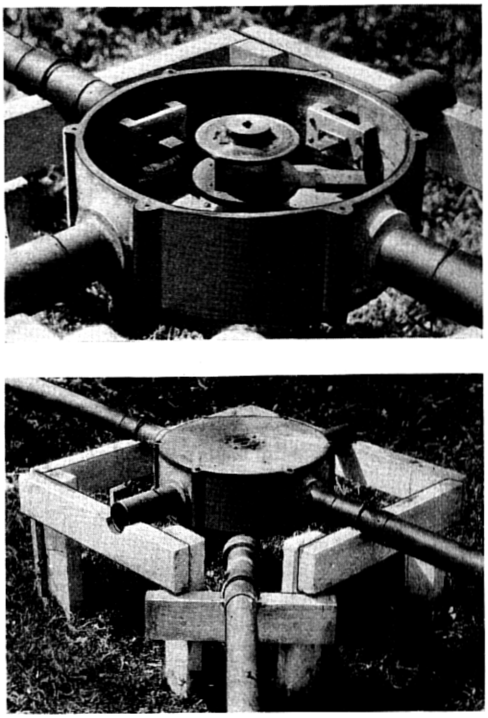

Fig. 20—An experimental selector switch for connecting several antenna lines to one transmitter line. The small coil antiresonates the capacity of the switch for the operating frequency associated with the particular contact to which it is connected.

The joints described above are required to accommodate a total annual variation of 0.5 inches. After one year's period of experimental operation the few faults found in a line containing these joints were nearly all traced to faulty construction at the braced support which clamps the lock joint.

Copper pipe lines may be too costly to permit the installation of more than one line per transmitter. In such cases a selector switch is required if several antennas are to be associated with one transmitter. Some of the details of such a switch may be obtained from the experimental arrangement shown in Fig.20.

The switch is an irregularity on the line and a source of undesired reflections. This difficulty may be corrected by making the design such that capacitive reactance of the switch predominates and then antiresonating this reactance with a suitable inductance. This scheme is effective provided the irregularity is not too great. In the latter event the corrective coil transforms the load impedance to a value different from the surge impedance so that the reflections arising from the mismatch are more serious than from the switch alone.

VII. OTHER APPLICATIONS OF TRANSMISSION LINES

In this section are described a number of transmission line applications to radio work some of which are feasible only at high frequencies because the wave-length is short.

Small concentric lines approximately 3/8-inch in diameter may be employed as radio-frequency wiring in radio stations. Such lines owing to the flexibility of the tubing may be snaked behind partitions in very much the same manner that armored or leaded conductors are installed. For this purpose refrigerator tubing has been found desirable because it is flexible and because it may be procured in long lengths. The inner conductor is insulated from the sheath by means of small porcelain beads spaced at intervals of approximately one inch. The beads are held in position by small metal ears extruded from the inner conductor. The beads fit loosely in the inner conductor so that the line may be bent into arcs as small as six inches radius. Construction details for small concentric lines may be obtained from Fig. 21.

Lines constructed from refrigerator tubing may be buried in the ground. Since only a few splices are necessary the possibility of faults arising from water seeping into the line are correspondingly small. A buried line constructed in this manner has been in service for more than a year without developing faults.

A number of the above-described lines may be terminated upon a jack board and circuits set up with patch cords as in telephone practice. Of course, the beads in the patch cords may be more closely spaced to assure flexibility and freedom from short circuits. The

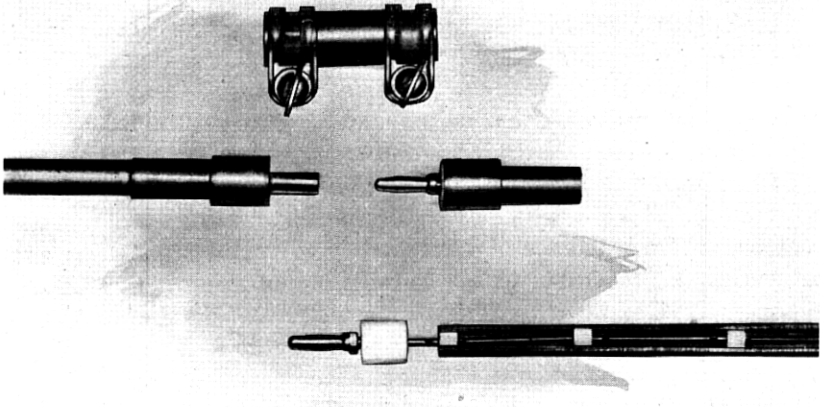

Fig. 21—Details of construction for small concentric lines suitable for station wiring and for patch cords. The plug-and-jack union is an effective scheme for temporarily connecting two small lines.

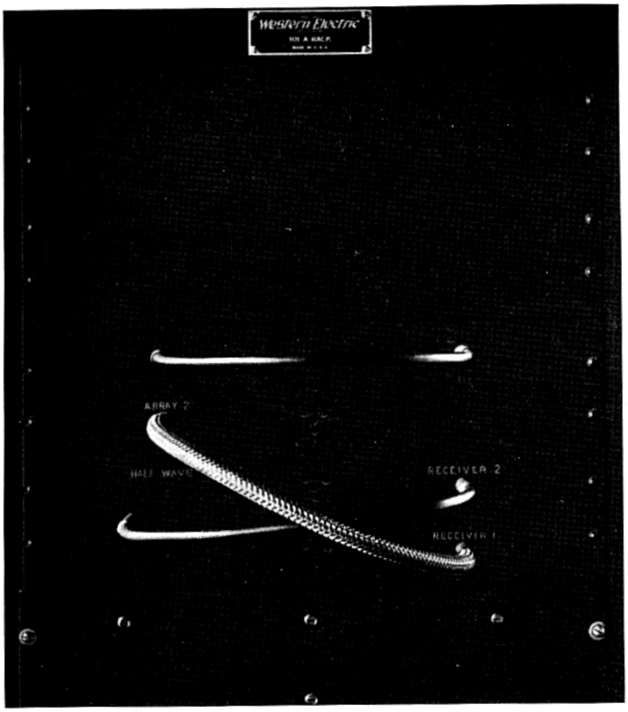

Fig. 22—An experimental radio-frequency jack board terminal for small concentric-tube lines. Path cords are constructed in the manner depicted in Fig. 21.

scheme is particularly advantageous where for operating reasons it is useful to connect any station antenna to a particular receiving unit. A board set up for this purpose is shown in Fig. 22.

Concentric-tube lines may be employed as standards of resistance when other standards become questionable. Since the agreement between theoretical and experimental values of radio-frequency resistance has been found very good at frequencies at high as 20 megacycles the theory may be considered adequate for much higher frequencies. One scheme for utilizing this situation so as to obtain an adjustable

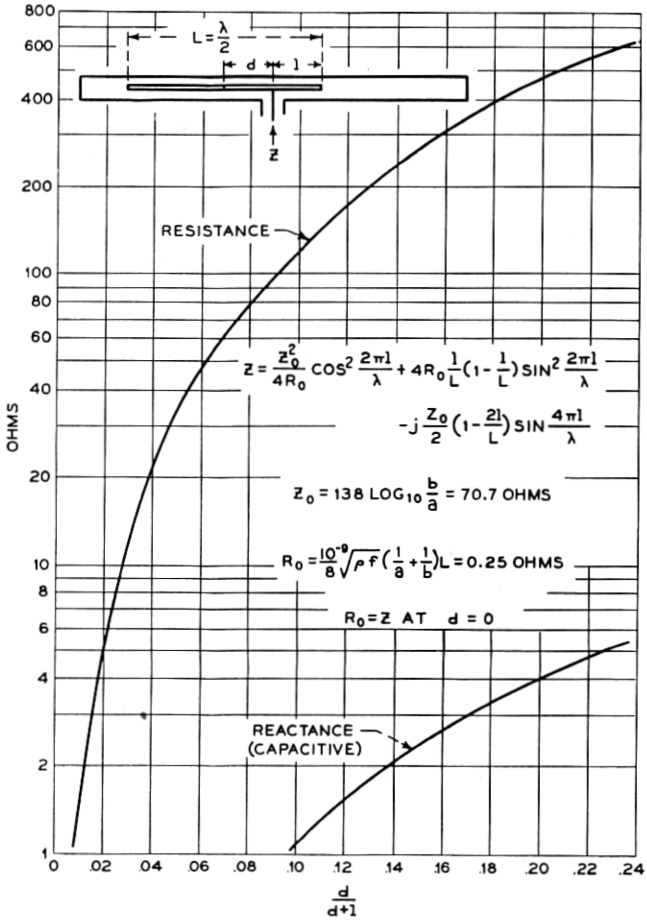

Fig. 23—A scheme for obtaining a calculable radio-frequency resistance standard which is essentially nonreactive and which is adjustable over wide limits of resistance.

radio-frequency resistance will be described. The scheme utilizes the resonant properties of a section of concentric-tube line of which the inner conductor is one-half wave-length long. The required resistance is obtained by a connection to the proper position on the inner conductor. The device is illustrated schematically on Fig. 23. It may be seen from the curves on this figure that the device is a means for obtaining a variable resistance which for most practical purposes is non-reactive. Additional advantages are that the device is rugged and that it may be designed to dissipate an appreciable amount of power.

For the purpose of testing transmitters and for other purposes in which the terminating network is required to dissipate several kilowatts, an iron wire line has been found to be of considerable utility. An iron wire line has the advantage that its impedance is almost independent of the frequency provided that the length of the line is sufficient. This impedance is very closely the characteristic impedance of the line. The far end of the line may be left either opened or closed. For operation at some one frequency the input impedance may be made more nearly equal to the characteristic impedance by means of a termination at the far end. For this purpose the scheme employing a short length of line as a parallel transforming impedance has been found very convenient.

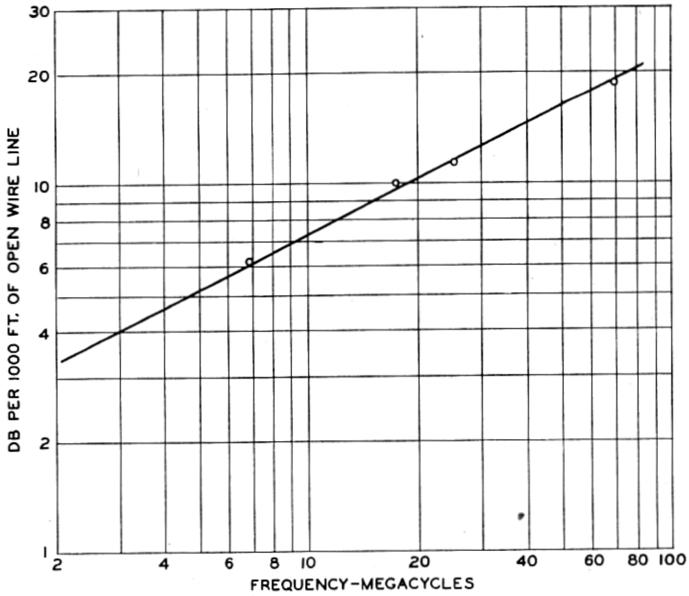

Fig. 24—The curve gives the attenuation in decibels for a balanced 600-ohm line constructed from No. 6 B & S iron wire.

An experimental attenuation curve for a 600-ohm line comprising 0.162-inch iron wire conductors is given in Fig. 24. The current entering the line was approximately one ampere. The measured resistivity for the iron is 12,300 e.m.u. The attenuation could be explained upon the basis that the permeability is 92.[20] This is not an unreasonable value. In fact it may be very desirable to obtain the permeability of iron at radio frequencies by forming the material into a transmission line and observing the line attenuation and direct-current resistance.

An iron wire line 1600 feet long has been in use at the Deal Laboratories for several years for the purpose of testing transmitters. This line successfully dissipates 15 kw. at 20 megacycles.

VIII. Conclusion

In conclusion a few remarks on the relative utility of open-wire and concentric-tube lines may assist in selecting the most desirable construction for a particular service. A definite discrimination between the two is not readily made because the economics of the entire radio plant are involved.

Concentric lines are more costly than open-wire lines. On the other hand, concentric lines permit the installation of a number of radio units within a single structure without incurring difficulties from cross-talk. The first cost and annual charges upon a compact installation may more than offset the cost of the lines when compared with an installation comprising several widely separated structures. Also, concentric lines may be constructed so as to be weatherproof.

There is little choice between the losses in open and concentric lines provided that a reasonable degree of current balance in the open-wire lines is maintained. In order to obtain balances the open-wire line terminal equipment both at the antenna and at the radio unit ends of the line must be carefully designed. The chief source of current unbalance difficulties resides in couplings between the antenna and an open-wire line. These may be materially reduced but cannot be completely eliminated. Another source may be unbalances with respect to neutral at the radio unit.

Complete isolation of the antenna from the line can only be obtained with shielded lines. Similarly, complete isolation from static and other noise sources for which discrimination by the antenna is obtained can only be effected by shielded lines. This is particularly important in reception. In this case small concentric-tube lines with losses as much as 2 db per 1,000 feet may be used provided that the noise level is reduced by a corresponding amount.

[20] P. P. Cioffi, Bell Telephone Laboratories, New York City, found an initial permeability of 95 for a sample of the above wire.

We wish to acknowledge the helpful suggestions which have been received in the course of this work from Messrs. H. T. Friis, J. C. Schelleng, and M. E. Strieby. Valuable advice on some of the mathematical questions encountered has been received from Mr. T. C. Fry.

Appendix

The following formulas for the power radiated by transmission lines were obtained by the conventional method of postulating the current distribution, calculating the electromagnetic fields and from the fields, the associated radiation by means of Poynting's theorem. As an independent check the same current distribution was postulated and the radiated power calculated following the methods of Pistolkors[21] and Bechman.[22]

Case I

An approximation for a line terminated in its characteristic impedance is a balanced two-wire line carrying a non-attenuated traveling wave. For this case the power radiated is:

$$\begin{aligned} P_1 = 120I^2 \Bigg[& \log_\epsilon (2L) - Ci(2L) + \frac{\sin (2L)}{(2L)} + 0.5772 - 1 \\ & - 2Ci(A) + \frac{\sin A}{A} - \frac{\sin (\sqrt{L^2 + A^2} - L) + \sin (\sqrt{L^2 + A^2} + L)}{2\sqrt{L^2 + A^2}} \\ & + Ci(\sqrt{L^2 + A^2} - L) + Ci(\sqrt{L^2 + A^2} + L) \Bigg] \text{ watts,} \end{aligned} \tag{1}$$

in which

$A = \frac{2\pi a}{\lambda}$,

$L = \frac{2\pi l}{\lambda}$,

$\frac{a}{\lambda}$ = line spacing in wave-lengths,

$\frac{l}{\lambda}$ = line length in wave-lengths,

I = r.m.s. value of current in each wire, and

$Ci(\ )$ = cosine integral.[23]

The equation simplifies considerably if it is assumed that a/λ is small so that:

$$\sin A \approx A \text{ and } L \gg A,$$

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[21] A. A. Pistolkors, *Proc. I. R. E.*, p. 562, March, 1929.

[22] R. Bechman, *Proc. I. R. E.*, p. 461, March, 1931.

[23] *See* Jahnke und Emde, "Funktionentafeln."

under which condition:

$$P_1 = 160I^2\left(\frac{\pi a}{\lambda}\right)^2 \text{ watts.} \tag{1a}$$

The numerical constant in (1a) differs somewhat from a result published some time ago by Carson.[24]

Case II

An approximation for an unterminated line is a balanced two-wire line bearing standing waves of the form:

$$I_x = I \cos\left[\frac{2\pi x}{\lambda} + \frac{2\pi m}{\lambda}\right].$$

For this case the power radiated is:

$$\begin{aligned} P_2 = 60I^2\Bigg[& \log_\epsilon(2L) - Ci(2L) + 2\cos M \cos(L - M)\frac{\sin L}{L} + 0.5772 \\ & - 2Ci(A) + \frac{\sin A}{A}\left[\cos^2 M + \cos^2(L - M)\right] \\ & - \cos^2 M - \cos^2(L - M) - 2\cos M \cos(L - \mathrm{M})\frac{\sin\sqrt{L^2 + A^2}}{\sqrt{L^2 + A^2}} \\ & + Ci(\sqrt{L^2 + A^2} - L) + Ci(\sqrt{L^2 + A^2} + L)\Bigg] \text{ watts,} \end{aligned} \tag{2}$$

in which $M = 2\pi m/\lambda$.

If as before it is assumed that the spacing is small and the line long the equation reduces to the following cases:

Case II-A

When the current is zero at both ends of the line, then,

$$\sin L = 0 \quad \text{and} \quad \sin M = \pm 1$$

and the radiated power is:

$$P_2 = 120I^2\left(\frac{\pi a}{\lambda}\right)^2 \text{ watts.} \tag{2a}$$

This agrees with a result published by Manneback.[25]

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[24] John R. Carson, *Jour. A. I. E. E.*, p. 789, October, 1921.

[25] Charles Manneback, *Jour. A. I. E. E.*, p. 95, February, 1923.

Case II-B

When the current is zero at one end and maximum at the other end of the line, then,

$$\sin M = \pm 1 \text{ and } \cos L = 0$$
$$\text{or } \cos M = 1 \text{ and } \sin L = \pm 1$$

and the radiated power is:

$$P_2 = 80I^2\left(\frac{\pi a}{\lambda}\right)^2 \text{ watts.} \tag{2b}$$

Case II-C

When the current is maximum at each end of the line, then,

$$\sin M = 0 \quad \text{and} \quad \cos L = \pm 1$$

and the radiated power is:

$$P_2 = 40I^2\left(\frac{\pi a}{\lambda}\right)^2 \text{ watts.} \tag{2c}$$

The approximation for the power radiated by unbalanced currents is essentially the case of a long wire parallel to a perfect earth. The approximation may be obtained from (2) by assuming that power is radiated only in one hemisphere, which divides the numerical constant by a factor of two and by writing for a the quantity $2h$, h being the height of the wire above ground. Equation (7) of the paper is written on the basis that ($\sin M = \pm 1$).

It is of interest to compare some of the above results with those for the case of a single conductor far removed from reflecting surfaces. If the wire is excited so as to bear standing waves of I r.m.s. amperes maximum value the radiated power is:

$$P_3 = 30I^2\Big[0.5772 + \log_e(2L) - Ci(2L) - \cos^2 M - \cos^2(L - \mathrm{M}) \\ + 2\cos M\cos(L - \mathrm{M})\frac{\sin(L)}{(L)}\Big] \text{ watts.} \tag{3}$$

If the wire is "terminated" so that there are no reflections from the ends a uniform current of I r.m.s. amperes may be assumed to exist along the wire. In this case the power radiated is:

$$P_4 = 60I^2\left[0.5772 - 1 + \log_e(2L) - Ci(2L) + \frac{\sin 2L}{2L}\right] \text{ watts.} \tag{4}$$