Constant Frequency Oscillators *

By F. B. LLEWELLYN

Summary—The manner in which the frequency of vacuum tube oscillators depends upon the operating voltages is discussed. The theory of the dependence is derived and is shown to indicate methods of causing the frequency to be independent of the operating voltages. These methods are applied in detail to the more commonly used oscillator circuits.

Experimental data are cited which show the degree of frequency stability which may be expected as a result of application of the methods outlined in the theory, and also show that the best adjustment is in substantial agreement with that predicted by theory. With a carefully built and adjusted oscillator the effects of normal variations in the operating voltages are negligible in comparison with the effects of temperature variations resulting from the changed operating currents. Methods of preventing these latter effects are not discussed in the present paper.

The appendix contains an analysis of the conditions under which the performance of an oscillator may be represented by the use of linear circuit equations.

IN recent years the commercial requirements of vacuum tube oscillators have grown more rigid. The tremendous increase in the number of radio broadcast stations with the consequent narrowing of frequency band available to each, the analogous demands by the carrier telephone, and the tendency toward higher frequencies where a small percentage frequency change defeats the universal effort to secure better quality, all have united in creating a need for very constant frequencies. This need has led to a study of methods for holding the frequency constant. The most notable of these is the piezo-electric crystal. However, it has been known for some time that certain oscillator circuits have the inherent property of maintaining their frequency quite constant even though not crystal controlled. Some of these circuits have the additional advantage of combining constant frequency at a given wave-length with the ability to maintain this constancy at other wave-lengths, thus giving a range of available frequencies, any one of which may be depended upon to stay constant.

The elements which cause the frequency of oscillators which are not crystal controlled to vary are such things as vibration, changing temperature, fluctuating voltage, and changing load. Vibration and temperature affect primarily the inductance and capacity in the circuit which naturally causes the frequency to change. Fluctuating voltages change the tube resistance, which in turn affects the frequency.

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Changing loads also change the frequency, since they take the form of variable resistance and reactance.

Considerable work has been done by various individuals to make the inductance and capacity of standard apparatus as free from the effects of vibration and temperature changes as possible. Work of that kind is not discussed here. Variable voltages occur in practically all installations and changing load impedance in many. These two things are the actual cause of the larger part of frequency variation in many installations.

Several vacuum tube circuits have been devised to surmount these difficulties. They may be divided roughly into two groups: first, those in which the attempt has been to minimize the change of frequency with battery voltage, and second, those in which the attempt has been to prevent the change. In the first group we have two types: first, circuits in which the effective resistance has been reduced to as low a figure as possible, and second, those in which a high impedance has been inserted between the tube and the tuned circuit in order to reduce the relative effect of changes in the tube. Considerable success has attended the efforts of a number of engineers led by J. W. Horton in these directions. More recently, circuits of the first type have been applied to the production of relatively high frequencies.[1]

The second group in which the attempt has been to prevent the frequency change developed from the work of Messrs. J. F. Farrington and C. F. P. Rose. They found that a certain critical value of an impedance between the vacuum tube and the tuned circuit apparently produced a constant frequency over a limited range when the battery voltages were varied. They experimented with various forms of networks for this stabilizing impedance and developed several in which the output power was not reduced by stabilization.

Theory

The writer attacked the problem from a theoretical standpoint and showed that in certain cases the mathematical procedure indicates means of making the oscillator frequency independent not only of a variable load resistance, but also of the battery voltages. The purpose of this paper is to develop the general theory and application of these circuits and to show how several circuits in particular may be made to produce practically constant frequency with customary variations of voltage and load resistance. The relations necessary to maintain the frequency constant at any given setting when it is desirable

[1] Ross Gunn, "A new frequency stabilized oscillator system," *Proc. I. R. E.*, **18,** September, 1930.

that the oscillator be operative over a range of frequencies are also indicated.

Before proceeding with a detailed description of the various specific embodiments necessary to secure independence of frequency and battery voltage, it will be well to lay down the physical conditions upon which the frequency of any vacuum tube oscillator depends.

In general, all such oscillators consist of or may be resolved into, a tuned electrical circuit or network to which is attached a vacuum tube. Irrespective of any particular circuit, the frequency of the oscillator is completely determined by the following quantities, the designations used here being uniformly employed throughout the subsequent analysis:

L, the self-inductance in the network
M, the mutual inductance in the network
C, the capacity in the network
R, the resistance in the network
r_p, the plate resistance of the vacuum tube
r_g, the grid resistance of the vacuum tube
μ, the amplification factor of the vacuum tube

Of these quantities, L, C, and M require little comment. They are merely symbolic of the elements of the electrical network. The quantity C includes the interelectrode capacities of the tube when they become of consequence. These three quantities are assumed to be constant, an assumption which has been found very reasonable in practice. The quantity R represents the resistance in the network. For the purpose of this discussion the oscillator is assumed to deliver only a small amount of power, being used most often in such a manner as to supply voltage to the grid of an amplifier tube. Consequently, the electrical network external to the vacuum tube may, and should, be constructed in such a manner as to include a minimum amount of resistance. Under these conditions the losses in the circuit have been found to be practically all the result of the internal resistances, r_p and r_g of the vacuum tube.

These two quantities, r_p and r_g, are very important, being principally responsible for changes in condition of the circuit as a whole. It should be realized that r_g has the same relation to the static values of grid current and potential that r_p has to the plate current and potential. The effect of varying the applied potential of the grid or plate, or of changing the filament current is directly to cause r_p and r_g to vary, usually in opposite directions. Further, when amplitude of oscillation varies, for which variation of battery voltages (grid, plate, and filament) are again principally responsible, both r_g and r_p vary.[2]

The quantity μ is the amplification factor and is used with its usual significance. It varies with battery potential but this variation is ordinarily very small, though not to be neglected.

It eventuates, from the above considerations, that if the reactive elements of the frequency determining circuit are constant, a permissible assumption, the frequency may be stabilized if adequate account is taken of changes in battery voltages and load resistances. This it is the purpose of the present paper to discuss.

Hartley Oscillator

Consider first the form of the Hartley oscillator shown in schematic form without indicating any special method of introducing the batteries, in Fig. 2. Figure 1 shows the circuit equivalent of several of the oscillators in the following figures when the impedances are represented in generalized form, and therefore will be employed for an analysis of the conditions necessary to secure independence of frequency and battery or applied voltages, and the results of this analysis will then be interpreted in detail in terms of the special circuit of Fig. 2. In Fig. 1 the impedances, Z_4 and Z_5, are inserted for the purpose of effecting the independence of frequency and battery voltages, and the values which they should have in order to accomplish this result are found by the following analysis:

From Fig. 1 we have the circuit equations when the assumed current conditions are as shown by the arrows:

$$
\begin{aligned}
\mu e &= I_1(r_p + Z_1 + Z_5) + I_2(Z_1 + Z_m) - I_3 Z_m, \\
0 &= I_1(Z_1 + Z_m) + I_2 Z_0 - I_3(Z_2 + Z_m), \\
0 &= -I_1 Z_m \qquad - I_2(Z_2 + Z_m) + I_3(r_g + Z_2 + Z_4), \\
e &= I_3 r_g. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (1)
\end{aligned}
$$

These equations are expressions of Kirchkoff's Law regarding the sum of the potentials in a closed mesh. The equations (1) effectively comprise only three simultaneous equations because the network has only three meshes.

In the above equation Z_0 is symbolic of the series impedance of the tuned circuit. Using the symbolism of Fig. 1,

$$Z_0 = Z_1 + Z_2 + Z_3 + 2Z_m. \qquad (2)$$

[2] The appendix to this paper contains a further discussion of the significance of r_p and r_g together with an analysis of the conditions under which oscillator networks may be treated by the use of linear circuit equations as is done in the following analysis.

Equations (1) may be rewritten in determinant form as follows:

$$\begin{vmatrix} (r_p + Z_1 + Z_5) & +(Z_1 + Z_m) & -(Z_m + \mu r_g) \\ +(Z_1 + Z_m) & Z_0 & -(Z_2 + Z_m) \\ -Z_m & -(Z_2 + Z_m) & (r_g + Z_2 + Z_4) \end{vmatrix} = 0. \tag{3}$$

This determinant form of (1) follows immediately from reducing (1) to three equations.

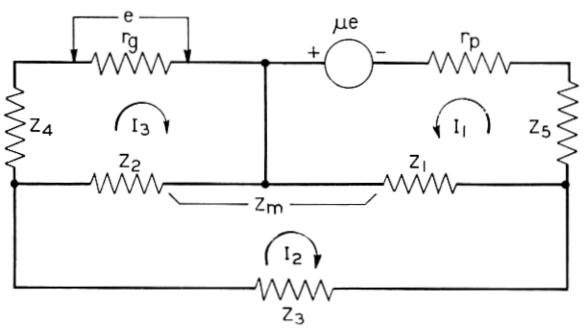


Fig. 1—Equivalent circuit network of Hartley or Colpitts-type oscillator.

In accordance with the theory of the operation of oscillators, discussed in the appendix, both the conditions necessary for oscillation to exist and the frequency of oscillation may be found from (3). That is:

$$\begin{aligned}(r_p + Z_1 + Z_5)Z_0(r_g + Z_2 + Z_4) &+ (Z_1 + Z_m)(Z_2 + Z_m)(\mu r_g + 2Z_m) \\ = Z_0Z_m(\mu r_g + Z_m) &+ (Z_1 + Z_m)^2(r_g + Z_2 + Z_4) + (Z_2 + Z_m)^2 \\ &(r_p + Z_1 + Z_5).\end{aligned} \tag{4}$$

The next step is to express each of the generalized Z's in the equivalent form of $(R + iX)$ where i stands for the imaginary quantity, $\sqrt{-1}$, and both R and X are real, representing, respectively, resistance and reactance. A great simplification results when it is recalled that the circuits external to the vacuum tube are assumed to have very little resistance, and that practically all of the losses in the network are caused by the tube resistances, r_g and r_p, so that these two are the only resistances which need be retained in the analysis. With this understanding, (4) becomes:

$$\begin{aligned}&[r_p+i(X_1+X_5)]iX_0[r_g+i(X_2+X_4)]-(X_1+X_m)(X_2+X_m)(\mu r_g+2iX_m) \\ &\quad = -X_0X_m(\mu r_g+iX_m)-(X_1+X_m)^2[r_g+i(X_2+X_4)] \\ &\quad -(X_2+X_m)^2[r_p+i(X_1+X_5)].\end{aligned} \tag{5}$$

In order for (5) to be true, both the real and the imaginary portions must separately be equal to zero. If (5) (which comes naturally from (3) with the given substitutions) is separated into its real and imaginary parts, the resulting two equations must be simultaneous and between them express the frequency and relative values which r_p and r_g must assume in order for oscillations to exist. The particular aim in the present case is to find whether values of X_4 or X_5 exist which will enable the frequency to be expressed in terms of the constants of the circuit external to the vacuum tube so that if r_p, r_g, and μ should vary, the frequency, being dependent upon the external circuit only, will remain constant.

From (5), then, the real and imaginary parts give the following two equations:

$$
\begin{aligned}
&- X_0[r_p(X_2 + X_4) + r_g(X_1 + X_5)] - \mu r_g(X_1 + X_m)(X_2 + X_m) \\
&\quad = - X_0 X_m \mu r_g - (X_1 + X_m)^2 r_g - (X_2 + X_m)^2 r_p. \qquad (6)
\end{aligned}
$$

$$
\begin{aligned}
&X_0[r_p r_g - (X_1 + X_5)(X_2 + X_4)] - 2X_m(X_1 + X_m)(X_2 + X_m) \\
&\quad = - X_0 X_m^2 - (X_1 + X_m)^2(X_2 + X_4) - (X_2 + X_m)^2(X_1 + X_5). \qquad (7)
\end{aligned}
$$

There are certain mathematical rules for finding whether the desired constancy of frequency may be obtained from the conditions given by (6) and (7). Without, however, going into detail in regard to these, it is easy to see from (7) that if X_4 and X_5 have such values as to satisfy the equation:

$$
\begin{aligned}
2X_m(X_1 + X_m)(X_2 + X_m) &= (X_1 + X_m)^2(X_2 + X_4) \\
&\quad + (X_2 + X_m)^2(X_1 + X_5) \qquad (8)
\end{aligned}
$$

(which is obtained by including all terms of (7) which do not contain X_0), then the frequency of oscillation is exactly that which will cause X_0 to become zero, and will remain so, no matter what values may be taken by r_p, r_g, and μ. In other words, the oscillation frequency is equal to the series resonant frequency of the tuned circuit.

It follows, then, that if the battery voltages were to vary, the frequency, being determined by the circuit elements external to the vacuum tube only, would remain constant. In regard to a changing load resistance, it is evident that if this were connected in parallel either with r_p or r_g, then the combination of the two resistances could be considered as a single resistance. It therefore follows that the same adjustment which causes the frequency to be independent of battery voltage is also the correct one to render the frequency independent of a variable load impedance when this impedance is resistive, only, and is connected in parallel either with the plate or grid resistance of the tube.

In order to complete the general demonstration, it remains to show that the values imposed on (7) by the condition of (8) do not require physically impossible values of r_p, r_g, and μ in order to satisfy (6) and thus maintain oscillation. To do this, assume that (8) is solved for either X_4 or X_5 and substitute in (6), remembering that X_0 is zero. The result is:

$$\frac{r_p}{r_g} = \mu\left(\frac{X_1 + X_m}{X_2 + X_m}\right) - \left(\frac{X_1 + X_m}{X_2 + X_m}\right)^2. \tag{9}$$

Inspection of this expression shows that the conditions required are physically possible, and it follows that the amplitude of oscillation increases or decreases until the effective values of r_p and of r_g, which are measures of the dissipation of energy on the plate and on the grid sides, take up the values specified by the conditions of (9). Thus, for instance, if X_1 and X_2 were approximately equal, then r_p would have to be $(\mu - 1)$ times as large as r_g before the oscillation amplitude settled down to a steady state value. To many who are accustomed to neglect the losses occurring on the grid side of a vacuum tube when dealing with oscillator problems, this low value of r_g will appear as somewhat unusual. In this connection, it may be pointed out that the low value of r_g is not in any way a special requirement imposed by the stabilizing reactances, X_4 and X_5, but is inherent in vacuum tube oscillators in general, unless particular conditions are arranged to render it otherwise. For instance, it is a well-known experimental fact that resistances of the order of 4000 ohms may be placed across the grid-filament terminals of an oscillator employing any of the more common types of three-element receiving amplifier tubes without stopping the oscillations, when a good low-loss tuned circuit is employed. In view of the fact that the amplitude of the oscillations is commonly limited by r_g, this is evidence that stable oscillations may be secured with values of r_g which are of the order of 2000 or 3000 ohms.

The demonstration may be made more rigid by the use of (6) and for the special case where $X_1 = X_2$ and $X_4 = X_5 = X_M = 0$, in which the stabilizing reactances have been omitted. For such a simplified circuit, it is found by elimination of X_0 between (6) and (7) that:

$$\frac{r_p}{r_g} = \mu\left[\frac{r_p r_g - X_1^2}{r_p r_g + X_1^2}\right] - 1.$$

Now, X_1 is of the order of 500 or 600 ohms at the most, while both r_p and r_g are at least enough larger than this in the case of the more commonly used vacuum tubes so that the expression for r_p/r_g is roughly equal to $(\mu - 1)$. Thus, in the simplest kind of vacuum tube circuit, it is seen that r_g is apt to be appreciably smaller than r_p, and by no means negligible in its effect.

To return to (8), which expresses the relation between X_4, X_5 and other circuit reactances which are necessary to cause the frequency to be independent of battery voltages, we note that, although (8) is still in generalized form, and is yet to be applied to the particular case shown in Fig. 2, the very significant fact that the oscillation frequency for this type of stability must be the series resonant frequency of the tuned circuit is a direct consequence of the requirements of the equation.

For application to the Hartley type of oscillator, the various terms of (8) have the following significance:

$$X_1 = \omega L_1,$$
$$X_2 = \omega L_2,$$
$$X_m = \omega M,$$

where $\omega = 2\pi \times$ frequency and X_4 or X_5 are to be determined. In the case of Fig. 2, where stabilization is accomplished on the plate side, we put X_4 equal to zero. Then solving (8) for X_5, we find:

$$X_5 = 2\omega M\left(\frac{L_1 + M}{L_2 + M}\right) - \omega L_2\left(\frac{L_1 + M}{L_2 + M}\right)^2 - \omega L_1.$$

X_5 is thus required to be negative, so that a capacitive reactance is necessary for plate stabilization of a Hartley-type oscillator. Thus putting

$$X_5 = -\frac{1}{\omega C_5}$$

and remembering that since $X_0 = 0$, the angular frequency is given by

$$\omega^2 = 1/C_3(L_1 + L_2 + 2M),$$

finally we get

$$C_5 = C_3\frac{L_1 + L_2 + 2M}{L_1 + L_2\left(\dfrac{L_1 + M}{L_2 + M}\right)^2 - 2M\left(\dfrac{L_1 + M}{L_2 + M}\right)}, \tag{10}$$

which is the value of capacity which should be inserted between the plate and the tuned circuit of a Hartley-type oscillator in order to cause the frequency to remain constant when the battery voltages are varied, and there is no reactance between the grid and tuned circuit.

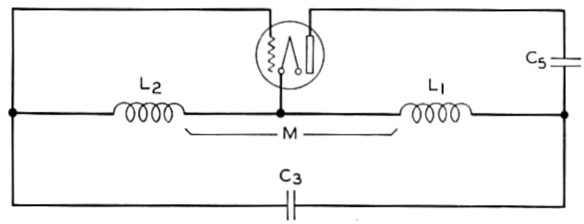


Fig. 2—Hartley oscillator, plate stabilization.

$$C_5 = C_3\left[\frac{L_0}{L_1 + L_2A^2 - 2MA}\right],$$

where

$$L_0 = L_1 + L_2 + 2M, \qquad A = \frac{L_1 + M}{L_2 + M}.$$

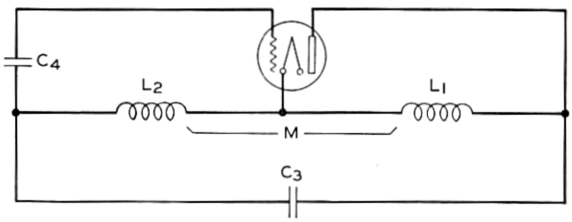


Fig. 3—Hartley oscillator, Grid stabilization.

$$C_4 = C_3A^2\left[\frac{L_0}{L_1 + L_2A^2 - 2MA}\right],$$

where

$$L_0 = L_1 + L_2 + 2M, \qquad A = \frac{L_1 + M}{L_2 + M}.$$

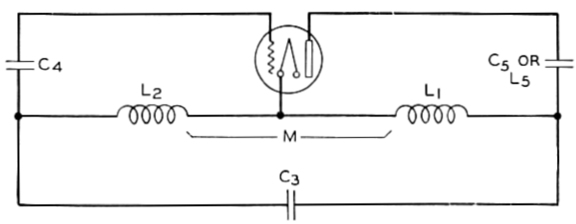


Fig. 4—Hartley oscillator, plate and grid stabilization.

$$\frac{1}{C_5} + \frac{A^2}{C_4} = \frac{1}{C_3}\left[\frac{L_1 + L_2A^2 - 2MA}{L_0}\right],$$

$$L_5 = L_0\frac{C_3}{C_4}A^2 - L_1 - L_2A^2 + 2MA,$$

where

$$L_0 = L_1 + L_2 + 2M, \qquad A = \frac{L_1 + M}{L_2 + M}$$

Of course in actual practice, it is necessary to provide a d-c path for the space current of the tube, which can be accomplished by shunting the condenser, C_5, with a high impedance choke.

It is often the case that a stopping condenser is desirable in the X_4 position, instead of a direct connection between grid and tuned circuit. This stopping condenser and the accompanying leak are advantageous inasmuch as it has been found by experience that an oscillator operating with a leak and condenser combination is inherently much more stable as regards change of frequency with change of battery voltage than an oscillator with a d-c low resistance path from grid to filament, even when a battery is employed to impose a negative bias on the grid. The explanation for this improved stability lies in the fact that the grid leak tends to keep the grid resistance, r_g, constant. It frequently happens, when the leak and condenser combination is used, that difficulty is experienced in avoiding "blocking" when a large enough condenser to have negligible reactance is employed. In such cases the required value of C_5 may be chosen in the manner indicated in connection with Fig. 4, which allows for a finite reactance between grid and tuned circuit, or else, as another alternative, the plate may be directly connected to the tuned circuit so that X_5 is zero, and stabilization may be accomplished by choosing the value of C_4 in accordance with the requirements then imposed by (8), which refer to Fig. 3 and necessitate the value of capacity shown in the figure. Another possible stabilizing arrangement is shown in Fig. 4, where either capacity or inductance may be used on the plate side, depending on the value of capacity at C_4 on the grid side. Yet another possible modification of Fig. 4 would be to use an inductance on the grid side. This would require a very small capacity on the plate side, and probably is less convenient than the arrangement indicated in the figure.

In all three of the cases considered thus far, the equations show that the value of the stabilizing capacity or inductance depends upon the values of L_1, L_2, M, and C_3 so that if the frequency of the oscillator were varied intentionally, by changing L_1, for instance, then a different value of stabilizing capacity or inductance would be required to secure independence of frequency and battery voltage at the new frequency. If, however, the circuit were so constructed that M were zero, and L_1 and L_2 were made so that they remained always equal to each other, then the value of the stabilizing element would depend upon C_3 only, and the frequency could be changed by varying L_1 and L_2 simultaneously without destroying the stabilizing adjustment.

Colpitts Oscillator

This property may be utilized to even greater advantage in connection with the Colpitts type of oscillator, which is illustrated in Figs. 5, 6, and 7 and will now be investigated with the aid of (8) in the same manner in which the relations necessary for stabilizing the Hartley oscillator were secured. Thus, for the Colpitts circuit:

$$X_1 = -\frac{1}{\omega C_1},$$

$$X_2 = -\frac{1}{\omega C_2}.$$

$$X_m = 0,$$

$$\omega^2 = \frac{1}{L_3}\left(\frac{1}{C_1} + \frac{1}{C_2}\right).$$

The values of the stabilizing elements required by (8) are given in Figs. 5, 6, and 7 which show several arrangements for the stabilizing impedance, as applied to the Colpitts-type oscillator. In particular, Fig. 7 shows a choice of either an inductance or a capacity on the grid side.

In Figs. 5 and 6 and in Fig. 7 when inductance is used on both plate and grid sides it is evident that if the condensers, C_1 and C_2, are connected together in a "gang" mounting so that when they are varied, the ratio of their capacities remains constant, then the frequency of the oscillator may be changed by changing C_1 and C_2 without disturbing the stabilizing adjustment which causes the frequency to be independent of battery voltages.

Feed-Back Oscillator

Figures 8, 9, and 10 show conventional drawings of the type of oscillator circuit known as a "feed-back" or sometimes as a "tuned input" circuit. In Fig. 8 stabilizing is accomplished on the plate side; in Fig. 9 on the grid side; and in Fig. 10 on both sides. A mathematical analysis analogous to that which was described in detail in connection with Fig. 1 gives the values of stabilizing impedances which are shown in the figures, and also indicates that the conditions for oscillation may be met when these values are employed.

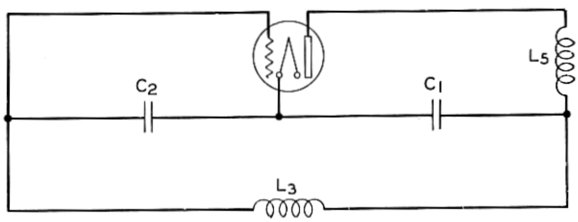


Fig. 5—Colpitts oscillator, plate stabilization.

$$L_5 = L_3 \frac{C_2}{C_1}.$$

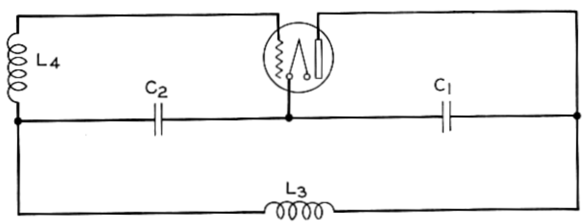


Fig. 6—Colpitts oscillator, grid stabilization.

$$L_4 = L_3 \frac{C_1}{C_2}.$$

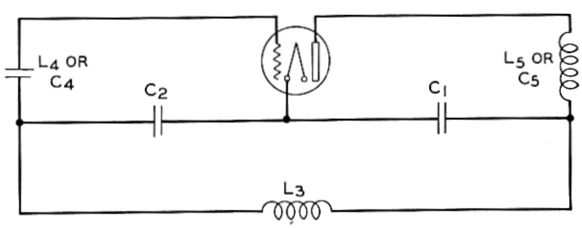


Fig. 7—Colpitts oscillator, plate and grid stabilization.

$$L_4 \left(\frac{C_2}{C_1} \right) + L_5 \left(\frac{C_1}{C_2} \right) = L_3,$$

$$L_5 = L_3 \frac{C_2}{C_1} \left[1 + \frac{C_2}{C_4} \left(\frac{C_2}{C_1 + C_2} \right) \right].$$

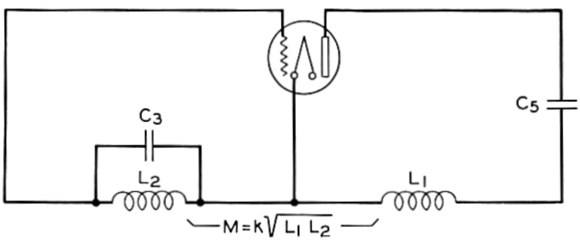


Fig. 8—Feed-back oscillator, plate stabilization.

$$C_5 = C_3 \frac{L_2}{L_1} \left(\frac{1}{1 - k^2} \right) \cdot$$

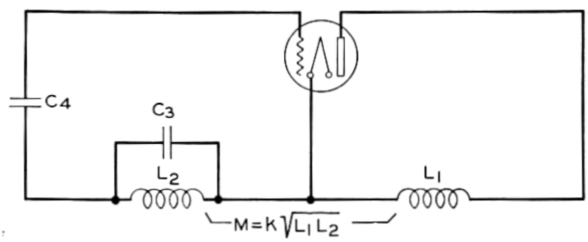


Fig. 9—Feed-back oscillator, grid stabilization.

$$C_4 = C_3 \left(\frac{k^2}{1 - k^2} \right) \cdot$$

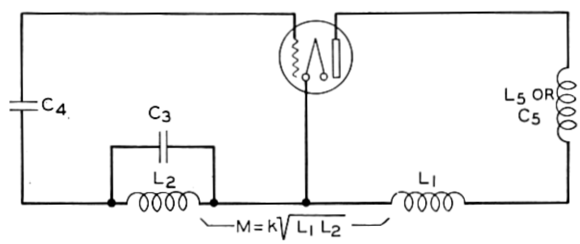


Fig. 10—Feed-back oscillator, plate and grid stabilization.

$$L_5 = L_1 \left[k^2 \left(1 + \frac{C_3}{C_4} \right) - 1 \right],$$

$$C_5 = C_3 \frac{L_2}{L_1} \left[\frac{1}{1 - k^2 \left(1 + \frac{C_3}{C_4} \right)} \right] \cdot$$

Reversed Feed-Back Oscillator

Figures 11, 12, and 13 show conventional drawings of the type of oscillator circuit known as a "reversed feed-back" or sometimes as a "tuned output" type of oscillator, with the application of stabilizing

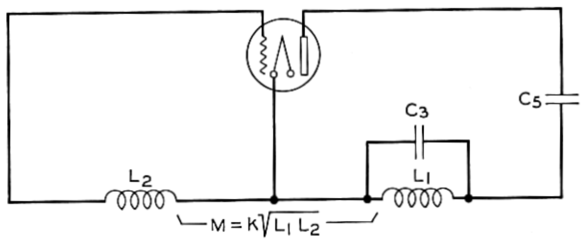


Fig. 11—Reversed feed-back oscillator, plate stabilization.

$$C_5 = C_3\left(\frac{k^2}{1 - k^2}\right).$$

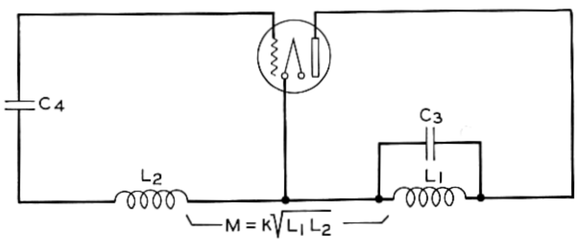


Fig. 12—Reversed feed-back oscillator, grid stabilization.

$$C_4 = C_3\frac{L_1}{L_2}\left(\frac{1}{1 - k^2}\right).$$

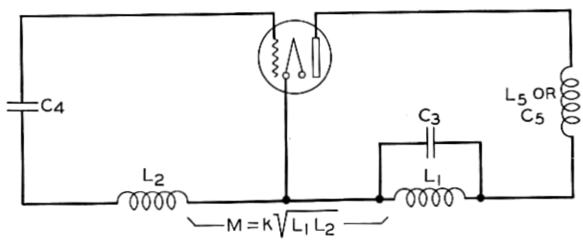


Fig. 13—Reversed feed-back oscillator, plate and grid stabilization.

$$L_5 = L_1\left[1 + \frac{1}{k^2}\left(\frac{L_1C_3}{L_2C_4} - 1\right)\right],$$

$$C_5 = \frac{C_3}{\frac{1}{k^2}\left(1 - \frac{L_1C_3}{L_2C_4}\right) - 1}.$$

impedances to cause the frequency to be independent of changes in battery voltages. In Fig. 11 the stabilizing impedance is placed between the plate and the tuned circuit; in Fig. 12 between the grid and coupling coil; and in Fig. 13 stabilization is accomplished by impedances placed in both positions. Again, the mathematical analysis gives the values of the stabilizing impedances as shown on the figures and indicates that oscillation is possible when these values are used.

Other Types of Oscillator Circuits

As an instance of the stabilizing of another general class of oscillator circuits which has wide application in a number of special cases, attention is drawn to the tuned-plate, tuned-grid circuit of Fig. 14. The

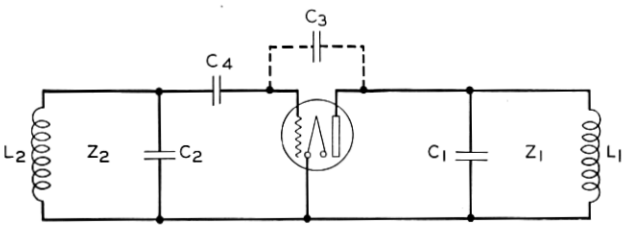


Fig. 14—Tuned-plate, tuned-grid oscillator with no magnetic coupling.

$$C_1 = \frac{L_2}{L_1}\left[C_2 + \frac{(1+\mu)C_3C_4}{C_4 + (1+\mu)C_3} \right] - C_3.$$

input and output circuits are shown in the drawing as consisting of condenser and inductance combinations connected in parallel. At any specified frequency, however, the parallel combination may be replaced

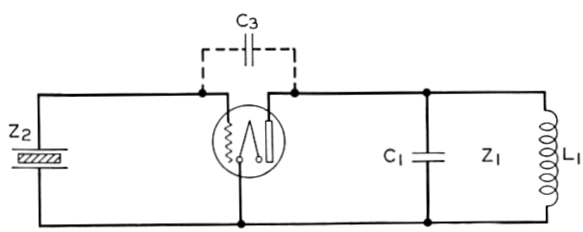


Fig. 14-a.

by a series circuit, or, in fact, by any form of network which has the same impedance, and none of the currents or voltages in the remainder of the circuit will be altered. In particular, the inductance and capacity shown on the input side in Fig. 14 may be replaced by a piezo-electric crystal, as shown in Fig. 14-a, having the same impedance at the operating frequency without affecting the currents and voltages in the remaining parts of the circuit.

It is well known that the frequency of such a piezo-electric oscillator is less affected by changes in battery voltage than is the frequency of the ordinary, nonstabilized electric oscillator. However, the battery voltage does influence the frequency of the piezo-electric oscillator to an extent which is undesirable for certain accurate types of work. It therefore becomes useful to apply stabilization to the piezo-electric oscillator. It will be shown that the stabilization may be accomplished by adjusting the size of the output tuning condenser to such a value that the impedance of the output circuit bears a certain critical relation to the impedance of the crystal, while at the same time, the circuit as a whole fulfills the conditions necessary for the existence of oscillations.

The same kind of stabilization is, of course, applicable to an electric oscillator having analogous relations between the input and output impedances. Thus, it is often possible to stabilize the Hartley oscillator by moving the connection between the filament and coil to different positions on the coil, until that one which gives the proper ratio of input to output impedances has been found. In the case of the Hartley and Colpitts oscillators, however, it is more often preferable to stabilize by the special circuit arrangements illustrated in Figs. 1 to 7, while, on the other hand, the tuned-grid, tuned-plate type of circuit lends itself readily to stabilization by adjustment of the output circuit.

Numerical expressions for the proper impedance relations may be obtained by noting that the circuit of Fig. 14 may be generalized into the circuit of Fig. 1 by regarding Z_4 and Z_5 as zero, while Z_2 comprises the whole input network which may consist of various arrangements of coils, condensers, grid leaks, and the like, and, in a similar fashion, Z_1 comprises the whole output network. The mathematical analysis given in connection with Fig. 1 may therefore be adapted to fit Fig. 14 immediately, and in place of (6) and (7) we have the two expressions:

$$X_0(r_gX_1 + r_pX_2) + \mu r_gX_1X_2 = r_pX_2^2 + r_gX_1^2, \tag{11}$$

$$X_0(r_pr_g - X_1X_2) = -X_1X_2(X_1 + X_2). \tag{12}$$

The requirements of (11) are:

$$r_p = r_g\frac{X_1}{X_2}\left[\frac{\mu X_2 + X_0 - X_1}{X_2 - X_0}\right], \tag{13}$$

which may be used to eliminate r_p in (12) and gives

$$X_0 r_g^2(\mu X_2 + X_0 - X_1) = X_2^2(X_0 - X_1 - X_2)(X_2 - X_0). \quad (14)$$

In order for the frequency to be independent of r_g, it is necessary for one of the factors on the left-hand side of the equation to be zero. This, however, necessitates that one of the factors on the right-hand side of (14) also should be zero. Investigation shows that the only pair of factors of (14) that may both be zero, and still be consistent with (13) is the following:

$$\mu X_2 + X_0 - X_1 = 0, \quad (15)$$

$$X_2 - X_0 = 0. \quad (16)$$

Elimination of X_0 between these two expressions results in the following relation:

$$(1 + \mu)X_2 = X_1. \quad (17)$$

The frequency is then given by the expression:

$$X_1 + X_3 = 0. \quad (18)$$

Equation (17) expresses the relation which is required between the reactances of the input and the output network in order to provide for a constant frequency with varying battery voltages.

In the application of this stabilization to a piezo-electric oscillator such as is shown in Fig. 14-a it sometimes happens that stability improves with decrease in the value of the output tuning capacity but oscillations cease before complete stabilization is secured. The explanation for this and its remedy may be obtained from (17) and (18) by supposing that the reactance, X_2, of the crystal may be represented by an antiresonant circuit, C_2 and L_2, in series with a capacity, C_4, while the output reactance, X_1, consists of the antiresonant circuit, C_1 and L_1. Thus, the value of C_1 which satisfies (17) and (18) is

$$C_1 = \frac{L_2}{L_1}\left[C_2 + \frac{(1 + \mu)C_3}{C_4 + (1 + \mu)C_3} C_4 \right] - C_3. \quad (19)$$

For discussion, the form which (19) takes in the absence of the stopping condenser, C_4, is:

$$C_1 = \frac{L_2}{L_1}[C_2 + (1 + \mu)C_3] - C_3.$$

This shows that a fairly large value of C_1 may be required when C_4 is absent, which places the tuning of the plate antiresonant circuit in a rather critical portion of its reactance characteristic. In order to avoid this, the introduction of a fairly small condenser at C_4 is advantageous. Thus, if C_4 were made somewhat smaller than C_3, then the value of C_1 required by (19) is roughly:

$$C_1 = \frac{L_2}{L_1}[C_2 + C_4] - C_3$$

which gives an appreciably smaller value of C_1 and results in stabilization with a much less critical adjustment than is the case when the stopping condenser is absent.

In all of the above analyses, the requirement of a capacity or an inductance is indicated by the fact that the signs come out right in the final equations. If the wrong type of reactive element were used, it would result, for example, that a negative inductance apparently would be required, which of course would indicate the requirements of a capacitance.

Another Type of Stabilization

A third general type of stabilization may be illustrated by considering a hypothetical oscillator having its plate circuit coupled back to its grid circuit by means of a transformer coil with a coefficient of coupling equal to unity. Methods of obtaining the equivalent effect

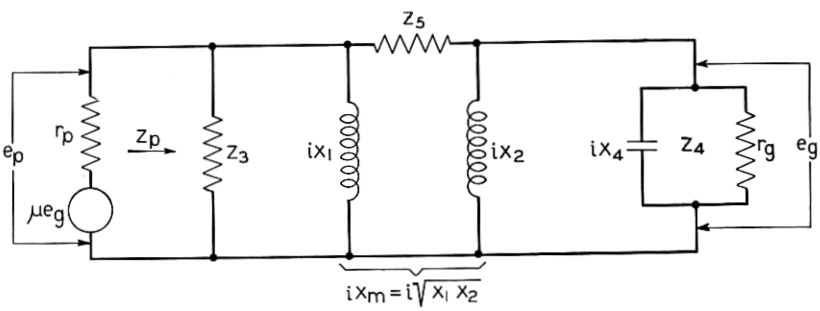


Fig. 15—Equivalent circuit of oscillator with unity coupling.

of such a coil under practical operating conditions will be described later, so that for the present it will be assumed that a unity coupled coil is on hand. The equivalent circuit diagram of the oscillator is shown in Fig. 15, where the primary and secondary windings of the coil are indicated at X_1 and X_2 respectively.

From the properties of unity coupled coils it follows that, no matter what impedances are hung across the coil, or connected between windings, the ratio of the voltage across the secondary to the voltage across the primary depends upon the coil reactances only, and not at all upon the attached impedances. In the circuit of Fig. 15 this ratio is given by the expression:

$$\frac{e_g}{e_p} = -\sqrt{\frac{X_2}{X_1}}. \tag{20}$$

In general, the voltage e_p may be expressed in terms of the impedance looking out of the plate-filament terminals of the tube. Thus

$$e_p = -\frac{\mu e_g Z_p}{r_p + Z_p} \tag{21}$$

where Z_p is the aforementioned impedance.

From (20) and (21) there results:

$$\frac{1}{Z_p} = \frac{1}{r_p}\left[\mu\sqrt{\frac{X_2}{X_1}} - 1\right]. \tag{22}$$

This equation completely expresses the operation of the oscillator in so far as impedance relations for the fundamental current component are concerned. From Fig. 15 ordinary circuit analysis shows that Z_p may be written

$$\frac{1}{Z_p} = \frac{1}{Z_3} + \frac{1}{Z_4}\left(\frac{X_2}{X_1}\right) + \frac{1}{iX_1} + \frac{1}{Z_5}\left(1 + \sqrt{\frac{X_2}{X_1}}\right)^2$$

so that (22) becomes:

$$\frac{1}{Z_3} + \frac{1}{Z_4}\left(\frac{X_2}{X_1}\right) + \frac{1}{iX_1} + \frac{1}{Z_5}\left(1 + \sqrt{\frac{X_2}{X_1}}\right)^2 = \frac{1}{r_p}\left[\mu\sqrt{\frac{X_2}{X_1}} - 1\right]. \tag{23}$$

The next step is to separate this into its real and imaginary components. We stipulate, as in the previous analyses, that the losses in the external circuit elements are small compared with those occasioned by the grid resistance of the tube. With this understanding, Z_4 may be separated into two parts, the one iX_4, in parallel with the other which constitutes the grid resistance r_g. Both Z_3 and Z_5 are taken as pure reactances. Thus the last expression yields the two equations:

$$\frac{1}{X_3} + \frac{1}{X_4}\left(\frac{X_2}{X_1}\right) + \frac{1}{X_1} + \frac{1}{X_5}\left(1 + \sqrt{\frac{X_2}{X_1}}\right)^2 = 0, \tag{24}$$

$$\frac{1}{r_g}\left(\frac{X_2}{X_1}\right) = \frac{1}{r_p}\left[\mu\sqrt{\frac{X_2}{X_1}} - 1\right]. \tag{25}$$

Equation (24) contains neither r_p, r_g, nor μ, so that the important conclusion can be drawn that the frequency of an oscillator with unity coupling between the plate and grid circuits depends only upon the inductances and capacities in the circuit, and not at all upon the tube parameters, r_p, r_g, and μ; provided, however, that the losses in the external circuit are small, and the harmonic voltages across the tube are small enough to allow r_p and r_g to be considered as pure resistances. In this connection, the interelectrode capacities may be grouped with the external circuit elements forming X_3, X_4, and X_5, so that no high-frequency difficulty is to be anticipated from them.

Equation (25) contains the relation between r_p, r_g, and μ necessary to insure the presence of oscillation. In practice, the amplitude of the oscillations builds up until this relation is satisfied.

The foregoing theory of the action of a unity coupled oscillator has led to an extremely useful and desirable result, namely, the independence of frequency and operating voltages. The point now remaining to be shown is how to get the unity coupling.

In attacking this question, the first thing to notice is that our theory does not require that the unity coupling condition,

$$M = \sqrt{L_1 L_2}$$

should be obtained. What actually is required is the much less rigid condition:

$$X_m = \sqrt{X_1 X_2} \tag{26}$$

where X_1 and X_2 are not limited to inductance alone.

Thus, imagine one of the impedances, say X_2, to consist of a coil L_2, in series with a condenser, C_2. We have then, by (26):

$$\omega^2 M^2 = \omega L_1 \left(\omega L_2 - \frac{1}{\omega C_2} \right) \tag{27}$$

or, writing

$$M = k\sqrt{L_1 L_2}$$

where k may now be less than one, we have from (27)

$$C_2 = \frac{1}{\omega^2 L_2 (1 - k^2)} \tag{28}$$

which gives the value of C_2 necessary to provide "unity coupling" at the operating frequency.

The value of X_2/X_1 is thus

$$\frac{X_2}{X_1} = \frac{\omega L_2 - \dfrac{1}{\omega C_2}}{\omega L_1} = \frac{\omega L_2 - \omega L_2(1 - k^2)}{\omega L_1} = k^2 \frac{L_2}{L_1}. \tag{29}$$

In practice, X_3, X_4, and X_5 would usually be capacities, to correspond to the circuit of Fig. 17. With this arrangement, and the relation given by (29) we have the frequency from (24):

$$\omega^2 = \frac{1}{L_1\left[C_3 + k^2C_4\frac{L_2}{L_1} + C_5\left(1 + k\sqrt{\frac{L_2}{L_1}}\right)^2\right]}. \tag{30}$$

The value of C_2 is thus written from (28) and (29) as follows:

$$C_2 = \left(\frac{L_1}{L_2}\right)\left[\frac{C_3 + k^2C_4\frac{L_2}{L_1} + C_5\left(1 + k\sqrt{\frac{L_2}{L_1}}\right)^2}{1 - k^2}\right]. \tag{31}$$

This is the general value of C_2 needed to stabilize the oscillator, and applies to any grid-stabilized oscillator where the unity coupling concept can be employed. In the case where C_5 and C_4 are small enough to be neglected we have the equivalent circuit of the reversed feed-back oscillator of Fig. 12, and for the value of the stabilizing capacity:

$$C_2 = \frac{L_1}{L_2}\frac{C_3}{1 - k^2}. \tag{32}$$

When the notation of Fig. 17 is reconciled with that of Fig. 12, this is in agreement with the conclusion reached for the reversed feed-back oscillator by the former method of analysis.

The present analysis has the twofold advantage of allowing the interelectrode capacities to be included, which results in (31) instead of (32); and of giving a more readily interpreted picture of the relation required for stability, namely the "unity coupling" condition of equation (26). Equation (31) is moreover applicable to the tuned-plate, tuned-grid type of oscillator, when there is magnetic coupling between the input and output circuits. Thus, in the particular instance when L_1 and L_2 are equal, as also are C_3 and C_4, we have from (31):

$$C_2 = C_3\frac{(1 + k^2)}{(1 - k^2)} + C_5\frac{(1 + k)}{(1 - k)}. \tag{33}$$

Hence, if tuning is done by "ganging" C_3 and C_4 together and varying them simultaneously, the stability may be maintained for all frequencies by making C_2 to consist of two parts: the one a fixed capacity equal to

$$C_5\frac{(1 + k)}{(1 - k)}$$

and the other a variable capacity "ganged" together with C_3 and equal to

$$C_3 \frac{(1 + k^2)}{(1 - k^2)} \cdot$$

In order to insure fulfillment of the requirements of this theory, it was mentioned above that the oscillator should be relatively free from harmonics, so that r_p and r_g may be taken as pure resistances. That this requirement can be successfully met may be demonstrated by reference to Fig. 15 and consideration of the means which would be employed if the circuit represented an amplifier with e_g impressed on the grid of a following tube instead of on the grid of the driving tube itself. In such an arrangement, it is well known that the distortion is least when the impedance looking out of the plate of the driving tube is made materially larger than the internal plate resistance of the tube itself; and, second, when the impedance looking back out of the grid of the driven tube is made materially smaller than the internal grid resistance of the tube itself. The conditions which determine how nearly these two requirements may be met in the oscillator tube are governed by (22). Thus

$$\frac{r_p}{Z_p} = \mu \sqrt{\frac{X_2}{X_1}} - 1. \tag{34}$$

This should be small in order that the first requirement mentioned above should be fulfilled. Hence $\mu k \sqrt{L_2/L_1}$ should exceed unity by as little as is consistent with reliable oscillation. When we come to consider the second of the requirements for decreasing harmonics we obtain an expression analogous to (34), namely:

$$\frac{Z_g}{r_g} = \mu \sqrt{\frac{X_2}{X_1}} - 1 \tag{35}$$

so that the requirement that the impedance looking back out of the grid should be less than the grid resistance is satisfied by the same condition as that required by (34); namely, that $\mu k \sqrt{L_2/L_1}$ should exceed unity by as little as is consistent with reliable oscillation.

From Fig. 17, by regarding C_3 and C_5 as zero, the feed-back type of oscillator is obtained. Again, when C_3 and C_4 are zero, the Hartley oscillator results. The stabilization both of the feed-back oscillator and the Hartley type by the method shown on the figure was not described in the analysis of Figs. 3 and 9 inasmuch as the present method places the stabilizing element directly in the tuned circuit, whereas the former method placed it between the tuned circuit and the vacuum tube.

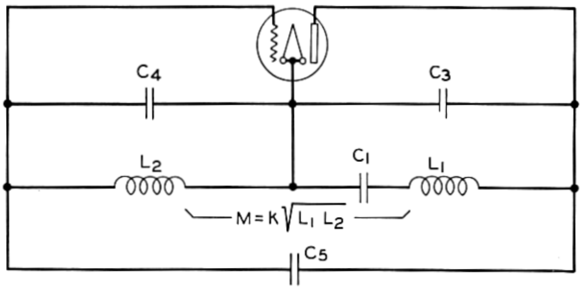


Fig. 16—General circuit of oscillator with unity coupling, plate stabilization.

$$C_1 = \frac{L_2}{L_1}\left[\frac{C_3k^2\frac{L_1}{L_2} + C_4 + C_5\left(1 + k\sqrt{\frac{L_1}{L_2}}\right)^2}{1 - k^2}\right].$$

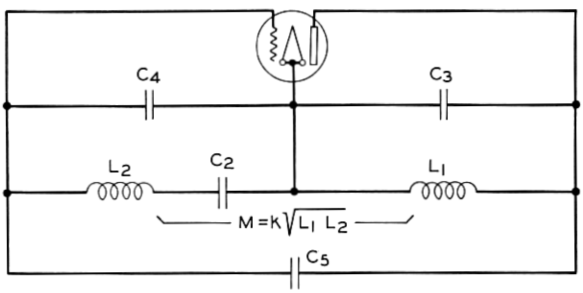


Fig. 17—General circuit of oscillator with unity coupling, grid stabilization.

$$C_2 = \left(\frac{L_1}{L_2}\right)\left[\frac{C_3 + k^2C_4\left(\frac{L_2}{L_1}\right) + C_5\left(1 + k\sqrt{\frac{L_2}{L_1}}\right)^2}{1 - k^2}\right].$$

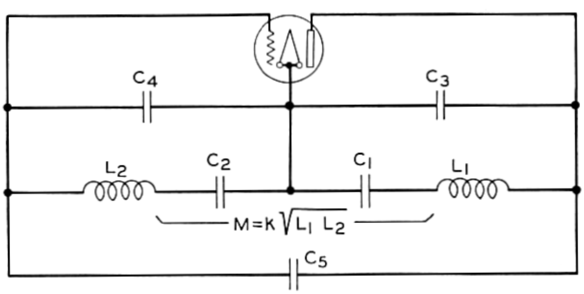


Fig. 18—General circuit of oscillator with unity coupling, plate and grid stabilization.

$$\omega^2k^2L_1L_2 = \left(\omega L_1 - \frac{1}{\omega C_1}\right)\left(\omega L_2 - \frac{1}{\omega C_2}\right).$$

Besides the general circuit of Fig. 17, where stabilization is accomplished by imposing a critical value on C_2, there is the alternative shown in Fig. 16, where the stabilizing element is C_1, a condenser located in series with L_1, and of such a size as to cancel the leakage reactance between L_1 and L_2. The formula giving this size in terms of the other circuit constants is shown in the figure, and the condition that the harmonic content should be small for circuits of the general type of Fig. 16 is that $(\mu/k)\sqrt{L_2/L_1}$ should exceed unity by as little as is consistent with reliable oscillation.

From Fig. 16 the circuits of various types of oscillators may be derived in the manner which was described in connection with Fig. 17. Of these circuits, the reversed feed-back and the Hartley were not described in connection with Figs. 11 and 12.

A combination of the features of Figs. 16 and 17 may be employed in the manner shown in Fig. 18 where condensers C_1 and C_2 are placed in series both with L_1 and L_2, respectively. In this case, the formula for the required size of C_1 and C_2 becomes quite cumbersome when expressed in terms of the other circuit elements, only. Since, however, the frequency is usually known approximately, we may use the relation

$$X_m^2 = X_1 X_2$$

for finding C_1 and C_2 in terms of ω and get:

$$\omega^2 k^2 L_1 L_2 = \left(\omega L_1 - \frac{1}{\omega C_1}\right)\left(\omega L_2 - \frac{1}{\omega C_2}\right). \tag{36}$$

As in the case of Figs. 16 and 17, so also may the circuit of Fig. 18 be modified to correspond to the reversed feed-back, the feed-back and the Hartley types of oscillators where stabilization is accomplished both on the plate and on the grid sides.

Experiment

Of course, Figs. 1 to 18 are intended to represent only the fundamentals of the corresponding circuits. For practical operation these circuits would have to include the usual stopping condensers, leak resistances, sources, choke coils, and accessories. These circuit elements should be so valued and introduced into the oscillator circuit as a whole as not to interfere with the relations required by the analyses, in order to maintain the stabilizing effects of the stabilizing impedances. As to the choke coils, this means merely that they must be what the name implies, that is, a substantially infinite impedance. In the case of a Hartley-type oscillator, where the reactance is chosen to be located in the grid leads instead of in the plate leads, a condenser must be used. This may replace the conventional stopping condenser. Where the reactance is in the plate lead for a similar type of oscillator, the stopping condenser in the grid lead should be large so as to have negligible impedance. Similar expedients are suggested for the impedances of the other types of oscillator circuits.

As typical of the general method whereby any of the simplified circuits of Figs. 1 to 18 may be elaborated into a conventional circuit of this kind, including the various adjunctive circuits, Fig. 19 should be

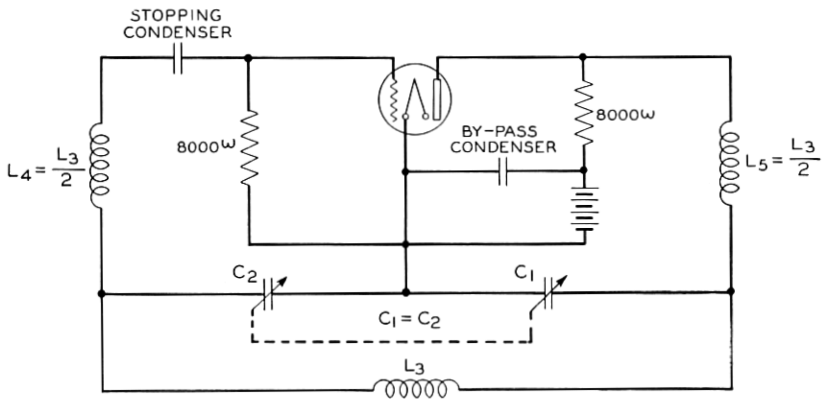


Fig. 19—Practical arrangement of oscillator with three series tuned coupled circuits.

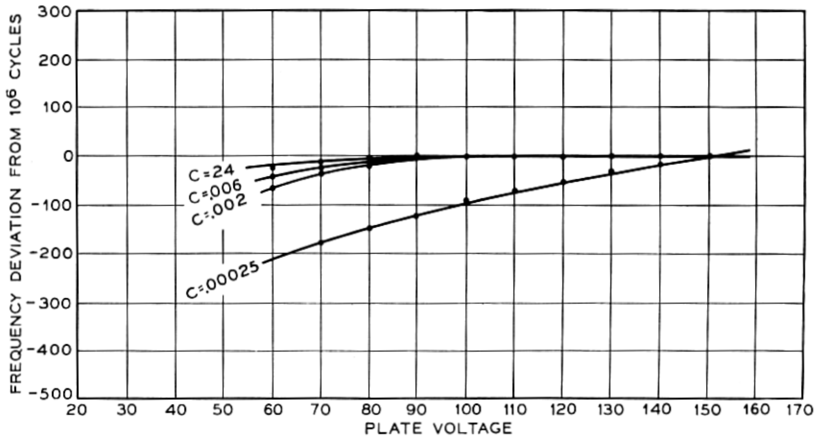
referred to. This figure illustrates a complete wiring diagram of the oscillator of Fig. 7 and shows an example of stabilization by means of the inductance L_5 in the plate circuit and the inductance L_4 in the grid circuit. In addition to satisfying the relation shown in Fig. 7, it may be noticed that the value of L_5 is such as to tune with C_1 to the oscillation frequency, and, similarly, the value of L_4 is such as to tune with C_2 to the oscillation frequency. Under such conditions a resistance of appreciable value may be introduced into the circuit of L_3 without affecting the frequency of the stabilization. The reason for this may be explained briefly as follows:

Consider a single series circuit formed of one of the three meshes of Fig. 19, for instance that composed of the elements, r_g, in parallel with the 8000-ohm leak, L_4, and C_2. This circuit is in series resonance at the frequency at which the circuit as a whole oscillates. Therefore it tends to introduce resistance impedance only into whatever circuits it is reactively coupled with. Thus, the effect of this circuit upon the adjacent circuit, L_3, C_1, C_2, with which it is coupled is to introduce resistance only. Similarly, if this last circuit operates at series resonance, only resistance is introduced into the plate circuit, L_5, C_1, and r_p, in parallel with the d-c feed of 8000 ohms, with which it is coupled. Hence, if the plate circuit likewise operates at series resonance, a change in resistance of any part of the circuit will change only the resistance into which the tube works and therefore will leave the frequency unaltered.

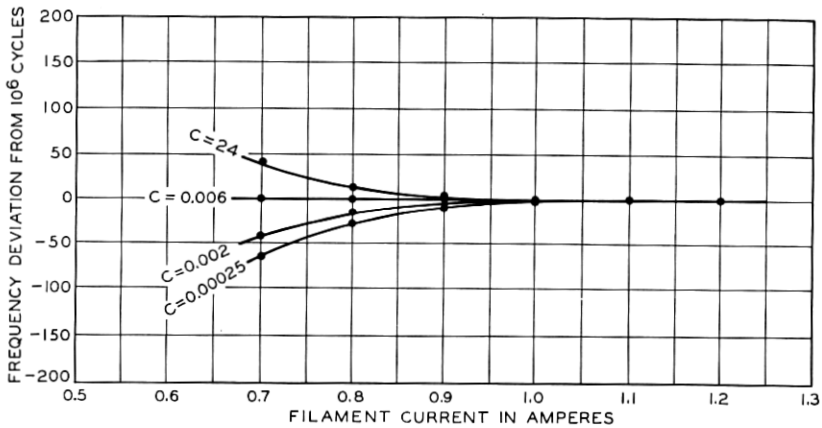
In a more general sense, any of the oscillator forms discussed may be stabilized even when the resistance in the external circuit is not inappreciable, the effect of the external resistance manifesting itself in two ways: first, a value of stabilizing reactance slightly different from that given in the above formulas may be required, and second, the frequency, instead of being absolutely independent of battery voltage variations, goes through a maximum or a minimum as the battery voltages change, the voltage at which this maximum or minimum occurs depending upon the exact value of the stabilizing reactance. An exact mathematical analysis of this more general case yields formulas for the stabilizing reactances which involve r_g or r_p and hence are not as useful even in a case where the resistance in the external circuit is of importance as are the formulas presented above. The latter may be used as first approximations in any event.

In practice it has been found that when ordinary precautions are taken to insure a low-loss external circuit, the relations given above hold very accurately and any variations in frequency then existing as a result of varying battery voltages may be traced to either one of two causes, both of which may be guarded against: first, the interelectrode capacities of the tube may be sufficient to enter into the impedance relations. In this event, a change in the form of circuit, such as the use of the tuned plate-tuned grid arrangement of Fig. 17, where the interelectrode capacities form a part of the external circuit, will eliminate the difficulty. Second, the harmonic currents caused by the nonlinear characteristics of the vacuum tube may introduce the effect of a reactive impedance back into the fundamental which may vary with battery voltage and so change the frequency. The remedy for this is to provide a low reactance path for the harmonics so that they have no opportunity to build up a reactive voltage across the tube, and also to use grid leaks and other such well-known devices for reducing the harmonic currents generated by the tube.

For the purpose of providing information as to the order of stability which may be expected from the several methods of stabilization outlined above, various quantitative experimental tests have been conducted. The general results of these tests may be summarized by saying that a close adherence to the theoretical requirements results in an oscillator whose frequency depends upon operating voltages to such a small extent that temperature effects become the predominating influence and special precautions must be taken in order to eliminate



(a)



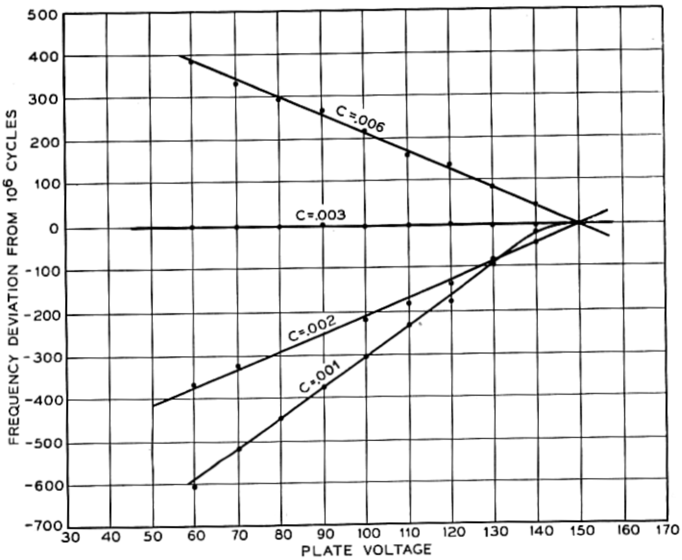
(b)

Fig. 20—Performance curves of reversed feed-back oscillator with tight coupling, grid stabilization.

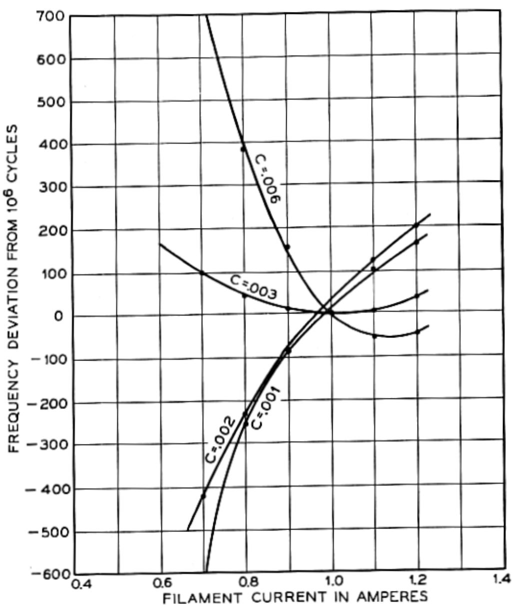
(a) Variation of frequency with plate potential.
(b) Variation of frequency with filament current.

them before data showing the dependence of frequency on operating voltages can be obtained.

For instance, the data for the curves shown in the accompanying Figs. 20 and 21 were secured by the following procedure: A plate



(a)



(b)

Fig. 21—Performance of reversed feed-back oscillator with loose coupling, grid stabilization.

(a) Variation of frequency with plate potential.
(b) Variation of frequency with filament current.

potential of 150 volts and a filament current of one ampere were selected as reference points. The frequency under these conditions was noted. A change to a different operating condition, say 140 volts plate potential and 1 ampere filament current, was made and the frequency measured as rapidly as possible, whereupon the operating voltages were returned immediately to their reference values and the frequency rechecked. Special care was taken to keep the room temperature very constant, but, even so, the heating of the parts of the oscillator circuit by the operating alternating and direct currents was sufficient to affect the frequency to an undesirable extent, requiring that the readings be taken with unusual rapidity in order to return the voltages to normal before the changed operating currents could appreciably affect the temperature of the coils, tube elements, and other parts of the circuit.

The final results, however, are consistent enough to be representative of what can be accomplished, and the two sets of curves shown in the figures bring out a result which was found to hold throughout the investigation; namely that the higher the coefficient of coupling in the coil used to secure feed-back, the less critical was the value of the stabilizing impedance. Thus, in Fig. 20 the coupling was as tight as it was possible to produce by winding the primary and secondary simultaneously upon a tube to form a single layer solenoid, while the coil used for Fig. 21 was made by removing about half the turns from the secondary of the same coil, thus providing for a step-down in voltage as well as a decrease in coupling. A possible explanation for the less critical adjustment required with tight coupling lies in the fact that the tightly coupled coil satisfies the condition for "unity coupling" as given by (26) over a range of frequencies whereas the loosely coupled coil satisfies the condition at only the frequency critically determined by the stabilizing capacity.

This would appear to indicate that the type of stabilization described in the theoretical part of the paper under the unity coupling concept offers certain practical advantages over those types where the stabilizing element is placed between the tuned circuit and the tube, as in the Colpitts oscillator of Figs. 5, 6, and 7, for example, where no magnetic coupling whatever is employed. Experiments with the Colpitts circuit have shown, however, that when the capacities in the tuned circuit are made large relative to the inductance, a very satisfactory degree of stabilization may be secured at the lower frequencies where interelectrode capacities may be neglected. A close inspection of the theory of stabilizing the frequency of the Colpitts oscillator shows an argument analogous to that of the coupled coils, namely that the smaller we make the inductance in the tuned circuit, and the larger we make the capacity, the frequency being kept constant, the greater will be the range of frequencies over which the condition, required by (8), that the series reactance of the tuned circuit be zero, is satisfied to an approximation sufficiently good for practical purposes, and hence the less critical will be the adjustment of the stabilizing impedance.

For the data in Figs. 20 and 21 a reversed feed-back type of oscillator was employed, having an elementary circuit similar to that of Fig. 12. A grid leak was placed in parallel with the stabilizing capacity, but care was taken to see that the resistance of the leak was always so high that its value did not affect the frequency. This was done by using a variable resistance and increasing its value until the frequency no longer changed. A large grid leak has always been found advantageous in securing constancy of frequency, but the size of the leak reaches a practical limitation determined by the time constant which produces the familiar "blocking."

The plate battery potential was fed through a choke in series with the plate inductance coil, the combination of choke and B battery being thoroughly by-passed with a large condenser.

For the purpose of checking the size of the stabilizing capacity to find its agreement with theory, an indirect method was used. The theory requires that

$$C_4 = C_3 \frac{L_1}{L_2}\left(\frac{1}{1-k^2}\right)$$

as shown in Fig. 12. The operating frequency was one megacycle, which made a direct measurement of the coupling coefficient, k, somewhat awkward, so that a method based on the "unity coupling" concept was employed. Thus from (26)

$$k^2\omega^2 L_1 L_2 = \omega L_1\left(\omega L_2 - \frac{1}{\omega C_4}\right) \tag{37}$$

may also be used for determining C_4. The primary, L_1, of the coil was connected through an impedance to a source of e.m.f. of one megacycle, and a vacuum tube voltmeter was placed across it. The condenser, C_4, was placed directly in parallel with the secondary, L_2. With this arrangement the impedance looking into the primary is

$$Z = i\omega L_1 + \frac{\omega^2 k^2 L_1 L_2}{i\left(\omega L_2 - \frac{1}{\omega C_4}\right)}$$

unless points very near the resonance point are considered. This last equation may be written

$$iZ\left(\omega L_2 - \frac{1}{\omega C_4}\right) = -\,\omega L_1\left(\omega L_2 - \frac{1}{\omega C_4}\right) + \omega^2 k^2 L_1 L_2. \qquad (38)$$

The condenser, C_4, was varied until the reading of the vacuum tube voltmeter became zero. This means that Z was zero, and hence (38) gives the value of C_4 required by (37) which is in turn the value needed to stabilize the oscillator.

How well this checked the actual values needed for the case shown in Figs. 20 and 21 may be seen by the following: For Fig. 21 the value of C_4 measured as above, was 4000 $\mu\mu$f. The experimental value was 3000 $\mu\mu$f. For Fig. 20 the measurement gave a rather broad zero on the vacuum tube voltmeter, which was, however, estimated at 8400 $\mu\mu$f. For a check, a measurement was made at 7 megacycles which gave a sharper zero, and a value of 120 $\mu\mu$f. This must be reduced to its equivalent value of 1 megacycle by multiplying by 7^2 which gives 5880 $\mu\mu$f. The experimental curves of Fig. 20 show a noncritical value of 6000 $\mu\mu$f. which is nevertheless in good accord with the above measurements, while in Fig. 21 the agreement is somewhat more striking.

As an example of stabilization of oscillators in the altogether different frequency region from 7 to 40 kc, the following table was taken from data kindly supplied by F. J. Rassmussen:

TABLE I

Frequency kc	L_1 mh	L_2 mh	Coupling k	Stabilizing Capacity μf	
				Experimental	Theoretical
12.5	3.176	6.891	0.131	0.0021	0.0022
40.0	3.210	7.010	0.134	0.0018	0.0023
12.5	3.178	6.974	0.129	0.0021	0.0022
40.0	3.211	7.094	0.133	0.0018	0.0023
7	2.498	0.500	0.720	0.4 to ∞	2.07
15	2.499	0.498	0.723	0.2 to ∞	0.46
15	1.409	0.161	0.695	0.2 to ∞	1.33
23	1.407	0.161	0.698	0.1 to ∞	0.58
23	0.665	0.093	0.677	0.1 to ∞	0.99
31	0.665	0.093	0.681	0.1 to ∞	0.57
31	0.487	0.076	0.663	0.1 to ∞	0.64

This table again emphasizes the less critical adjustment required when the coupling is tight, as in the last seven rows.

It is hoped that the foregoing data and comments will serve as a guide to design methods for constant frequency oscillators, in so far as dependence of the frequency on operating voltages is concerned. Combinations and permutations of the various circuits dealt with will occur to the designer who requires special arrangements to fit special cases. The generalized circuit of Fig. 18 is suggested as being adaptable to meet the most widely varying conditions. This is particularly true at very high frequencies, since all the interelectrode capacities are included in the circuit of that figure.

The popular "push-pull" type of circuit may likewise be generalized to correspond to several of the fundamental circuits illustrated in the figures, and may be stabilized by the methods indicated. However, because of the nonuniformity of vacuum tubes and the added complication of the circuit, no advantage has been obtained by its use, so that the single tube circuits are to be preferred wherever special conditions do not require the push-pull type.

Appendix

The complete and rigorous mathematical relations for oscillation circuits containing vacuum tubes have seldom been discussed in connection with their practical application to useful circuits. In the case of the stabilization of oscillators against changes in battery voltages it is important to base the theory upon as strictly rigorous a mathematical foundation as possible, yet at the same time to be able to express the results in readily useful terms. It will be shown that this desirable result may be attained by a proper interpretation of the meaning of the internal impedances, r_p and r_g, of the vacuum tube.

To show this in the shortest and most obvious way, a simple series circuit will be considered. Let the circuit consist of a resistance, R, a condenser, C, and an inductance, L, all connected in series with a vacuum tube which may be taken as having a "negative resistance" characteristic. In order to increase the generality of the demonstration, a sinusoidal driving voltage, E, of angular frequency, ω, is also allowed to act on the circuit. By Kirchkoff's Law, the current in the circuit is expressed by the equation

$$E = RI + L\frac{dI}{dt} + \frac{1}{C}\int Idt + V \tag{1}$$

where V is the drop across the vacuum tube. As a general expression for V in terms of the current the following expression may be used:

$$V = V_0 + A_1I + A_2I^2 + A_3I^3 + \cdots. \tag{2}$$

We are interested in the "steady state" solution, and accordingly a Fourier series will be the most general form which can be assumed for the current. It is convenient to write the series in the following form:

$$I = \sum_{n=-\infty}^{\infty} \frac{b_n}{2} \epsilon^{in\omega t} \tag{3}$$

or, for brevity, in the symbolic form

$$I = \sum I(n\omega) \tag{4}$$

where the summation is understood to extend from minus infinity to plus infinity. Substitution of (4) and (2) into (1) gives:

$$\begin{aligned} E = {} & R\sum I(n\omega) + L\sum in\omega I(n\omega) + \frac{1}{C}\sum \frac{I(n\omega)}{in\omega} \\ & + V_0 + A_1\sum I(n\omega) + A_2\sum\sum I(n\omega)I(m\omega) \\ & + A_3\sum\sum\sum I(n\omega)I(m\omega)I(l\omega) + \cdots. \end{aligned} \tag{5}$$

For the component of fundamental frequency, ω, we get

$$\begin{aligned} E(\omega) = {} & RI(\omega) + Li\omega I(\omega) + \frac{I(\omega)}{Ci\omega} + A_1 I(\omega) \\ & + A_2 \sum_{n+m=1} I(n\omega)I(m\omega) + A_3 \sum_{n+m+l=1} I(n\omega)I(m\omega)I(l\omega) + \cdots \end{aligned} \tag{6}$$

where the summation terms involve the products of all frequency components which beat together to give the fundamental, as indicated. In order to put the last expression in symmetrical form, it is convenient to multiply and divide each of the summation terms by $I(\omega)$ so that we may write

$$\begin{aligned} E(\omega) = I(\omega) \Bigg[& R + Li\omega + \frac{1}{Ci\omega} + A_1 + \frac{A_2}{I(\omega)} \sum I(n\omega)I(m\omega) \\ & + \frac{A_3}{I(\omega)} \sum I(n\omega)I(m\omega)I(l\omega) + \cdots \Bigg]. \end{aligned} \tag{7}$$

This expression exhibits the terms in square brackets in the form of an impedance, and shows that the vacuum tube may be treated as an ordinary linear circuit element if it is considered as having the impedance

$$Z = A_1 + \frac{A_2}{I(\omega)} \sum I(n\omega)I(m\omega) + \frac{A_3}{I(\omega)} \sum I(n\omega)I(m\omega)I(l\omega) + \cdots. \tag{8}$$

Of course, the numerical value of such an impedance cannot be found from this expression alone, but in oscillator analysis there is no necessity for its numerical evaluation. The very important fact that the nonlinear elements in a circuit network may be replaced by equivalent impedances so that the ordinary circuit analysis can be employed has been demonstrated. It is possible to tell something about the form of the tube impedance from (8). Thus, the first term, namely A_1, is a real quantity and contributes a part of the total effective resistance of the tube. All of the remaining terms are, in general, complex, depending upon the phases of the different harmonic currents. Thus, the conclusion is reached that a nonlinear resistance may be reduced to an equivalent linear impedance, but that this impedance has a reactive as well as a resistive component in the general case. There is at least one important instance where the equivalent linear impedance is resistive only. This occurs when the impedance in the circuit external to the vacuum tube contains resistance, only, to all of the harmonic currents.

With the general conception of the impedance of the vacuum tube, described above, the fundamental component of (1) becomes

$$E = \left[R + i\omega L + \frac{i}{i\omega C} + r + iX \right] I \tag{9}$$

where the tube impedance is represented by $r + iX$.

When the driving voltage, E, is zero, as in the case of oscillators, then for a finite current to exist the oscillation conditions are:

$$\left. \begin{aligned} R + r &= 0 \\ \omega L - \frac{1}{\omega C} + X &= 0 \end{aligned} \right| . \tag{10}$$

In the treatment of oscillator networks employed in the foregoing paper the quantities, r_p and r_g, are used in the sense of the resistance, r, in (9) and (10) of this appendix. The reactive component, X, of the tube impedance has been neglected in the paper, for the reason that all of the circuits discussed are of such character that the reactance of the external circuit to the harmonic currents may be made quite low, and the nonlinearity of the vacuum tube characteristics is not such as to cause excessive production of harmonics.

In the case of the dynatron type of oscillator, where the harmonic currents are especially strong, it has been found by experiment that the reactive component of the tube impedance cannot be neglected, but that it is, in fact, altogether responsible for the variation in frequency with battery voltages which is characteristic of the dynatron oscillator.