Observations on Modes of Vibration and Temperature Coefficients of Quartz Crystal Plates [1]

By F. R. LACK

The characteristics of piezo-electric quartz crystal plates of the perpendicular or Curie cut are compared with parallel or 30-degree cut plates with reference to the type of vibration of the most active modes, the frequency of these modes as a function of the dimensions, and the magnitude and sign of the temperature coefficients of these frequencies.

It is pointed out that the two principal modes of the perpendicular cut plate appear to be of the longitudinal type, the high-frequency mode being a function of the thickness while the low-frequency mode is a function of the width (along the electric axis). Both modes have a negative temperature coefficient of frequency. Of the two corresponding modes of the parallel cut plates a shear vibration is responsible for the high frequency. This frequency has a positive temperature coefficient. The low-frequency mode is of the longitudinal type and has a negative temperature coefficient.

Considering only the high-frequency vibration of these plates it is observed that there are characteristic variations of the frequency and temperature coefficient with the ratio of dimensions of the plate and the temperature, which are peculiar to the parallel cut plate. These variations can be attributed to a coupling of the shear and longitudinal modes.

It is then shown that if the parallel cut plate be treated as a group of coupled oscillatory systems with appropriate temperature coefficients the usual coupled system analysis will explain the curves of frequency *vs.* dimensional ratio, frequency *vs.* temperature, and temperature coefficient *vs.* dimensional ratio that are characteristic of this plate. This analysis offers an explanation of the low temperature coefficients which can be produced by a proper choice of the dimensional ratios.

WITH the increasing demands of the radio industry for a high degree of carrier-frequency stability, considerable attention has been focused recently on the piezo-electric quartz crystal as a circuit element in frequency generating systems. The low damping of these mechanical oscillators, combined with their piezo-electric properties makes them particularly suitable for frequency control where a high degree of constancy is required. The frequency stability of the quartz plates prepared in the usual manner, is however, often not sufficient for many of the demands for constant frequency. For instance such a crystal plate does not compare favorably as a substandard of frequency with a good astronomical clock. To meet the demands for frequency substandards as well as many other practical problems concerning frequency generation in the communication art, it becomes necessary to devise methods for improving the frequency stability of these crystal systems. This involves a study of the many factors upon which this stability depends.

A crystal plate constitutes an extremely complex vibration system with a large number of degrees of freedom which are for the most

[1] Presented April 3, 1929, before Institute of Radio Engineers.

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part combinations of certain fundamental types of vibration. The ultimate frequency stability attained with a given crystal-controlled frequency generator is then a function of the equivalent electrical characteristics of the combination vibration set up in the crystal plate as well as the constants of the rest of the generator circuit. In particular, the frequency change in a crystal oscillator with changes in tube constants or attached load is a function of the equivalent electrical decrement of the vibration which the crystal happens to be executing. Further, the temperature coefficient of frequency of the crystal oscillator depends largely upon the temperature coefficient of frequency of the crystal vibration, which in turn depends upon the change with temperature of the various mechanical elastic constants that are called into play by this vibration.

The general relation between stress and strain, which in an ordinary isotropic medium involves only two constants, in crystal quartz requires six.[2] The choice of a particular constant or constants that enter into a given mode of vibration depends upon the orientation of the plate with respect to the original crystal axes, and the particular type of vibration, whether longitudinal, torsional, etc.

It is to be expected, therefore, that there will be a variation among the characteristics of the modes of vibration of plates cut in a different fashion, as well as between the different modes of a given plate. In practice we have found considerable difference in the magnitude of the electric and electrothermal constants, between the various modes of vibration of a given crystal plate, even when the vibration frequencies are within a few hundred cycles of each other.

To secure uniformity of results with respect to frequency stability it becomes necessary, therefore, to study the various possible modes of vibration of these crystal plates in detail, and set up certain criteria by which it will be possible to produce plates that will vibrate in a definite mode whose characteristics are known.

The theoretical aspects of this problem offer considerable difficulty, for it will be remembered that the classical case of the vibrations of an isotropic plate whose edges are free has as yet only been solved approximately,[3] and with the extension of the theory made necessary by the crystalline nature of quartz, the complexity of the problem is considerably increased, with the possibility of a complete solution very remote.

Using long rods or bars of crystal, instead of plates, other investi-

[2] Voigt's "Kristallphysik," pp. 749–755, or Love's "Mathematical Theory of Elasticity," Chap. VI.

[3] Rayleigh, "Theory of Sound," Chap. X and X_A.

gators[4] have been able to set up the three types of vibration (longitudinal, flexural, and torsional) common to isotropic bars. Moreover, the formulæ for these vibrations in isotropic material can be used to determine the frequency of the quartz rods to a good first approximation.

Returning to the problem of the plate, if the experimentally determined facts concerning plates of certain definite orientations are examined, it will be seen that they suggest the treatment of the plate as a special case of a bar. A résumé of these facts will illustrate this point and at the same time indicate the effect of orientation on the character of the modes of vibration.

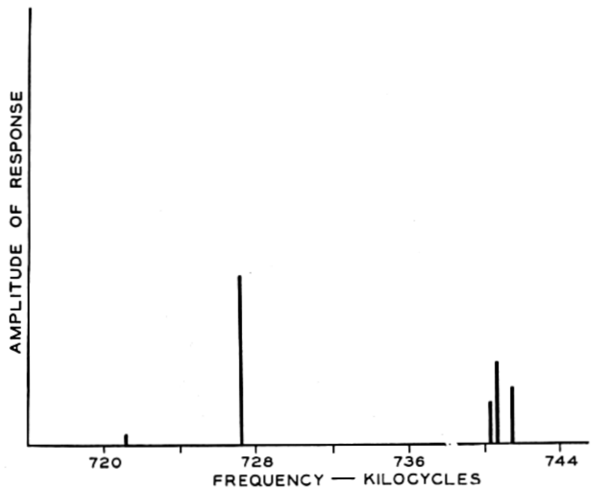

Fig. 1—Response frequencies of 32 x 47 x 2.760 mm. parallel cut crystal plate in the region of the major high frequency.

In general, a quartz crystal plate cut with any orientation with respect to the crystal axes will respond to a large number of frequencies. A plot of these frequencies showing their spacing and the relative magnitudes of response[5] may be termed the frequency spectrum of the plate. Fig. 1 shows part of the high-frequency region of such a spectrum. In these frequency spectra there are usually one or more frequencies at which the crystal will react with sufficient voltage to drive a vacuum tube in the usual crystal oscillator circuit.

[4] Cady, *Proc. I.R.E.* 10, p. 83, 1922. Harrison, *Proc. I.R.E.* 15, p. 1040, 1927 Giebe, *ZS. f. Phys.* 46, p. 607, 1928.

[5] The amplitude of response in this case is the maximum amplitude of current through the crystal at constant voltage, which in turn is a measure of the equivalent series resonant impedance of the crystal system.

The relation between these major response frequencies and the dimensions of the plate for the two principal orientations can be outlined as follows:

Curie or Perpendicular Cut

When the crystal plate is so cut that its major surfaces are parallel to the optic axis and perpendicular to an electric axis (the Curie or perpendicular cut, see Fig. 2) there are two major response frequencies, one high and one low.[6] The high frequency is a function of the

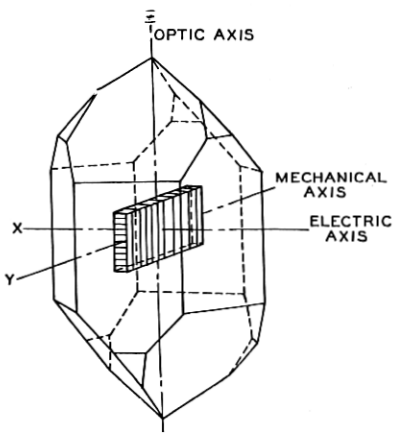

Fig. 2—Orientation of a perpendicular or Curie cut plate with respect to the crystal axes.

thickness of the plate and to a good approximation is given by the expression

$$f = \frac{K}{t}, \tag{1}$$

where t is the thickness in millimeters and $K = 2.860 \times 10^6$. If the plate could be considered as a bar of length t then the frequency of a simple longitudinal vibration would be given by the expression

$$f = \frac{1}{2t}\sqrt{\frac{E_{xy}}{d}}, \tag{2}$$

where E_{xy} is Young's modulus in the X-Y plane and d is the density. If the numerical values[7] of E_{xy} and d are substituted in the above

[6] For this discussion the low-frequency flexural vibration of the type described by Harrison will not be considered.

[7] For numerical values of the elastic constants and the density of quartz, see Sossman, "The Properties of Silica," the American Chemical Society Monograph Series.

expression it is found that the same value for K is obtained as that of equation (1).

The low frequency is a function of the width, the dimension parallel to the Y axis, and is given by the same expression as equation (1) with the same value of K, the width in millimeters being substituted for the thickness.

For this type of crystal plate there are then two possible major modes which appear to be of the longitudinal type and depend upon the same elastic constant. (Young's modulus in the X-Y or equatorial plane has the same magnitude in any direction.)

The temperature coefficient of both these frequencies is negative, which is in agreement with the temperature coefficient of Young's modulus for the equatorial plane.[8]

The Parallel or 30-degree Cut

When the crystal plate is so cut that its major surfaces are parallel to both the optic and electric axes (the parallel or 30-degree cut, see Fig. 3) this 30-degree shift in orientation from the perpendicular

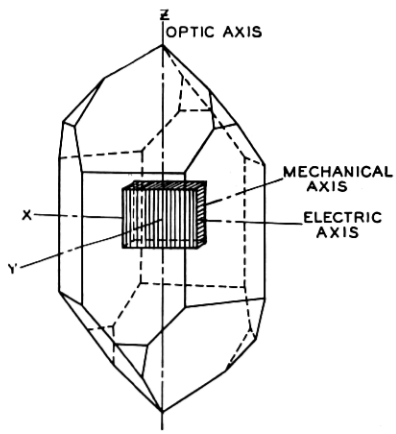

Fig. 3—Orientation of a parallel or 30-degree cut plate with respect to the crystal axes.

changes the characteristics in some important respects. As before there is a high and a low principal frequency, but in this case the high frequency sometimes occurs as a doublet (two response frequencies a kilocycle or so apart).

For thin plates of large area the high frequency is a function of the thickness of the plate and is given by the approximate expression

[8] Perrier & Mandrot, *Mem. Soc. Vaudoise Sci. Nat.* (1923), 1, pp. 333–364.

$$f = \frac{K}{t}, \tag{3}$$

where t is the thickness in millimeters and K is now 1.96×10^6. It will be noted that this constant differs from that found in the case of the perpendicular cut crystal. Moreover the temperature coefficient of this frequency is positive.

These facts lead one to believe that this is not a simple longitudinal vibration. Cady [9] has pointed out that if it be considered as a shear vibration in the X-Y plane the frequency can be calculated using the appropriate shear modulus.[10]

The low frequency is a function of the width, the dimension parallel to the electric or X axis, and is given by the same expression and constant as the frequencies of the perpendicular cut plate. It has the same characteristic negative temperature coefficient.

For these parallel cut plates there are then two possible major modes which, however, differ in type of vibration and sign of temperature coefficient.

Limiting this discussion to the high-frequency region, it is seen that these parallel and perpendicular cut plates have different frequency-thickness constants and temperature coefficients of opposite sign. On closer examination it is found that there is an additional difference which involves the variation of the magnitudes of these frequency-thickness constants and temperature coefficients with the ratio of width to thickness of the plate.

For the perpendicular cut plate the frequency-thickness constant changes but little with the size of the crystal. The same is true for the temperature coefficient, and from recent measurements on a number of sizes of plates the magnitude of this coefficient lies between minus 20 and minus 35 cycles in a million per degree centigrade.

The parallel cut plate, on the other hand, has a frequency-thickness constant which for any but thin plates of large area varies considerably with the width. The temperature coefficient also varies with the width, and is in addition a function of the temperature. This coefficient has a wide range of values whose limits are approximately plus 100 cycles in a million per degree centigrade and minus 20 cycles in some special instances, with all possible intermediate values including zero. Then, as has been mentioned before, these parallel cut

[9] Cady, *Phys. Rev.*, 29, p. 617, 1927.

[10] If it could be shown that the shear modulus of this plane had a positive temperature coefficient it would substantiate this assumption, but there is no information at present available regarding the effect of temperature on the elastic constants other than for the two values of Young's modulus.

crystals frequently have two high-frequency modes of vibration within a kilocycle or so of each other and will start on either of these modes if the circuit constants are changed slightly. These two modes usually have widely different characteristics.

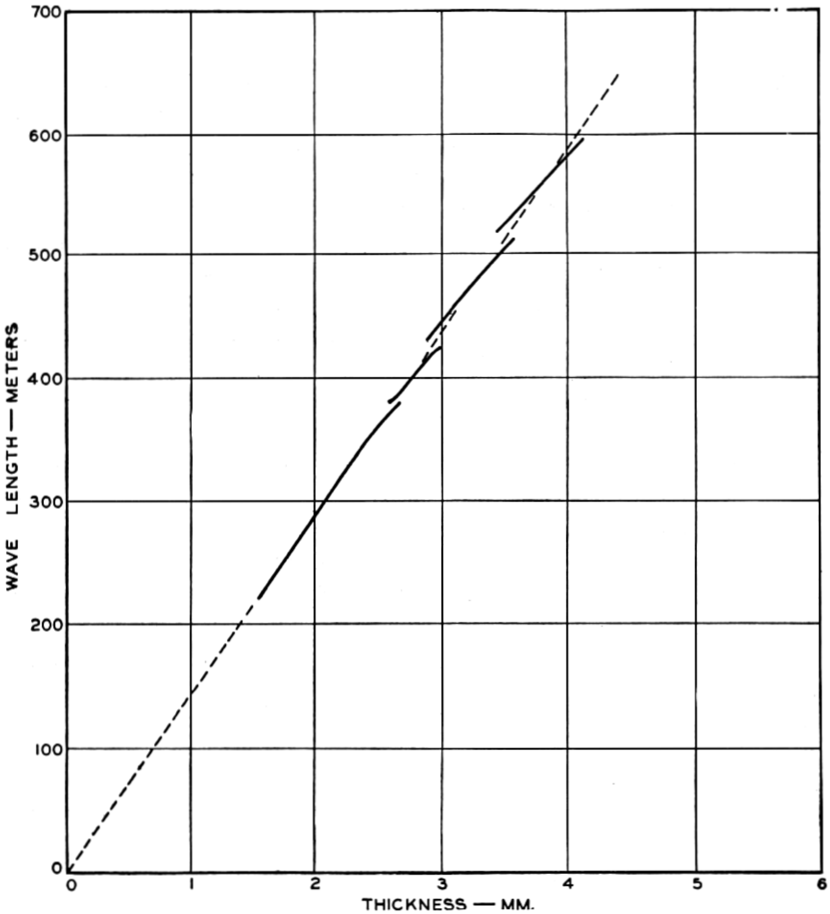

Fig. 4—The wave-length at which a 2.5 cm. square parallel cut crystal plate will operate in an oscillator circuit as its thickness is progressively reduced.

Apart from this seemingly erratic variation it has been the experience of this laboratory that the parallel cut crystal will oscillate more readily in the Pierce type of oscillator circuit. For this reason this type of crystal has been used for a number of purposes and these observed variations have been the object of considerable study.

As a result of this work an explanation has been evolved to account for these variations. This explanation not only suggests reasons for the above mentioned phenomena, but, what is more important, it indicates the procedure by which it is actually possible to produce crystals having negligible temperature coefficients. Before outlining this theory the experimental facts which served as its foundation will be discussed in detail.

Frequency-Thickness Constant as a Function of Dimensions

When work on the production of parallel cut crystals in the broadcast frequency band was first started, it was found that it was very difficult to grind crystals for certain low frequencies using a 2.5 cm. square plate because of discrete jumps in frequency for a small reduction of thickness. Fig. 4 is a typical curve showing the wavelength [11] as a function of the thickness for a 2.5 cm. square crystal. This curve should be a straight line (for from equation (3) it is evident that $\lambda = K't$) but it will be noted that there are certain discontinuities

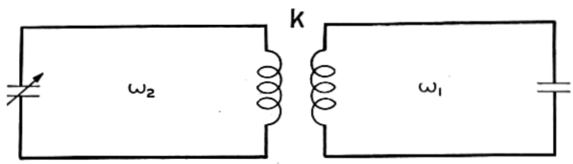

Fig. 5.

at the upper end. It was found that these discontinuities were present at frequencies that could be identified with harmonics of the frequency the crystal would have if it were vibrating in the direction of its length along the electric axis.

This was the first definite indication obtained in the Bell Telephone Laboratories that the longitudinal vibration of the crystal in the direction transverse to the applied field could affect the frequency supposed to depend only on the thickness. It was checked by further work on crystals of other dimensions, and in each case the position of these discontinuities was found to depend on the width of the crystal.

The presence of a resonant system whose frequency depends upon the width is evidently responsible for this phenomena, this system affecting the frequency of the vibration along the thickness through some form of mechanical coupling. At the suggestion of Mr. R. A. Heising of the Bell Telephone Laboratories, an explanation of these experimental facts was developed based on the treatment of the plate

[11] In plotting the change in rate of vibration of a crystal plate as a function of the dimension, it is more convenient to use wave-length instead of frequency because of the direct linear relation between the dimensions and the wave-length.

as a system of coupled circuits.[12] Consider the two coupled oscillatory circuits shown in Fig. 5 having the uncoupled angular frequencies ω_1 and ω_2, then the frequencies of the coupled system in the absence of damping will be given by the usual expression [13]

$$\omega = \frac{\sqrt{\frac{1}{2}(\omega_1^2 + \omega_2^2) \pm \frac{1}{2}\sqrt{(\omega_1^2 - \omega_2^2) + 4k^2\omega_1^2\omega_2^2}}}{\sqrt{1 - k^2}}, \tag{4}$$

k being the coupling.

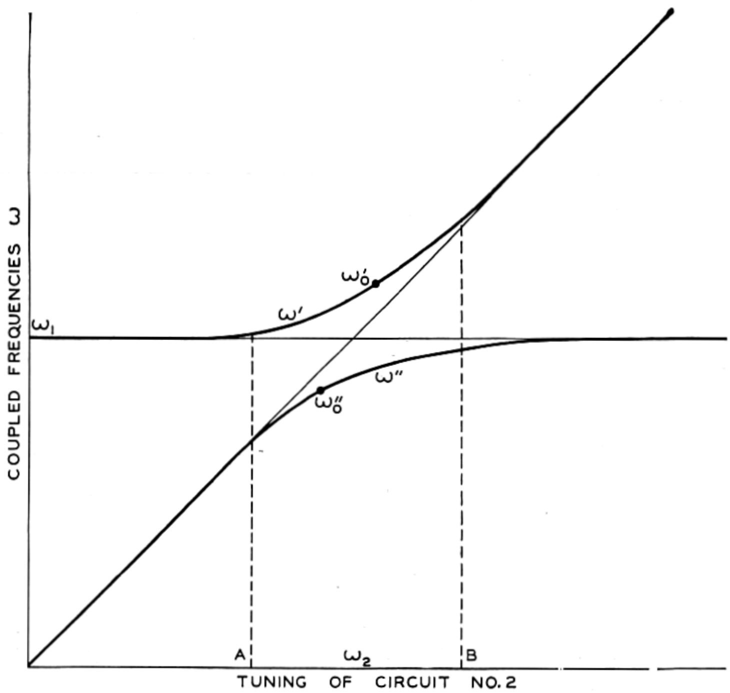

Fig. 6—Angular frequencies of a system of two coupled circuits as a function of the tuning of one circuit, the tuning of the other circuit being fixed.

If these two frequencies be plotted as a function of the tuning of the second circuit, i.e., ω_2, the familiar set of curves shown in Fig. 6 results.

Suppose now other circuits are added to the system as shown in Fig. 7, each additional circuit being fixed at a harmonic of the uncoupled frequency of circuit No. 2, and so linked with this circuit mechanically that the group is tuned as a whole.

[12] The term "circuit" is introduced here to describe a mechanical oscillatory system because many readers are accustomed to think in terms of electrical circuits.
[13] See Pierce, "Elec. Osc. & Waves," Chap. VII.

There are now two possible combinations depending upon which circuit or group of circuits is kept fixed while the other is varied. If the case in which the second group of circuits is kept fixed be examined first, it will be seen that as the frequency of circuit No. 1 is varied it will come into tune successively with each of the circuits of the second group. The result will be a series of coupling curves with the characteristic reaction illustrated by Fig. 6 repeated at each coincident point. If it be assumed that the coupling decreases as the order of the harmonic increases, then the magnitude of the reaction also decreases. This is illustrated by Fig. 8, which shows the coupling curves of such a system plotted in terms of the equivalent electrical wave-length.

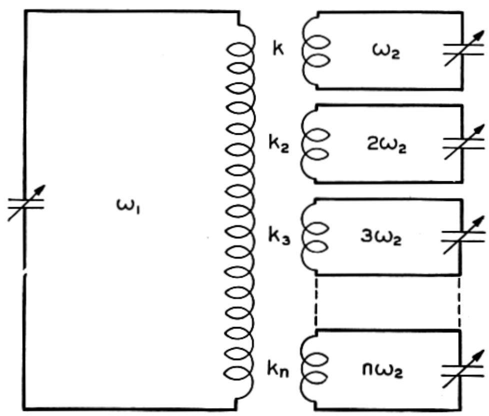

Fig. 7.

Returning to the crystal plate, if the vibration in the direction of the thickness be identified with circuit No. 1 while the width vibration and its harmonics be identified with circuit group No. 2, then Fig. 8 should represent what happens to the crystal wave-length as the thickness is reduced. Comparing Figs. 4 and 8, it is seen that this is true in a restricted region but that the wave-lengths which depend upon the width vibration do not continue much beyond the coupling region in the experimental curves. This is to be expected, for these wave-lengths which depend upon a harmonic of a vibration transverse to the applied field are more difficult to excite than the fundamental in the direction of the field. This particular point is discussed further in connection with temperature coefficients.

If now the case be examined in which the tuning of the second group of circuits is varied, it will be seen that the coupling curves are slightly different in character. The curves for this case are illustrated by Fig. 9 which shows the wave-lengths of this system as a function of the tuning of circuit group No. 2. Fig. 10 shows the wave-lengths at which a parallel cut plate will oscillate plotted as a function of the width. The similarity between this experimentally

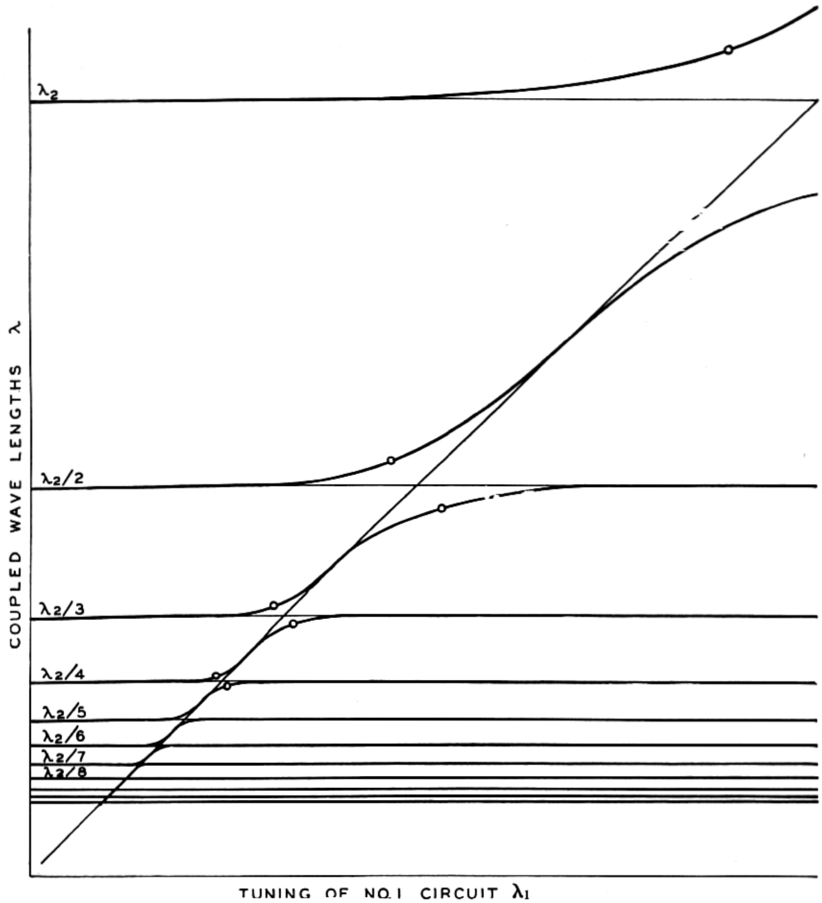

Fig. 8—Wave-lengths of the system of coupled circuits of Fig. 7 as a function of the tuning of circuit No. 1, the tuning of circuit group No. 2 being fixed.

determined curve and Fig. 9 is at once apparent. There is one anomalous segment of a curve between the 7th and 8th harmonics, the line AB; but it is possible that this is caused by the coupling of some third free period which has not been considered, perhaps a high order harmonic of a flexural vibration. In general, however, the curves of wave-length versus thickness for these crystal plates are of such a character as to indicate that the analogy between the two systems of coupled circuits and the crystal modes of vibration is sufficiently good to serve as a useful guide.

If, then, these parallel cut crystal plates are considered as a system of coupled circuits the reason for the variation of the frequency-thickness constant with dimensional ratios and the presence of the frequency doublets is at once apparent. With the coupling at the various harmonics determined, the character of these variations can

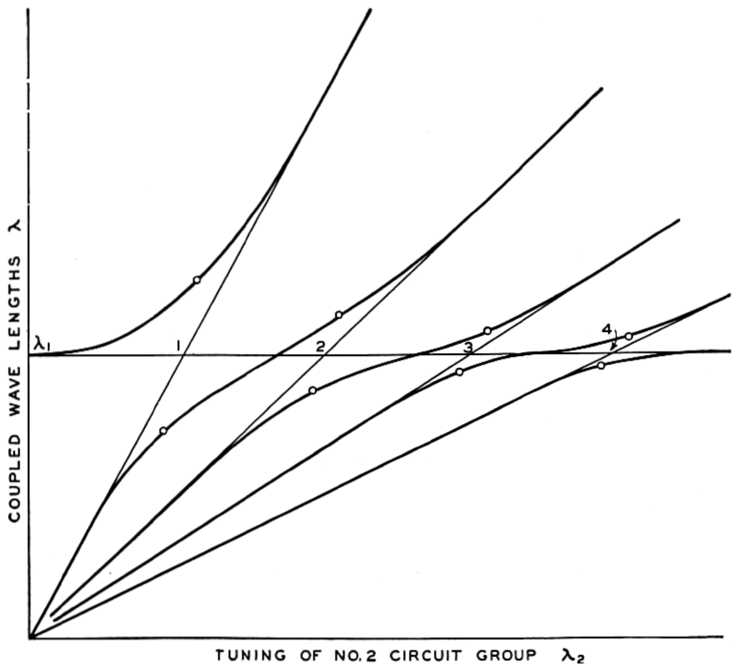

Fig. 9—Wave-lengths of the system of coupled circuits of Fig. 7 as a function of the tuning of circuit group No. 2, the tuning of circuit No. 1 being fixed.

be predicted. Given an experimentally determined series of coupling curves similar to Fig. 10, the coupling at the nth harmonic can be determined from the expression

$$k_n = \frac{\left(\frac{\lambda'}{\lambda''}\right)^2 - 1}{\left(\frac{\lambda'}{\lambda''}\right)^2 + 1}, \tag{5}$$

where λ' and λ'' are the wave-lengths of the coupled system at the point where $\lambda_1 = n\lambda_2$.

Temperature Coefficient as Function of Dimensions and Temperature

As mentioned above, when the temperature coefficients of these parallel cut crystals were studied it was found that there was considerable variation between plates having the same thickness but slightly different areas, and the temperature coefficient of a given plate was found to be a function of the temperature. To illustrate this last point a typical frequency-temperature curve for a parallel

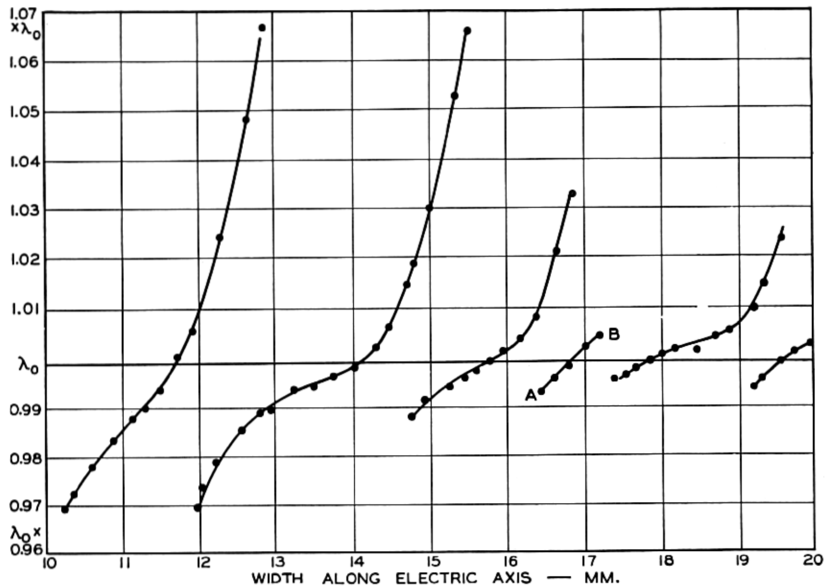

Fig. 10—The wave-lengths at which a parallel cut crystal will operate in an oscillator circuit as its width (the dimension along the electric axis) is progressively reduced, the other dimensions being fixed.

$\lambda_0 = 153 \times$ thickness.

Thickness along mechanical axis = 1.64 mm.

Length along optic axis = 54.8 mm.

cut crystal is shown in Fig. 11. It will be noted that the frequency increase is linear until a given temperature is reached, at which point the curve flattens off and then begins to reverse. Just beyond the point of reversal the frequency jumps to a new value and, if the curve is continued, the frequency increases again at the same rate as originally. This type of frequency-temperature curve is common to a large percentage of parallel cut crystals, the only difference being the width of the flat part of the curve and the temperature at which the discontinuity occurs.

Mr. W. A. Marrison of the Bell Telephone Laboratories first suggested that low temperature coefficients could be obtained with parallel cut crystals by utilizing the coupling of two modes of vibration having individual coefficients of opposite sign. Several of this type of low temperature coefficient crystals were produced by Marrison

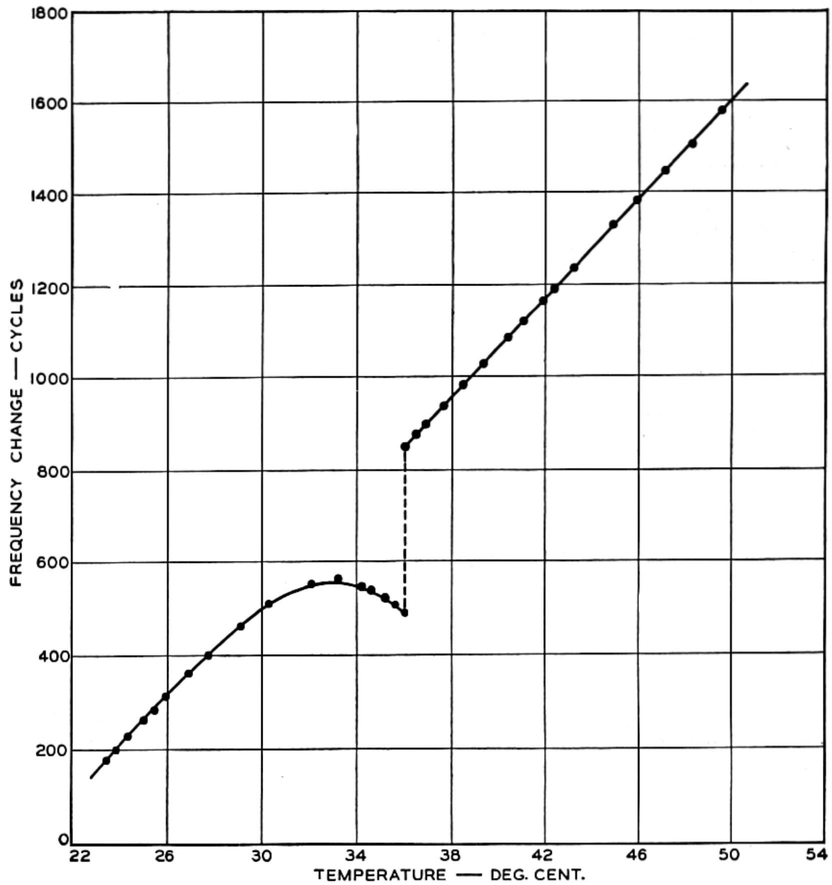

Fig. 11—The frequency change of a 32 x 47 x 2.760 mm. parallel cut crystal with temperature.

and are described in his concurrent paper "A High Precision Standard of Frequency."

If Heising's coupled circuit analysis is extended to include the effect of temperature and the proper temperature coefficients with due regard to relative magnitude and sign are identified with each circuit, the change in temperature coefficient with dimensional ratio and temperature can be explained. In addition, the dimensional ratios or tuning points which yield zero temperature coefficients for a given temperature can be predicted if the coupling is known.

Referring again to Fig. 6, suppose the two coupled circuits have temperature coefficients of opposite sign, circuit No. 1 being positive and circuit No. 2 negative, for ω_2 less than ω_1 say at the point A, ω' has a positive and ω'' a negative temperature coefficient. For a value of ω_2 greater than ω_1 say at B, ω' now has a negative and ω'' a positive temperature coefficient, ω' and ω'' having interchanged rôles. Somewhere between therefore, both ω' and ω'' must have had a zero temperature coefficient. Returning to equation (4), if this expression for ω be differentiated with respect to the temperature, regarding k, the coupling as constant, and the result placed equal to zero, the condition that ω is independent of temperature [15] is obtained as follows:

$$\omega^2 = \frac{\omega_1^2\omega_2^2(m - n)}{(m\omega_1^2 - n\omega_2^2)}, \tag{6}$$

where

$$m = \frac{1}{\omega_1}\frac{d\omega_1}{dT} = \text{temperature coefficient of circuit No. 1},$$

$$n = -\frac{1}{\omega_2}\frac{d\omega_2}{dT} = \text{temperature coefficient of circuit No. 2};$$

now let $Q = n/m$ then equation (6) becomes

$$\omega^2 = \omega_2^2\frac{1 - Q}{1 - Q\left(\dfrac{\omega_2}{\omega_1}\right)^2}, \tag{7}$$

solving equation (7) for ω_2/ω_1 replacing ω^2 by its value from equation (4)

$$\left(\frac{\omega_2}{\omega_1}\right)^2 = \frac{k^2(1 - Q)^2}{2Q} + 1 \pm \sqrt{\left[\frac{k^2(1 - Q)^2}{2Q}\right]^2 + \frac{k^2(1 - Q)^2}{Q}},$$

which when k is small becomes

$$\left(\frac{\omega_2}{\omega_1}\right)^2 = 1 \pm \frac{k(1 - Q)}{\sqrt{Q}}. \tag{8}$$

This equation gives the tuning points, or the values of ω_2 at which the angular frequencies of the coupled system, ω' and ω'', will have

[15] Dr. F. B. Llewellyn of the Bell Telephone Laboratories is responsible for this analysis.

zero temperature coefficients, in terms of the ratios of the uncoupled temperature coefficients and the coupling.

Referring again to Fig. 6, ω_0' and ω_0'' represent the values of ω' and ω'' which would have zero temperature coefficient provided m is greater than n, that is, the temperature coefficient of ω_1 is greater in magnitude than that of ω_2. Carrying this idea over to the case of the group of circuits for which the curves of Figs. 8 and 9 are drawn,

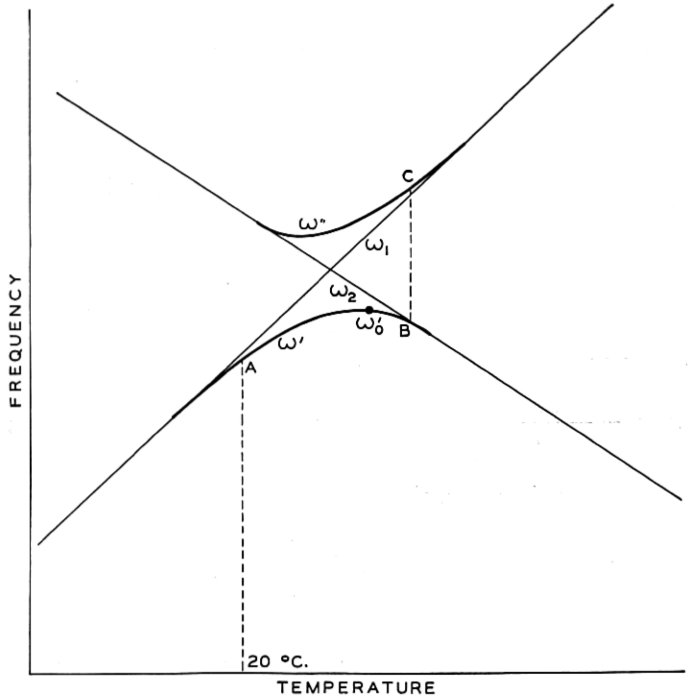

Fig. 12—Effect of temperature on the angular frequencies of a system of two coupled circuits having temperature coefficients of opposite sign.

there will be points of zero coefficient in the neighborhood of each coupling point as indicated by the circles on the curves.

The above conditions for zero temperature coefficient only apply if the coefficient as a function of the tuning be examined in the region of some given temperature. If the temperature is varied over a considerable range a small change in the tuning of both circuits is effected, one having its frequency raised, the other lowered. The result of this tuning on the frequencies of the coupled system can be illustrated by Fig. 12, which shows the tuning with temperature on a magnified scale. In this figure, the lines ω_1 and ω_2 represent the change in frequency of these circuits with temperature if there were no coupling between them. The change of the frequencies of the coupled system with temperature is shown by the curves ω' and ω''. It will be seen that both these frequencies pass through regions of zero temperature coefficient.

Such frequency-temperature curves can be derived graphically by a construction similar to that shown in Fig. 13. This figure illustrates what happens when the tuning of both circuits No. 1 and No. 2 is varied, and consists of a series of the usual coupling curves (the

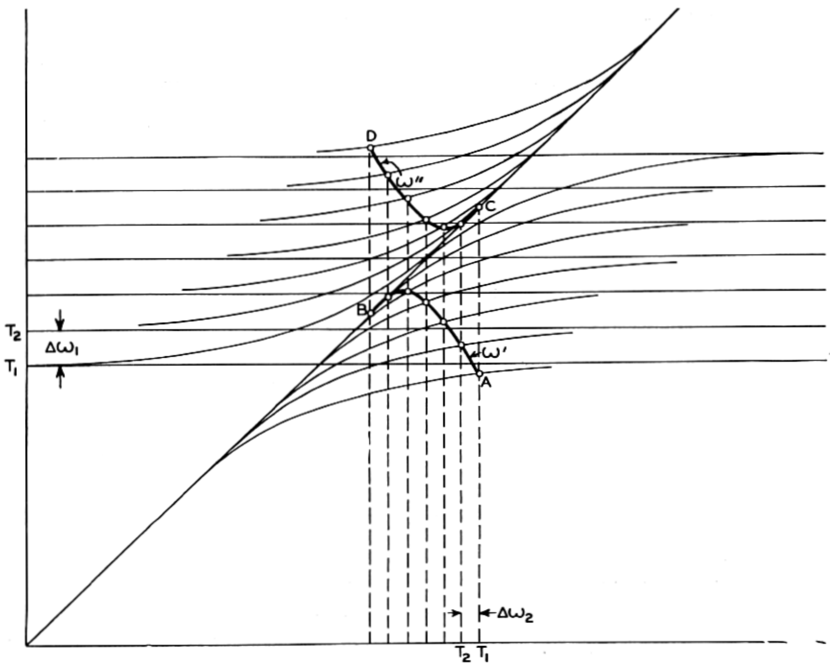

Fig. 13—Effect on the angular frequencies of a system of two coupled circuits as the individual circuits are tuned simultaneously in opposite directions.

coupled frequencies plotted as a function of ω_2), each set of curves of the series being drawn for a different value of ω_1. When the temperature is increased from T_1 to T_2, the uncoupled frequency of circuit No. 2 is reduced by an amount $\Delta\omega_2$ and the uncoupled frequency of circuit No. 1 is increased by an amount $\Delta\omega_1$. The result is the frequencies ω' and ω'' move from curve to curve in the direction shown by the lines *AB* and *CD*.

Now if the variation of the temperature coefficient of a crystal plate when used in an oscillator circuit be examined at a given temperature as the width is changed (which amounts to a change in the tuning of the transverse vibration), it will be seen that the experimental results are in accord with the above treatment. Fig. 14 shows the temperature coefficient of the two frequencies of a crystal plate at 58° C. as its width is progressively reduced in the neighborhood of the 5th harmonic of the transverse vibration. These curves show how the temperature coefficients change sign in this region. The dotted sections of the curves are extrapolated, for owing to the rapid reduction in activity once a coupled frequency acquires a negative coefficient,

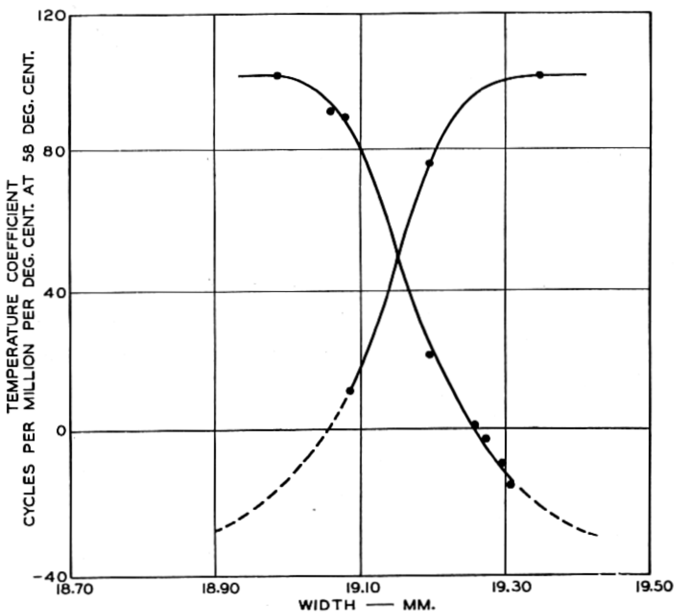

Fig. 14—The change of temperature coefficient of a parallel cut crystal at 58° C. as the width is progressively reduced in the region where the fifth harmonic of the vibration in the direction of the width coincides with the frequency of the vibration in the direction of the thickness.

data on the crystal plate used as an oscillator are difficult to obtain in this region.

Returning to the experimentally determined curve of frequency versus temperature for a parallel cut crystal plate shown in Fig. 11, this can also be explained with the aid of the above analysis. Referring to Fig. 12, if it be assumed that at 20° C. the crystal is oscillating with a frequency A, this is in the region where this particular frequency has a positive temperature coefficient. As the temperature increases the frequency increases in the direction of B, passing through a maximum at the point ω_0' where it has zero coefficient, and then decreases. As the frequency decreases the activity of this particular mode decreases rapidly and finally the crystal "hops" frequency to point C on the ω'' curve. From this point on the frequency with

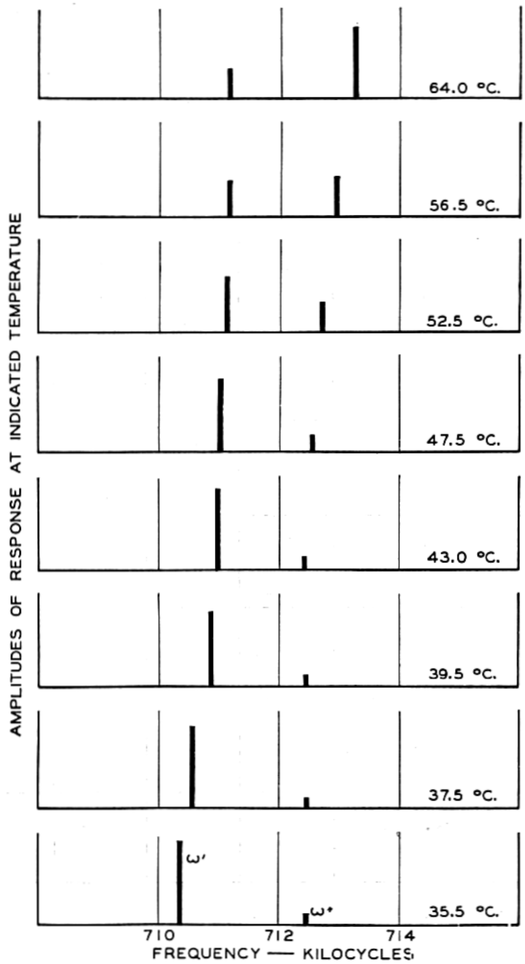

Fig. 15—The response-frequency spectra of a parallel cut crystal plate at different temperatures illustrating the interchange of activity between the two frequencies as the frequencies of the two modes of vibration pass through a coincident value.
Length of plate along optic (Z) axis = 47 mm.
Width along electric (X) axis = 19.35 mm.
Thickness along mechanical (Y) axis = 2.75 mm.

temperature increases, for this frequency has a positive temperature coefficient in this region.

If it were not for the decrease in activity of the period with the negative temperature coefficient it is to be expected that the crystal frequency, instead of "hopping," would continue to decrease with increase in temperature. In some instances (for low order of harmonics) the crystal frequency will decrease for a few degrees, and it

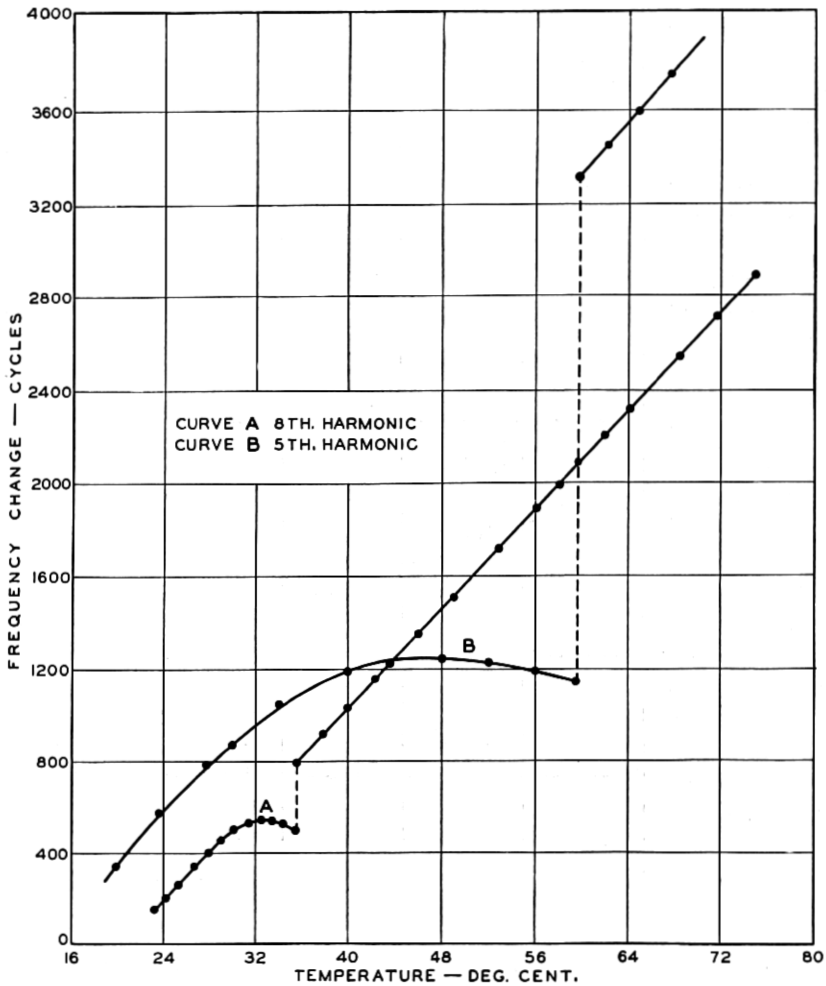

Fig. 16—The frequency change with temperature of two parallel cut crystals of different width.

Curve *A* region of eighth harmonic
(width = 32.0 mm.)
Curve *B* region of fifth harmonic
(width = 19.35 mm.)

The other dimensions of the plates are identical.

is found that the magnitude of the negative coefficient for this region approximates that to be expected for the transverse vibration alone. In general, however, a frequency jump occurs just after the zero temperature coefficient region is passed.

This interchange of activity of these two periods as they interchange temperature coefficients can be studied in detail by examining the changes in the spectrum of a crystal at different temperature levels. Fig. 15 shows a series of spectra of a crystal taken for different temperatures in the region of zero temperature coefficient, the dimensions of the crystal being unchanged. These spectra illustrate the rapid decrease in activity of the frequency ω' after it passes through zero temperature coefficient while at the same time ω'' increases and assumes the place of major activity vacated by ω'.

The assumption that the coupling increases with the decrease in the order of the harmonic finds confirmation in the experimentally determined facts as computed from curves of the type shown by Fig. 10. As the coupling increases the temperature range for which there is no frequency change with temperature increases, that is the region of zero temperature coefficient becomes extended. To illustrate this, Fig. 16 shows two curves of frequency versus temperature, one for the coupling of a fifth harmonic, the other for an eighth.

It would, of course, be desirable to extend the zero temperature coefficient range over the limits of temperature to be expected in normal operation. This necessitates tight coupling of the two modes which in turn demands a dimensional ratio in the neighborhood of unity. The cross sectional area of such a plate in the direction of its thickness and width approaches a square in shape which, for high-frequency crystals, is of very small dimensions.

Before concluding it should be noticed that since both modes of the perpendicular cut crystals have a negative temperature coefficient, it is to be expected that it would be impossible to obtain zero temperature coefficient crystals with this orientation. This seems to be true as far as our experience with those crystals is concerned.