A Braun Tube Hysteresigraph

By J. B. JOHNSON

In this paper apparatus for observing hysteresis loops of magnetic materials is described. It combines a cathode ray oscillograph with a vacuum tube amplifier and an electrical integrating circuit consisting of condenser and resistance. The device describes the B–H curve for alternating magnetization in the frequency range of five to perhaps several thousand periods per second. The specimens may be either long strips or closed rings. Alternating flux as low as one maxwell may be readily observed.

The operation of the apparatus is analyzed so as to account for the effects of finite time constants of the amplifier and integrator, of conductance in the condensers, of demagnetization by current in the search coil and by the stray fields of coils and specimen, and of eddy currents in the specimen.

THE use of the cathode ray oscillograph for delineating magnetic hysteresis curves has proved a convenience in a number of studies of magnetic phenomena.[1] The essential advantage of the method lies in the speed with which the hysteresis loop is traced. Complete curves are drawn in rapid succession by the oscillograph tube, and any change in these curves resulting from altered mechanical or magnetic conditions of the specimen can immediately be observed and recorded. Furthermore, the area bounded by these curves represents the total energy loss in the specimen corresponding to the particular kind of magnetic cycle that is used, and not the hysteresis loss alone as is the case in the curves derived by the slower point-by-point methods.

In the present article is described an apparatus combining a cathode ray oscillograph with an electric circuit, for magnetic measurements. A fairly extensive analysis of the operation of the device is presented in order to show how accuracy may be maintained in the measurements and the probable errors estimated.

The apparatus has been in use since 1924, particularly for observing the magnetic properties of various alloys. It is so designed that by suitably choosing the circuit constants it can be used for obtaining the hysteresis curves of magnetic materials from the saturation value down to fields where the amplitude of alternating flux is about one maxwell. The purposes for which the apparatus has been employed are illustrated by the hysteresis curves reproduced in Plates I and II.

---

[1] K. Ångström, *Phys. Zeits.*, 1, p. 121, 1899; *Phys. Rev.*, 10, p. 74, 1900.
E. Madelung, *Ann. d. Phys.*, 17, p. 861, 1905; *Phys. Zeits.*, 8, p. 72, 1907.
C. W. Waggoner and F. A. Molby, *Phys. Rev.*, 17, p. 427, 1921.
Y. Niwa, J. Matura, and J. Sugiura, "Researches of the Electrotechnical Laboratory (Ministry of Commerce, Tokyo)," No. 144, May, 1924.

The hysteresis curves of Plate I (1) are those of a ribbon of Armco iron. The curves were made with 25 cycle magnetization and at four different magnetizing fields, the highest field being sufficient to saturate the sample. The effects of frequency and of permeability on the eddy current loss in materials are shown in the next two figures of this plate. Two similar ribbons were used. The one was permalloy magnetized to saturation, Plate I (2); the other, Plate I (3), was Armco iron at a low magnetizing field. The curves were made at three different frequencies, those marked with the letters *a*, *b* and *c* corresponding to magnetization at 25, 200 and 1,000 cycles, respectively. A comparison of the magnetic properties of pure iron and permalloy is made in the two figures, (4) and (5). In each figure the letter *a* indicates the curve for iron, *b* that for permalloy. The former figure was made at a field strength high enough to saturate the iron, the latter at a field strength which saturated the permalloy but not the iron. The last figure of this plate, (6), shows the influence of tension on the magnetic properties of a ribbon of permalloy containing 65 per cent nickel. The curve *a* was obtained without tension, *b* with a moderate tension on the ribbon. The curves of Plate II reproduce the magnetization cycles of a sample of Armco iron at various temperatures, from room temperature to the recalescence point. At the temperature of about 790° C., ferromagnetic properties were gone and the only flux that is indicated existed in the uncompensated air space of the search coil.

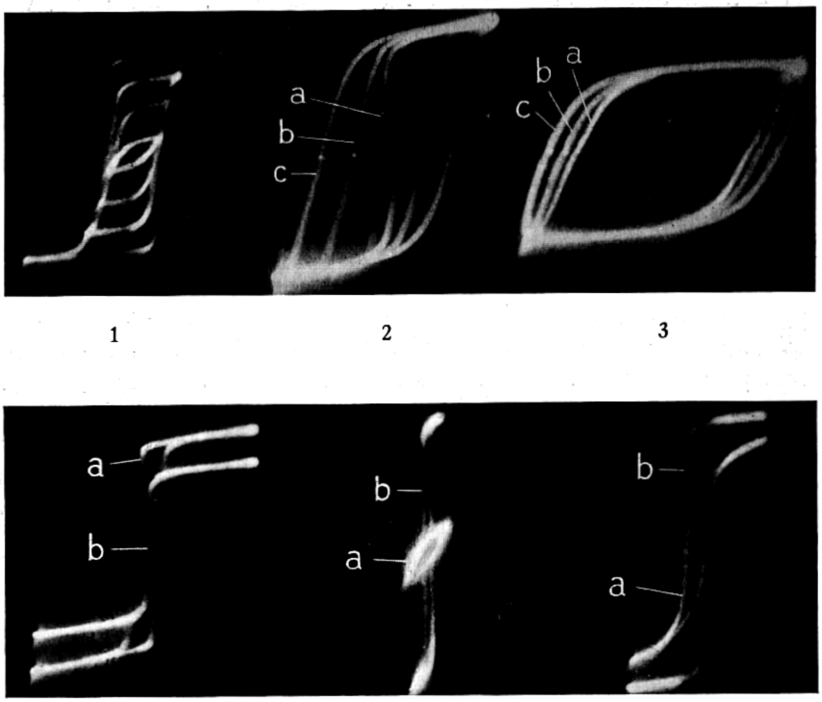
I. Description of Method

A change in magnetic flux in a specimen is usually measured either as a change in the field in a relatively short air gap, or as the time integral of the potential set up in a search-coil surrounding the specimen. The second method is illustrated by the use of the ballistic galvanometer as an integrating instrument, employed in most magnetic measurements. In the present case, however, the integration is accomplished by a purely electrical circuit. The integrating element consists of a resistance and condenser in series with the search coil surrounding the sample of material.[2]

---

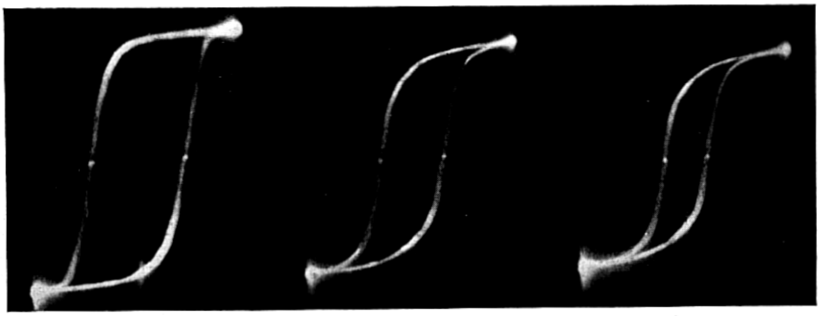
[2] This type of integrating circuit is one of at least ten simple combinations of resistance, capacity and inductance which can be used for obtaining the cyclic integral of current or potential. Some of these have been described in connection with hysteresis measurements by E. L. Bowles, *Jl. A. I. E. E.*, 42, p. 849, 1923; O. E. Charlton and J. E. Jackson, *Jl. A. I. E. E.*, 44, p. 1220, 1925; W. Kaufmann, *Zeits. f. Phys.*, 5, p. 316, 1921. W. Kaufmann and E. Pokar, *Phys. Zeits.*, 26, p. 597, 1925. K. Krüger and H. Plendl, *Zeits. f. Hochfr.*, 27, pp. 155–161, 1926; W. Neumann, *Zeits. f. Phys.*, 51, p. 355, 1928. The circuit chosen here was used by Bowles and by Charlton and Jackson in connection with a mechanical oscillograph, and by Krüger and Plendl in connection with a Braun tube.

Plate I. Hysteresis curves of iron and permalloy under various conditions.

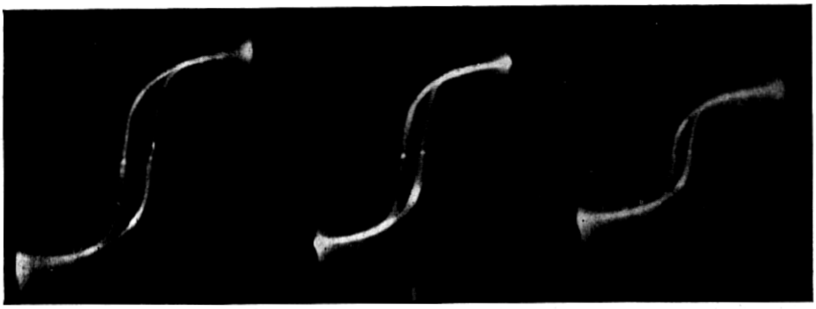


1. Armco iron at various fields.
2. Permalloy at (*a*) 25~, (*b*) 200~, (*c*) 1,000~; high field.
3. Armco iron at (*a*) 25~, (*b*) 200~, (*c*) 1,000~; low field.
4. Armco iron (*a*) and 78 per cent permalloy (*b*); high field.
5. Armco iron (*a*) and 78 per cent permalloy (*b*); low field.
6. 65 per cent permalloy, (*a*) without tension, (*b*) with tension.

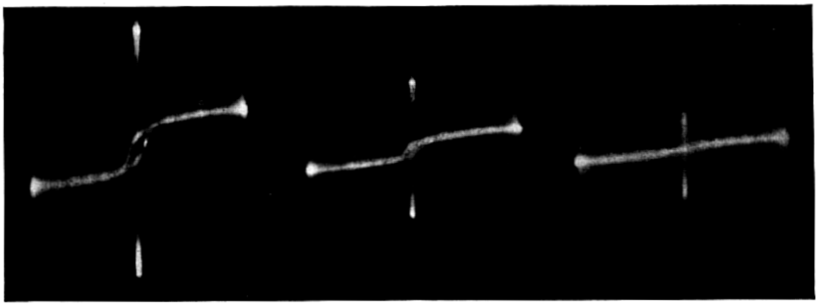
Plate II. Magnetic cycle of Armco iron at various temperatures.



a. 20° C. *b.* 380° C. *c.* 555° C.



d. 690° C. *e.* 733° C. *f.* 757° C.



g. 775° C. *h.* 788° C. *i.* 795° C.

The principle of the arrangement whereby the cyclic magnetization curve is recorded on the oscillograph tube, will be made clear by reference to Fig. 1. The specimen of magnetic material M, is magnetized by alternating current flowing through the primary winding P. The varying magnetic flux in the specimen induces in the secondary winding or search coil T a voltage which is proportional to the *rate of change of flux*. This voltage is applied to the integrating circuit consisting of the resistance R and the condenser C. When the resistance is large compared with the impedance of the condenser, the current in this circuit is limited largely by the resistance and it is, therefore, proportional to the voltage applied by the search coil. The charge on the condenser, and therefore the voltage across its terminals, is proportional to the time integral of the current in the circuit. The *voltage of the condenser* is, therefore, proportional to the *magnetic flux in the sample.*

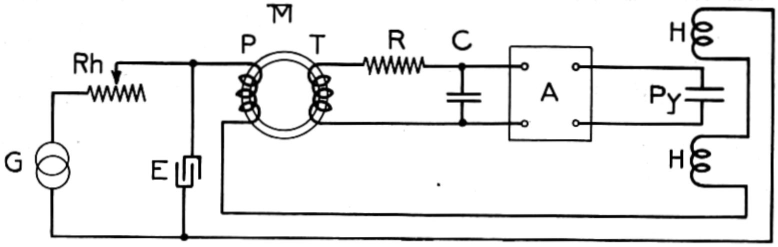


Fig. 1—Elementary diagram of the circuit.

This voltage is amplified by the distortionless amplifier A, the output side of which is connected to one pair of deflector plates of the oscillograph tube. While the deflection of the indicating spot thus follows the *flux* in one direction, deflection in a line at right angles to this and proportional to the *magnetizing field* is produced by the magnetic field of the deflector coils H which are connected in series with the magnetizing winding P. The spot then traces out a path on the screen during each cycle of current which is the hysteresis diagram for the sample, and which by suitable calibration yields quantitative results.

The voltage on the integrating condenser at any time is given by the relation

$$e = 10^{-8} \int_{-\infty}^{t} \frac{NS}{RC} \frac{dB}{dt}\, dt = \frac{NS}{RC} B \times 10^{-8} \text{ volts,} \qquad (1)$$

where R and C are the resistance and capacity of the integrator, N is the number of turns in the search coil, S is the cross-sectional area of the sample and B the flux density. The voltage e is amplified and applied to the deflector plates of the oscillograph. This part of the apparatus is calibrated in terms of a small alternating voltage of known amplitude applied to the amplifier, which produces a deflection of d cm. per volt on the oscillograph tube. An ordinate b on a hysteresis curve therefore indicates an induction of

$$B = \frac{b}{d}\frac{RC}{NS} \times 10^8 \text{ gauss.} \tag{2}$$

The calibration for magnetizing field is done by measuring the deflection produced by a known direct current passed through the coils H, and the magnetizing field in the coil P is then calculated from the dimensions of the coil in the usual way in terms of the current indicated by the oscillograph.

The greatest difference between the actual circuits used and the simple one shown in Fig. 1 is that the amplifier is constructed with the push-pull arrangement in order to reduce distortion. This makes necessary the maintenance of symmetry on the two sides of the circuit so that the apparatus is really two similar circuits in parallel as shown in Fig. 2.

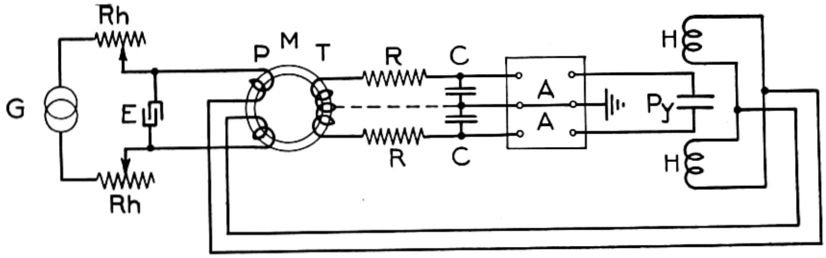


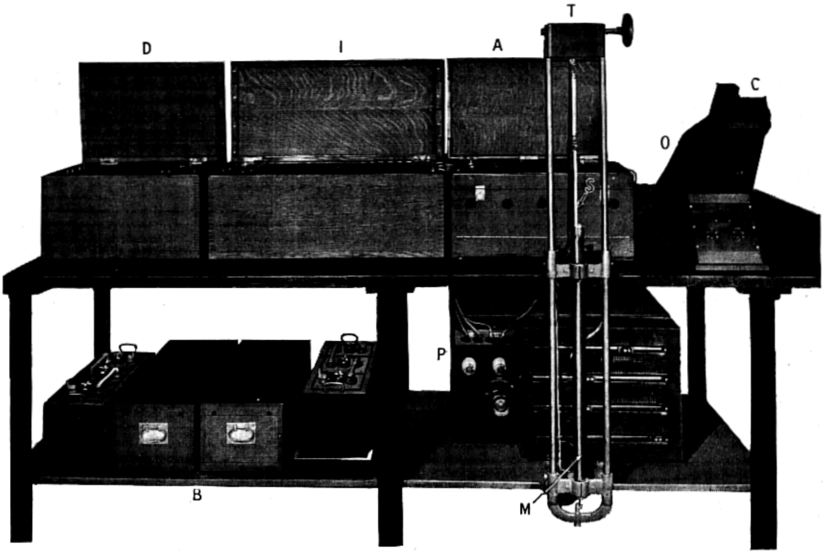
Fig. 2—Diagram of the symmetrical circuit.

The fact that the integrator and amplifier are thus connected makes no difference in the final results of the calculations, if for N is used the total number of turns on the search coil and for the other constants only the values on one side of the system.

II. DESCRIPTION OF THE APPARATUS

For greater flexibility the apparatus has been made in a number of separate units which will now be described. A photograph of the complete assembly is reproduced in Plate III.

PLATE III. Apparatus assembly.



A—Amplifier
B—Batteries
C—Camera
D—Calibration Set
I—Integrator
M—Magnetizing Coil
O—Oscillograph Tube Box
P—Power Unit
S—Sample
T—Tension Rack

1. THE INTEGRATOR.

The integrator unit contains two banks of paper condensers of 100 mf. each, variable in steps of 1 mf., 3 mf., 6 mf., and 9 steps of 10 mf. The two resistances are each variable by factors of about two from 12,000 ohms to 1,000,000 ohms. These combinations are thought to cover any requirements that are likely to arise.

2. THE AMPLIFIER.

Four stages of Western Electric 102-D tubes on each side, resistance-capacity coupled, make up the amplifier. The time constant of each coupling unit (stopping condenser and grid leak) is 8 seconds, which is sufficient for frequencies down to 10 p.p.s. or less. The amplifier is suitably shielded against electromagnetic, mechanical and acoustic shocks.

3. THE OSCILLOGRAPH.

A box contains the cathode ray oscillograph tube[3] and its controls, except the batteries. The coils *H* are mounted on a hard rubber holder which can be clamped securely to the tube. A camera attachment contains a Dallmeyer F. 1.9 lens with which photographs of the pattern can be made on plates or film pack.

The oscillograph tube is connected to the amplifier unsymmetrically since one side of the amplifier leads to the common deflector plates and the batteries of the oscillograph tube, the other side only to a deflector plate. The oscillograph and its batteries must, therefore, be so placed that the conductance and capacity to ground and to the rest of the apparatus are small. When this is done no distortion results from the lack of symmetry.

4. THE CALIBRATION SET

The calibration box contains the apparatus for applying a small sinusoidal voltage of known amplitude to the amplifier and oscillograph. The set takes current from the same source that supplies the magnetizing current. A low-pass filter admits only current of the fundamental frequency, and a thermocouple provides for measuring the output current of the filter. This current passes through a potentiometer from which the calibrating voltage is applied to the amplifier. The potentiometer is variable in several steps, each reducing the current amplitude by a factor of about two.

---

[3] Western Electric 224-B tube. J. B. Johnson, *J. O. S. A.* and *R. S. I.*, 4, p. 701, 1922.

5. THE POWER UNIT.

The alternating magnetizing current is derived from a low speed dynamotor. The machine operates on 24 volt storage battery power, and delivers at low load, nearly sinusoidal current of 24 volts *amplitude*. Resistance in the field circuit permits regulation of the frequency between 10 and 30 cycles, while rheostats in each side of the output of the machine serve to regulate the current for the magnetizing coil.

Mounted in the power unit and connected directly across its output terminals there is an electrolytic condenser of about 1,200 mf. (*E* of Figs. 1 and 2). This condenser serves three important functions: *a*. It smoothes out ripples in the magnetizing current which would, if they were large enough, produce secondary loops in the hysteresis curve. *b*. It makes the power unit a source of potential having low impedance to a.c. so that the magnetizing current is in part determined at any moment by the counter e.m.f. of the magnetizing coil, rather than wholly by external impedances. This being so, the magnetizing current is retarded in the steep parts of the hysteresis curve of the sample where the counter e.m.f. is great. Eddy currents do not, therefore, build up in the sample nearly so much when the condenser is in the circuit as when it is not. *c*. The spot on the oscillograph tube being thus slowed down on the steep parts of the hysteresis curve and correspondingly speeded up on the saturation parts results in a much more uniform brightness of pattern which can be observed more readily.

6. MAGNETIZING COILS, SEARCH COILS, AND SAMPLES.

The samples which have been used are mostly in the form of either flat rings stamped from sheet metal, or long straight strips of thin tape. The rings are about four inches in diameter so as to fit into a toroidal furnace made for magnetic testing.[4] When the furnace is used the windings are applied on the furnace, 45 turns for the magnetizing winding and 90 turns for the search coil. When used without the furnace a similar number of turns are wound directly on the sample. The number of turns in the search coil being small the cross-sectional area of the sample is made correspondingly large, about 1/4 square inch in order to apply sufficient voltage to the integrator.

The apparatus for use with the single ribbon samples consists of straight magnetizing and search coils. A glass tube about $1\frac{1}{4}$ inches in diameter carries two parallel windings 30 inches long, for producing the magnetizing field. At the center of the length of this coil and

---

[4] The furnace will be described by Mr. G. A. Kelsall in a forthcoming number of the Journal of the Optical Society of America and Review of Scientific Instruments.

side by side are placed two equal search coils, connected in series opposing. These coils are 10 inches long, and in one assembly carry 30,000 turns each, in another 7,000 turns each. The sample, about 40 inches long, is placed in the axis of one search coil, the other coil serving to compensate for the air space of the first. This assembly is mounted in a brass frame to which is attached a windlass and spring scale for applying tension to the sample when so desired. The effect of the earth's magnetic field on the specimen is made negligible by either passing a direct current through one of the magnetizing windings in series with a high inductance, or by passing a direct current through the two coils from a battery in parallel with the electrolytic condenser.

III. Mathematical Analysis of the Apparatus.

The performance of the apparatus will be analyzed for two simple cases representing extremes in the properties of the magnetic material. In the first case it will be assumed that the flux in the search coil is proportional to the magnetizing field, a condition that is approached with hard materials at low field strengths. In the second case the assumption will be made that the material becomes saturated perfectly in the positive or negative direction, as the magnetizing field passes through zero. This is the limiting case of soft materials at high alternating field strengths.

1. Circuit Corrections for the Case of Small Magnetizing Fields.

The circuit of the integrator is represented in Fig. 3. The condenser is shunted by an effective resistance r made up of the grid leak resistance r' and the condenser resistance r''. The conductance of condensers is a function of frequency and temperature and possibly of other factors. As a function of frequency it may be written

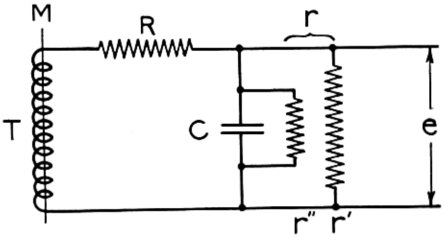


Fig. 3—Equivalent circuit of the integrator.

$$G = \frac{\omega C}{Z_1} + \frac{\omega^2 C}{Z_2} + \cdots. \tag{3}$$

If the terms in powers of ω higher than the first are neglected, the resistance is

$$r'' = \frac{1}{G} = \frac{Z}{\omega C}, \tag{4}$$

where Z is the resistance of a one farad condenser at $1/2\pi$ cycles per second.

In terms of the rate of change of induction, B, in the sample and the output voltage e of the integrator, the differential equation of the circuit in Fig. 3 is

$$\frac{de}{dt} + \omega J e = -NS\frac{dB}{dt}, \tag{5}$$

where

$$J = \frac{1}{RC\omega}\left[1 + \frac{R}{r'} + \frac{RC\omega}{Z}\right].$$

Let us now assume that the magnetizing field is sinusoidal,

$$H = H_1 \cos \omega t;$$

that

$$B = \mu H, \ \mu \text{ being considered constant;}$$

and that

$$e = a \cos \omega t + b \sin \omega t.$$

Making these substitutions in equation (5) and solving for a and b by equating coefficients of like terms, we get

$$e = -\frac{NS}{RC}\mu H_1 \frac{1}{1 + J^2}\ [\cos \omega t - J \sin \omega t]. \tag{6}$$

The value of J is less than .01 in this apparatus so that J^2 can be neglected compared with unity. The negative sign is merely a matter of convention, since the curve can be turned by reversing one pair of terminals. The sign of the $\cos \omega t$ term will therefore be taken positive henceforth, and we have

$$e = \frac{NS}{RC}B\left[\cos \omega t - \frac{1}{RC\omega}\left(1 + \frac{R}{r'}\right)\sin \omega t - \frac{1}{Z}\sin \omega t\right]. \tag{7}$$

The value of e therefore departs from that for a perfect integrator by two terms, one depending upon the time constant of the integrator, the frequency, and the value of the grid leak resistance of the first amplifier stage; the other term depending only upon the conductance of the integrating condenser.

This voltage e is applied to the first stage of the amplifier, resulting in a voltage e_1 at the output of this stage which is then applied to the second stage, and so on. Fig. 4 represents the first stage of the amplifier, ρ_1 being the internal resistance of the vacuum tube whose amplification constant is μ_1, r_1 the external plate resistance, R_1 the grid leak resistance for the next tube, C_1 the coupling capacity having the effective leak resistance r_1''. Solving for e_1 by the same method that was used for e and omitting negligible terms gives the result

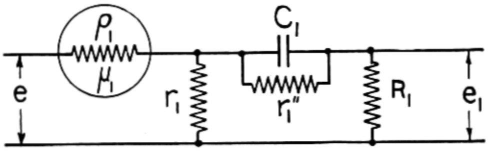


Fig 4—Equivalent circuit of one stage of the amplifier.

$$\left.\begin{aligned} e_1 = \frac{NS}{RC}\,\frac{B\mu_1R_1r_1}{(R_1+r_1)(\rho_1+r_1)-r_1^2} \\ \left[\cos\omega t-\left(\frac{1}{r'C\omega}+\frac{1}{RC\omega}+\frac{1}{R_1C_1\omega}+\frac{2}{Z}\right)\sin\omega t\right] \end{aligned}\right\}. \tag{8}$$

Similarly, the output from s stages of the amplifier is

$$e_s=\frac{NS}{RC}B\left[\prod_1^s M_s\right]\left[\cos\omega t-\left(\frac{1}{r'C\omega}+\sum_0^s\frac{1}{R_sC_s\omega}+\frac{s+1}{Z}\right)\sin\omega t\right], \tag{9}$$

where

$$M_s=\frac{\mu_sR_sr_s}{(R_s+r_s)(\rho_s+r_s)-r_s^2}\doteq\mu_s\frac{r_s}{\rho_s+r_s}\,.$$

The product from 1 to s contains only amplifier constants, while the summation from 0 to s means that the values for the integrator are included.

According to equation (9) the circuit introduces errors which at low fields makes the spot describe an ellipse instead of a straight line with positive slope. This ellipse is traced in the clockwise direction, the opposite direction to that in which the apparatus traces hysteresis loops. Since in all practical cases the hysteresis loop at low frequencies approaches an ellipse traced in the counter-clockwise direction, the effect of finite time constants and condenser resistance is to make the figure traced by the apparatus narrower than it ought to be.

Fig. 5 illustrates the nature of this distortion. The straight line aa' is the curve for the ideal material and perfect apparatus for which

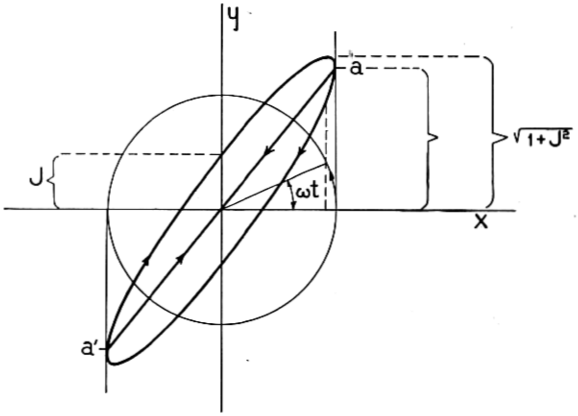


Fig. 5—Distortion produced by integrator and amplifier at low flux densities.

the maximum amplitude in the B direction may be taken as unity. The ellipse is the curve

$$\left.\begin{aligned} x &= \cos \omega t, \\ y &= \cos \omega t - \sum_0^s J_s \sin \omega t, \end{aligned}\right\} \qquad (10)$$

which the apparatus traces. The point of tangency of the ellipse with a line parallel to the B axis is at the point a and always remains there. The intersections with the B axis are at $\pm \Sigma J_s$ and the height exceeds that of the point a by the usually very small quantity $\sqrt{1 + (\Sigma J_s)^2} - 1$.

In the present system the constants are: $R_s = 2 \times 10^6$ ohms; $C_s = 4 \times 10^{-6}$ farad; $r' = 2 \times 10^6$ ohms. We may set $R = 10^5$ ohms, $C = 20 \times 10^{-6}$ farad, $\omega = 100$, and assume four stages of amplification. For the type of condensers used in the integrator and amplifier, $Z = 250$. The vertical width of the ellipse is then 3 per cent of the total height, of which .5 per cent is caused by the time constant of the integrator, .5 per cent by the time constant of the four amplifier couplings (including the output circuit), and 2 per cent by the conductance of the condensers in the circuit. The conductance therefore contributes the largest error in the present system, showing that the time constants have been made large enough for practical purposes.

2. CIRCUIT CORRECTIONS FOR THE CASE OF LARGE MAGNETIZING FIELDS.

When the induction in the specimen changes abruptly between negative and positive saturation as the magnetizing field passes through zero, it may be represented by the Fourier series

$$B = B_m \frac{4}{\pi} \sum_{1}^{n} (-1)^{n-1} \frac{1}{m} \cos m\,\omega t, \tag{11}$$

where B_m is the saturation value, $n = 1, 2, 3$, etc., and $m = 2n - 1$. It is assumed that the magnetizing field has the constant fundamental frequency $\omega/2\pi$, but it need not be sinusoidal. Each term of this series can be treated as was the single term in the expression of B for the small fields. When this is done for the integrator and amplifier and the terms are again summed, the output voltage is given by

$$\begin{aligned} e_s &= \frac{NS}{RC} B_m \frac{4}{\pi} \left[\prod_{1}^{s} M_s \right] \sum_{1}^{n} \left[(-1)^{n-1} \left\{ \frac{\cos m\,\omega t}{m} \right.\right. \\ &\quad \left.\left. - \left(\frac{1}{r'C\omega} + \sum_{0}^{s} \frac{1}{R_s C_s \omega} \right) \frac{\sin m\,\omega t}{m^2} - \frac{s+1}{Z} \frac{\sin m\,\omega t}{m} \right\} \right] \\ &= \frac{NS}{RC} B_m \left[\prod_{1}^{s} M_s \right] \left[\alpha - \left(\frac{1}{r'C\omega} \quad \sum_{0} \frac{1}{R_s C_s \omega} \beta - \frac{s+1}{Z} \gamma \right) \right], \end{aligned} \tag{12}$$

where

$\alpha = 1$ from $\omega t = -\frac{\pi}{2}$ to $+\frac{\pi}{2}$;

$\alpha = -1$ from $\omega t = \frac{\pi}{2}$ to $3\frac{\pi}{2}$; (Fig. 6)

$\beta = \omega t$ from $\omega t = -\frac{\pi}{2}$ to $+\frac{\pi}{2}$;

$\beta = (\pi - \omega t)$ from $\omega t = \frac{\pi}{2}$ to $3\frac{\pi}{2}$; (Fig. 7)

$\gamma = \frac{4}{\pi} \sum_{1}^{n} (-1)^{n-1} \frac{\sin m\,\omega t}{m}$; (Fig. 8)

$m = 2n - 1$.

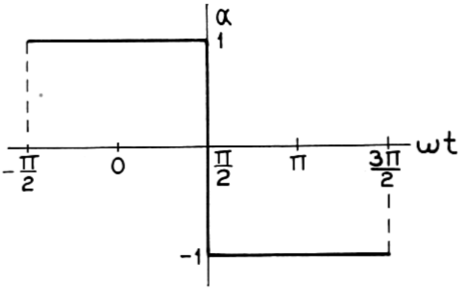


Fig. 6—The value of α during one cycle.

The value of γ is finite for all values of ωt except $\pi/2$ and $3(\pi/2)$ where it becomes infinite. Near these values, therefore, the equation fails for the reason that the higher powers of ω were neglected in the expression for the condenser conductance (Eq. 4).

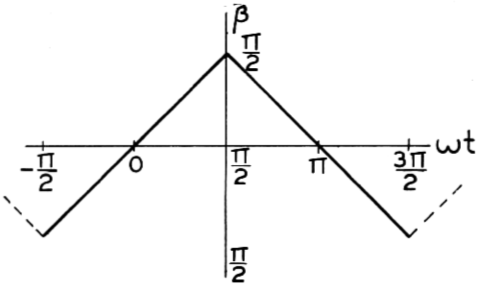


Fig. 7—The value of β during one cycle.

Let the heavy line *abob′a′*, Fig. 9, be the trace of the ideal curve

$$\left.\begin{aligned} x &= \cos \omega t, \\ y &= \frac{4}{\pi}\sum_{1}^{n}(-1)\,\frac{\cos m\,\omega t}{m}. \end{aligned}\right\} \qquad (13)$$

The curve followed by the value of e_s in equation (12) is then of the form shown by the light line *ac′a′ca*, which again is traced in the clockwise direction. When the apparatus is used for an actual material having positive hysteresis, the loop is made narrower at the *H* axis by this distortion, but the amount of the decrease in width

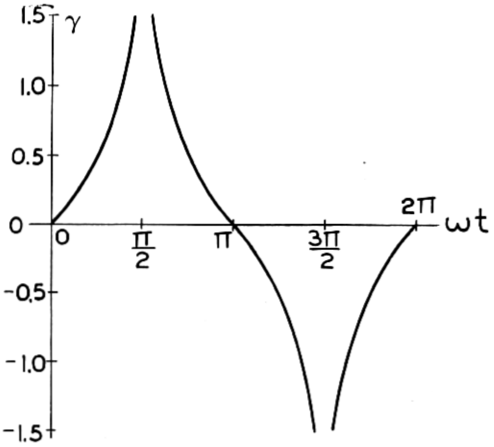


Fig. 8—The value of γ during one cycle.

can not be determined analytically without more accurate knowledge of the condenser conductance as a function of frequency. The value of equation (12), therefore, lies chiefly in giving the distortion to be expected along the saturation parts of the curve, where the curve is made narrower in the B direction and in extreme cases crosses itself as shown in Fig. 10.

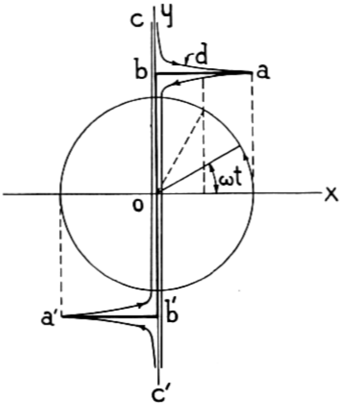


Fig. 9—Distortion produced by the integrator and amplifier for an ideally saturated material.

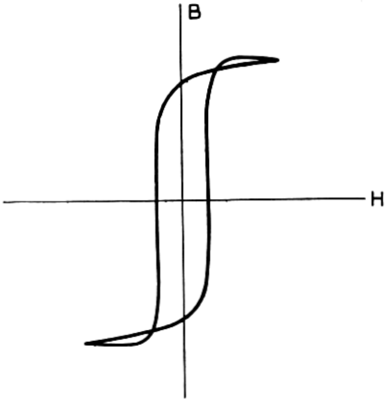


Fig. 10—Distortion produced by the integrator and amplifier with a normally saturated material.

The amount of the distortion will be estimated for the same circuit constants as were used above, but with only three stages of the amplifier. The distorted curve coincides with the ideal one at the points a and a', Fig. 9. At the point d, where $\omega t = \pm 60°$, we have $\alpha = 1$, $\beta = 1.05$ and $\gamma = .84$ for $\omega t = 60°$; and $\alpha = 1$, $\beta = -1.05$,

$\gamma = -.84$ for $\omega t = -60°$. Calculation by equation (12) gives for the vertical spread of the curve at this point 2.3 per cent of the double height of the curve, of which 1 per cent is contributed by the time constant factors and 1.3 per cent by the condenser conductance. It is seen that of the errors discussed in this and the preceding section, those caused by the conductance of the condensers are the larger.

3. DEMAGNETIZATION BY CURRENT IN THE SEARCH COIL.

The voltage induced in the search coil by the change of flux in the sample creates a current in the search coil circuit. The magnetic field set up by this circuit in the search coil opposes changes in the field due to the current in the magnetizing winding and so makes the hysteresis loop appear wider than it actually is.

At low fields the effect may be calculated with some degree of accuracy. Let the field induced by the magnetizing coil be $H_1 \cos \omega t$, and let the search coil contain a sample which has the initial permeability μ and the cross section S. The field set up by the search coil current is then

$$-\frac{4\pi}{10}\frac{N}{l}\cdot\frac{NS}{R}\frac{dB}{dt}\times 10^{-8}$$

and the total field is, very nearly,

$$H = H_1\left(\cos \omega t + \frac{4\pi}{10}10^{-8}\frac{N^2S}{lR}\mu\omega \sin \omega t\right).$$

The induction is

$$B = \mu H = \mu H_1\left(\cos \omega t + \frac{4\pi}{10}\times 10^{-8}\frac{N^2S}{lR}\mu\omega \sin \omega t\right), \qquad (14)$$

while the magnetizing field which the tube indicates due to the influence of the deflector coils is $H_1 \cos \omega t$. The spot therefore traces an ellipse in the *counter-clockwise* direction, so as to add to the effect of hysteresis in the material.

Assuming the values $N = 30{,}000$, $l = 30$, $R = 100{,}000$, $\mu = 10{,}000$ (permalloy), $S = .005$, $\omega = 100$, the width of the ellipse along the B axis is about 6 per cent of its total height. This is a considerable error, so that when working with so high a permeability a high value of amplification may have to be used while the factor N^2S/lR is decreased.

When the magnetizing field is such as to saturate the sample the calculation of the effect of search coil current is more uncertain.

Let the true cycle of magnetization of the sample be that shown in Fig. 11. Let the magnetizing field be sinusoidal, $H' = H_1 \cos \omega t$, so

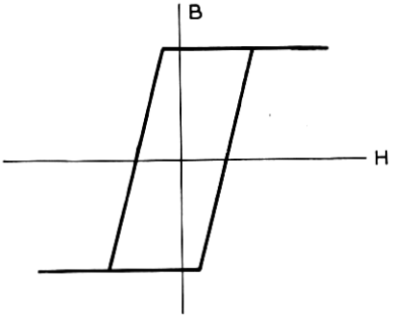


Fig. 11—Simplified hysteresis loop.

that the rate of change of the field in the region where the induction is variable is nearly

$$\frac{dH'}{dt} \doteq \omega H_1.$$

When the induction is varying there is a reverse magnetizing field set up by the current in the search coil of the strength

$$\Delta H = \frac{4\pi}{10} 10^{-8} \frac{N^2 S}{lR} \frac{dB}{dt} = \frac{4\pi}{10} 10^{-8} \frac{N^2 S}{lR} \frac{dB}{dH} \frac{dH}{dt} \cdot$$

The value of dH/dt is not known, since it itself involves ΔH, but its maximum value is $(dH'/dt) = \omega H_1$. The reverse field is, therefore, at the most

$$\Delta H = \frac{4\pi}{10} 10^{-8} \frac{N^2 S}{lR} \frac{dB}{dH} \omega H_1. \tag{15}$$

The calculation of ΔH has been done for a number of cases. With the ring shaped samples having search coils of 90 to 135 turns the value of ΔH was .1 per cent of the coercive force H_c for iron, and 4 per cent of H_c for permalloy. When a permalloy having a thin hysteresis loop with the slope $(dB/dH) = 200{,}000$ is used in the search coil of 30,000 turns, the value of ΔH is nearly as large as H_c so that the curve is approximately doubled in width. In such a case the factor N^2S/lR must be made smaller, preferably by decreasing N^2 since this does not involve a corresponding increase in the amplification.

It must be remarked that these formulæ give the maximum errors, and that in fact the errors are considerably less. The condenser across the magnetizing coil permits the magnetizing current to take other than sinusoidal form, so that the value of dH/dt is much smaller than ωH_1.

4. Demagnetizing Factor of Magnetizing Coil and Sample.

The errors under this heading pertain to straight coils and samples and include the open end correction of the magnetizing coil, the error due to the non-uniformity of the magnetizing field of the open ended coil, and the demagnetizing factor of the finite length of sample.

Calculations by L. W. McKeehan and measurements by P. P. Cioffi [5] on coils and samples of nearly the same dimensions as those used in this apparatus, have shown that the errors in H and B which result from neglecting the above factors amount to less than three per cent. The exact correction to be applied for any particular sample is not readily calculated, but the sample can be assumed to be sufficiently long if displacing it a few centimeters in the coils results in no appreciable change in the hysteresis loop.

5. Eddy Currents and Magnetic Viscosity in the Sample.

Currents induced in the sample flow in such a direction as to oppose the applied magnetizing force, with the result that to reach any given density of magnetization a higher applied field is required when eddy currents are present. The loss of energy in eddy currents is added directly to the hysteresis loss, so that the dynamic hysteresis curve is different from the static one. Qualitatively this difference manifests itself as a widening in the H direction of the recorded loop at intermediate frequencies, and a shrinkage in the B direction when the frequency is so high that skin effect prevents the induction from reaching its full value before the applied field has receded appreciably from its maximum.

In dealing with the distortion introduced by eddy currents, the procedure must be governed by the nature of the problem in hand. If the total energy loss corresponding to a given magnetic cycle is sought, no eddy current correction is required. If it is desired to reproduce as nearly as possible the static hysteresis loop of a specimen, then the frequency of magnetization should be chosen low enough so that, with the aid of the condenser in parallel with the magnetizing coil, the eddy current effect is not appreciable. If, finally, the object of the work is to detect differences between the static and the dynamic hysteresis curves, the presence of eddy currents must be reckoned with. The following analysis, based upon a simplified static magnetization cycle and an assumed sinusoidal time variation of the magnetizing field, makes possible a rough estimation of the apparent increase in the coercive force H_c introduced by the eddy currents.[6]

Let Fig. 12 represent the cross-section of a rectangular lamination

---

[5] P. P. Cioffi, *Jl. Opt. Soc. Am.* and *Rev. Sc. Inst.*, 9, p. 53, 1924.

[6] A similar result has been obtained and used by Neumann, l.c.

of iron having the thickness d and the width c. The sample is being magnetized in the direction perpendicular to the plane of the paper. The magnetization produces eddy currents and we shall assume that the elements of induced current flow in circuits like the shaded element of area at the distance x, $-x$ on the horizontal axis and the distance $(c/d)x$, $-(c/d)x$ on the vertical axis, with the corresponding widths dx and $(c/d)dx$. The current density i_x which is induced in the elementary circuit is

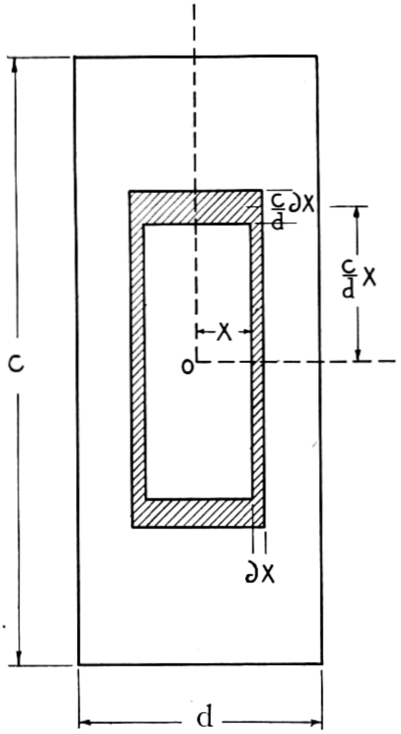


Fig. 12—Cross-section of specimen.

$$i_x = \frac{e}{r} = 10^{-8}\,\frac{d\varphi_{cx}}{dt}\,\frac{1}{\rho}\,\frac{cd}{4(c^2+d^2)}\,\frac{1}{x}\cdot \tag{16}$$

The flux φ_{cx} is the total flux enclosed by the elementary circuit, and ρ the specific resistance of the iron. Within the elementary area itself the flux $d\varphi_x$ is the sum of the fluxes produced by the applied field and by the field due to the eddy currents external to the elementary area. If it is now assumed that the applied field is just passing through the value H_c and that for a small variation in H about this point the flux is proportional to $H - H_c$, the flux threading the elementary circuit is

$$\varphi_{cx} = \int_0^x d\varphi_x = \int_0^x 8\,\frac{c}{d}\,x dx\,\frac{dB}{dH}\left[H - H_c - \frac{4\pi}{10}\int_x^{d/2} i_x d_x \right]. \tag{17}$$

The value of dB/dH is obtained from the static loop. It does not depend upon x or t and is for the moment assumed independent of H. The total flux φ in the sample is, therefore,

$$\varphi = 8\,\frac{c}{d}\,\frac{dB}{dH}\,(H - H_c)\int_0^{d/2} x dx - 8\,\frac{c}{d}\,\frac{4\pi}{10}\,\frac{dB}{dH}\int_0^{d/2} x dx \int_x^{d/2} i_x dx \tag{18}$$

$$= cd\,\frac{dB}{dH}\,(H - H_c) - \varphi_2. \tag{19}$$

The term φ_2 may now be expanded by successively substituting in equation (18) the value of i_x from equation (16). The result is the series

$$\varphi = cd\,\frac{dB}{dH}\,(H - H_c) - \frac{\pi}{4}\,\frac{10^{-9}}{\rho}\,cd^3\,\frac{(dB)^2}{(dH)}\,\frac{dH}{dt} + \varphi_r. \tag{20}$$

where φ_r is the sum of higher terms and where the factors have been simplified by regarding d small compared to c.

Now the coercive force obtained by the dynamic method is the value H_a of the field at the point of the curve where the total flux φ is zero. The apparent increase in coercive force caused by eddy currents can at this point be obtained from equation (20), giving

$$\Delta H_c = H_a - H_c = \frac{\pi}{4}\,\frac{10^{-9}}{\rho}\,d^2\,\frac{dB}{dH}\,\frac{dH}{dt}\,. \tag{21}$$

We may now inquire what, in a qualitative way, is the distortion of the hysteresis loop predicted by equation (21).

a. The widening of the loop is proportional to the square of the thickness of the lamination.

b. If the magnetizing force is kept sinusoidal the value of dH/dt is proportional to H_m. Since also dB/dH increases with H_m the value of ΔH_c increases at a greater rate than the first power of H_m.

c. For narrow loops and sinusoidal magnetizing current ωH_m is a sufficiently close approximation to dH/dt, while with loops which are wide because of hysteresis loss $(dH/dt) = \omega\sqrt{H_m^2 - H_c^2}$. If now the curve is widened by eddy currents, these values of dH/dt are too large and the correction to be applied is smaller than that given by inserting these values in equation (21).

d. For small values of ΔH_c this correction is proportional to the frequency of the magnetizing field, through the factor dH/dt. When the correction becomes appreciable compared to H_m the effect considered under *d* becomes active and ΔH_c increases at a slower rate than the first power of the frequency.

e. The value dB/dH is not constant as was assumed but depends upon H and attains a maximum near $H = H_c$. The effect of eddy currents is to make the magnetization non-uniform over the cross-section of the sample, and the average value of dB/dH is then considerably less than the maximum value. This effect again makes the value of ΔH_c increase at a slower rate than the first power of the frequency and of H_m.

f. When conditions are such that ΔH_c is small, a sufficient approximation to its value is obtained by deriving dB/dH from the dynamic loop so that the correction can be made without reference to the static curve.

Besides the eddy current effect, widening of the hysteresis loop has also been ascribed to what has been called magnetic viscosity, a property of the material which causes the induction to lag behind the actual magnetizing force. The existence of this phenomenon has been affirmed by some experimenters and denied by others. Not enough is known about it to make its discussion here pertinent.[7]

6. Non-Linearity of the Amplifier.

The push-pull construction of the amplifier compensates to some extent for curvature of the tube characteristics. In order to have full compensation, the tubes should be matched so that the tubes of each pair work on a part of their characteristic having the same steepness and curvature.

Furthermore, the last pair of tubes, the tubes working into the oscillograph, should have a practically straight characteristic over about 30 volts on each side of the average plate voltage, or in the case of 102-D tubes about 1.5 volts on each side of the average grid voltage.

7. Observational Errors.

Because of the width of the fluorescent trace on the screen, there is a probable error of about .2 mm. in measuring the width or height of the hysteresis loop on the tube. This corresponds to probable errors of the order of 1 per cent in B and 2 per cent to 3 per cent in H_c. Probable errors of 1 per cent are also introduced in the calibration

---

[7] For a recent consideration of time-lag in magnetization see R. M. Bozorth, *Phys. Rev.*, 32, p. 124, 1928.

measurements. The thermocouple reading involves an error of perhaps 1 per cent. The wave shape factor of the calibrating voltage must in general be considered, but in this apparatus the distortion is probably made negligible by the filter.

The deflections on the tube are not absolutely proportional to the deflecting forces, but since the calibrations are made over approximately the same distances as the hysteresis measurements the error thus introduced must be small.

Altogether, the probable observational errors may amount to as much as 4 per cent.