The Bell System Technical Journal

April, 1929

Electrons and Quanta[1]

By C. J. DAVISSON

The experiments by the author and L. H. Germer, by G. P. Thomson and by others from which the wave properties of electrons are adduced are briefly described. The agreement between the results of these experiments and the prediction of L. de Broglie is pointed out. The wave and corpuscular properties of electrons are compared with the similar properties of light quanta.

WHEN I discovered on looking over the announcement of this meeting that Professor Compton is to speak on "X-rays as a Branch of Optics," I realized that I had not made the most of my opportunities. I should have made a similar appeal to the attention of the Society by choosing as my subject, "Electrons as a Branch of Optics." And a very good case can be made out that electrons should be so regarded. During the last few years we have come to recognize that there are circumstances in which it is convenient, if not indeed necessary, to regard electrons as waves rather than as particles, and we are making more and more frequent use of such terms as diffraction, reflection, refraction and dispersion in describing their behavior. If this in itself is not enough to mark electrons as a branch of optics, it is sufficient at least to establish a certain community of ideas between the subjects of optics and electronics which cannot but be of interest to the members of this Society.

The evidence that electrons are waves is similar to the evidence that light and X-rays are waves. A beam of electrons is scattered by a grating—either the lattice grating of a crystal or an ordinary optical grating—and the intensity of scattering, as measured by the current density of electrons proceeding in different directions, is such as can be explained by assuming as is done in optics that what we are dealing with is the superposition of trains of scattered waves proceeding from the grating elements. In other words, current density of scattered electrons displays in these experiments the same type of spacial distribution as flux density in the analogous experiments in optics, and the observations are given a similar interpretation—an interpretation, that is, in terms of the interference of coherent wave-trains. The

---

[1] Presented at the Michelson Meeting of the Optical Society, Washington, D. C., November 1–3, 1928.

standard methods of optics are at once available for calculating the wave-lengths of electrons of various speeds. We do not hesitate to make these calculations, nor do we hesitate to attach physical significance to the results.

The experiments by which these phenomena are revealed have been made by Dr. Germer and myself in New York, by Thomson and Reid in Scotland, by Rupp in Germany, by Rose in England, by Nishikawa and Kikuchi in Japan, and by Szczeniewski in France. The subject is being actively cultivated at present, and there may be still other experiments of which I have not yet heard. I do not propose to give a detailed description of any of the investigations. The type of result and the methods of treating the data are so exactly those of optics—including, of course, X-rays as one of its branches—that the details would hardly interest you. I shall have something to say later on about the general nature of the results, but to begin with I shall attempt a brief account of certain theoretical speculations in which these results were more or less definitely anticipated.

It is a remarkable circumstance, and one that attests to the exceptional insight and daring of Louis de Broglie, that these newly discovered properties of the electron were suspected, and a definite hypothesis concerning them was formulated, two or more years before any of the experiments I have mentioned had been performed; even the exact relation between the speed and wave-length of the electron was accurately predicted. It is true that Einstein had at an even earlier date made use of the idea that an assemblage of gas molecules may for certain theoretical purposes be regarded as equivalent to a system of standing waves, but de Broglie seems first to have seen clearly that the duality of wave and corpuscular properties to which we are becoming reconciled in the phenomena of light might be characteristic also of electrons and material particles in general.

If light and X-rays behave in certain circumstances as if they are particles, why should there not be circumstances in which particles behave as waves? This question was suggested to de Broglie, not by any idea of the general fitness of things nor by any sense of symmetry in the universe, but by the realization, which he shared with others, that the laws of classical mechanics had been so amended in the Bohr atom model as to have become all but non-existent. It was generally felt that the Bohr atom had become too artificial to be acceptable, and that the real trouble arose from an unwarranted extrapolation of classical mechanics to systems of atomic size.

To understand the hypothesis on which de Broglie hoped to build a new model of the atom it will be necessary to have clearly in mind the evidence that light is in some sense corpuscular. This idea had its inception in Einstein's speculations in regard to Planck's theory of the distribution of energy in the spectrum of a black body radiator. It was conceived that the energy radiated by one atom remained in some way localized in space, and could be delivered in toto to another atom or resonator suitably constituted to receive it. From this hypothesis Einstein predicted the relation which was later found to obtain between frequency of radiation and maximum electron energy in photoelectric emission, and the idea has become more and more essential to the understanding of the photoelectric phenomenon as the facts concerning it have been more and more fully revealed to us by experiment. Energy in amounts $h\nu$ is absorbed from radiation of frequency ν, not by slow accretion from waves, but instantaneously as from particle impact. The picture seemed clear enough; the energy of a beam of light is carried by corpuscles, each corpuscle transporting the amount $h\nu$. If we were willing, for the sake of pursuing this idea further, to disregard the whole gallery of interference phenomena with which it apparently conflicted, we could show that if the corpuscles possess energy $h\nu$ they must possess also momentum $h\nu/c$—this relation being required to explain the observations on light pressure in terms of impinging particles.

Thus the corpuscular theory seemed to be required to explain the photoelectric phenomena, and it might be made to explain also the phenomenon of radiation pressure. On the other hand the light corpuscle seemed a strange and ephemeral sort of particle, lacking that continuity in time which we attribute to electrons and atoms. Apparently it was manufactured within an atom for the express purpose of carrying away a part of its energy and was later destroyed in another part of the material universe to which this quantum of energy was delivered. It was difficult to regard an apparently transient entity of this sort as a particle in good standing to be classed with electrons and alpha rays.

If a certain suspicion still attaches to the light quantum in respect to its continuity, this suspicion has at any rate been considerably allayed, and the reputation of the quantum as an authentic particle correspondingly enhanced, by the discovery of the Compton effect, and again quite recently by the discovery of the Raman effect. In the first of these phenomena we see the quantum surviving an encounter with an essentially free electron, with which it exchanges energy and momentum in accordance with the ordinary laws of elastic collision; in the latter we see the quantum preserving its identity through an encounter with a molecule to which it imparts a part only of its energy.

If we are required by these recent developments to accept quanta as actual particles which carry the energy and momentum of light, how, in terms of such particles, are we to explain interference? How are we possibly to get on without waves? We even depend upon the waves to supply us with information concerning the energy and momentum of the quanta. One way in which it has been proposed to resolve the difficulty is to relegate the waves to the comparatively unimportant rôle of supplying the laws of motion of the quanta. Let us assume, for example, that when a stream of quanta passes through a narrow slit the particles do not continue in straight lines as Newton supposed, but that they spread out in such a fashion that the current density of quanta proceeding in different directions is proportional to the intensity of the light proceeding in these directions as calculated on the wave theory. In making this assumption we have given over classical mechanics and explained diffraction—or at least described it—by setting up a form of wave mechanics in its place.

With this rather crude and incomplete picture before us of light quanta being guided in their motion by waves, it is not difficult to imagine the general trend of de Broglie's speculations. de Broglie sensed that electrons like quanta might have waves to guide them—to supply the laws of their motion. That the ordinary laws of mechanics are adequate to describe the motions of electrons in discharge tubes is not inconsistent with this view, for it is well known that these laws are adequate also for a corpuscular theory of light to within the accuracy with which the phenomena are described by geometrical optics. It is only when one tries to explain diffraction that the simple corpuscular theory fails him. de Broglie envisaged a similar situation in regard to electrons—a range of small scale phenomena requiring a wave theory for their proper description. Assuming the frequency of these hypothetical waves to be given by the total energy of the electron divided by h, de Broglie was able to show that the length of the waves would be given by h divided by the momentum of the electron—and this as it happens is just the relation which obtains between the wave-length and momentum of quanta.

The goal toward which de Broglie was striving, as I have mentioned, was a new theory of the atom, and he was able to point at once to a suggestive relationship which exists between the lengths of these hypothetical electron-waves and the lengths of the circular orbits in the Bohr atom. The permitted orbits are just those which contain an integral number of these electron wave-lengths. But it was Schroedinger, as we all know, who elaborated these ideas into a comprehensive wave theory of mechanics, and showed the tremendous possibilities of this theory in explaining the properties of the atom as revealed to us by the data of spectroscopy. At last we have an atom endowed with a constitution rather than a set of by-laws.

The success of the Schroedinger theory in explaining in a natural way the stationary states of the atom and the various rules governing transitions between these states, not to mention numerous others of its successes, must be taken as very strong evidence in favor of the fundamental idea upon which the theory is based—namely, that the duality of wave and corpuscular properties which characterizes light is characteristic also of electrons. If the evidence supplied by these data lacks something in the matter of directness, this deficiency is made good by the experiments on the scattering of electrons by crystals about which I am supposed to be speaking.

If I have been a long time in coming to the point, the time has not been wasted, for with the picture before us of the energy and momentum of a beam of light being carried by a stream of quanta for which the waves serve only to supply the laws of motion, a workable theory of the scattering of electrons is at once at hand—to a first approximation we merely read "electrons" for "quanta," and there we are. The observations on electron scattering are consistent with the view that the electrons are being guided by waves in just the way we have imagined quanta to be guided in the phenomena of optical diffraction. The only real difficulty seems to be that in the light phenomena it is not easy to believe in the particles, while in the electron phenomena it is hard to have faith in the waves.

Before going further I should like to point out that we now have two wave-lengths associated with an electron of given speed: one is the length of the X-ray waves which will be generated if the whole of the kinetic energy of the electron is converted into radiation and the other is the length of this new de Broglie wave, the so-called phase wave. The first of these wave-lengths is inversely proportional to the energy of the electron while the second is inversely proportional to its momentum. In terms of the equivalent voltage V of the kinetic energy of the electron, the lengths of the two waves are given in Ångstrom units by the formulæ

$$\lambda_x = \frac{12,350}{V} \qquad \text{and} \qquad \lambda_p = \left(\frac{150}{V}\right)^{1/2}$$

and their ratio is given approximately by

$$\frac{\lambda_x}{\lambda_p} = \frac{1000}{V^{1/2}} \cdot$$

For values of V below, say, 10,000 volts the X-ray wave-length is much greater than the corresponding de Broglie wave-length.

The lengths of de Broglie waves of electrons which have been accelerated through potential differences comparable with 100 volts are the same as the lengths of moderately hard X-rays. For this reason crystal diffraction of de Broglie waves is observed with electrons of relatively low speeds—speeds corresponding to 100 volts or less—whereas, to observe the same phenomenon with X-rays, the tube producing the radiation must be operated at potential differences comparable with 10,000 volts.

The first clear evidence of the diffraction of X-rays was obtained when Laue and his collaborators investigated the scattering of X-rays—of X-ray quanta, shall we say—by a single crystal of zincblende. The analysis of this phenomenon led to the prediction and discovery of the Bragg reflection as a special case of crystal diffraction, and later on to the prediction and discovery of the special case of diffraction by aggregates of small crystals of random orientation. All three of these types of diffraction have now been observed with electrons. The Laue type of diffraction, and also the Bragg type, have been observed and investigated by Dr. Germer and myself. Diffraction by the crystal aggregates has been studied by Thomson and Reid, by Ironside and by Rupp. And observations by the Bragg method have been made also by Szczeniewski and by Rose.

I must now modify to a certain extent the picture of electron diffraction which I suggested to you a while ago. It is not quite true, as I suggested, that the only difference between the diffraction of light waves and the diffraction of electron waves is that in one case the pattern is formed by light quanta and in the other by electrons. In our investigation of the Laue type of diffraction we find, for example, that the streams of electrons which issue from the crystal do not coincide exactly in direction with the streams of quanta which would issue from the same crystal if the experiment were made with X-rays. In the case of X-ray diffraction the streams of quanta proceed from the crystal in the directions of regular reflection from important sets of atom planes, or nearly so. It is recognized that the Laue beams do not, in general, lie precisely in these directions because of a very slight refraction of the rays by the crystal. The situation in regard to electrons seems to be that electrons also are refracted and much more strongly than X-rays. The refractive indices of a metal such as nickel for electrons of low speed depart from unity much more widely than do the indices for X-rays of equal wave-length. It is a consequence of this difference that the departure from the simple law governing the directions of beams, which in the case of X-rays is negligible, is in experiments with low-speed electrons marked and important.

Fortunately, we are not prevented by this complication from arriving at perfectly definite values of wave-lengths from observations on the Laue type of electron diffraction, and these wave-lengths turn out to be in acceptable agreement with the values of h/mv, as predicted by de Broglie.

Further evidence of electron refraction is contained in the observations we have made on the electron analogue of the Bragg X-ray reflection beam. And from the data of these experiments we have constructed a dispersion curve for nickel which displays some of the features to be expected from certain theoretical considerations. In conjunction with these measurements we have made additional determinations of electron wave-lengths, and these agree within one per cent or less with the values calculated from de Broglie's formula.

In the similar experiments made by Szczeniewski and by Rose, no certain evidence of electron refraction has been found. This may be due to some important difference in regard to refraction between bismuth and aluminium, the crystals employed in their experiments, and nickel, the crystal upon which our measurements were made. On the other hand Rupp has found evidence of refraction for a number of metals in measurements which he has made on the diffraction of low-speed electrons by crystal aggregates.

Electron diffraction differs from X-ray diffraction also in the matter of resolution. The X-ray beams are ordinarily extremely sharp because of the very great number of elements comprised in the diffracting lattice. Much broader beams are met with in electron diffraction—particularly in the diffraction of low-speed electrons—and occurrences of the beams are much less critical in wave-length. These characteristics are explained by the slight penetration of the electrons—and therefore of the electron waves—into the crystal; the effective number of scattering centers is small and the resolving power of the grating is correspondingly low.

The diffraction of electrons by crystal aggregates has been studied in Aberdeen by G. P. Thomson, who first observed this phenomenon, and by Rupp in Göttingen. Thomson has worked with thin polycrystalline foils of various metals and with high-speed electrons for which the refractive indices are practically unity. The results which he has obtained are in perfect agreement with those obtained in the corresponding experiments with X-rays—electrons of a given wave-length form exactly the same series of diffraction rings as would be formed by X-rays of the same wave-length.

Those of us who are studying electron diffraction are most fortunate in having before us a perfect model for our experiments and a fund of valuable data in the vast amount of work that has been done in the last fifteen years on the diffraction of X-rays. It is for this reason that, in spite of a rather difficult technique, so many and such varied results have been obtained in less than two years. Already we have passed on from crystal diffraction to diffraction by optical gratings. The first results of this sort were reported a month or so ago by Rupp and are in agreement with our expectations. Electrons are diffracted by an optical grating as if they were waves of length h/mv.

I have still to mention the beautiful but puzzling results which have been obtained in Japan by Nishikawa and Kikuchi. It is too bad to have to conclude my remarks with mention of the only results so far obtained which are distinctly puzzling. Nishikawa and Kikuchi have been studying the scattering of high-speed electrons by thin sheets of mica and calcite. The method of their experiment is identical with that of the original Laue experiment except that the heterogeneous beam of X-rays is replaced by a homogeneous beam of electrons. The results, as I have mentioned, are puzzling. If the incident beam is homogeneous, as stated, it is equivalent to a beam of monochromatic waves, and no diffraction pattern—or at most a very simple one—should be observed; and yet, when extremely thin sheets of mica are employed, elaborate and beautiful patterns of sharply defined spots are obtained—and patterns which cannot be readily explained even on the assumption that the incident beam contains a large range of wave-lengths, instead of a single wave-length only. When the speed of the incident beam is changed, the form of the pattern remains the same but its scale factor is altered. This also is unlike anything observed with X-rays. The results are such as might be expected if the diffracting system were a two-dimensional mesh rather than a three-dimensional lattice.

When somewhat thicker sheets of mica are used, the pattern of sharply defined spots is replaced by an array of rather fuzzy rings and lines. Again the observations are contrary to our expectations, and their explanation is far from obvious.

It may be significant that these are the only experiments, so far reported, in which the diffracting material is an insulator. But whether the clue lies here or elsewhere, it is highly unlikely, I think, that the explanation of these results will conflict with the conception we now have of electrons which are sometimes particles and sometimes waves.