A Method of Rating Manufactured Product

By H. F. DODGE

SYNOPSIS: This paper outlines a method of rating manufactured product. In the particular form here described, the rate has been found very useful for measuring the quality of communication equipment and materials entering the plant of the Bell System. While the primary object is control of quality of finished product, it is proving useful for measuring the workmanship of individual operators and groups of operators engaged in similar production work. Particular attention is directed to the statistical aspects of the rate to show how it can assist in controlling quality.

GIVEN a product whose quality is dependent on a number of diverse characteristics, the following questions and others similar to them frequently require answers. Has quality been satisfactorily controlled? Is there any general trend in quality either upward or downward? How does current quality compare with that of a year ago?

These are questions of importance to the manufacturer. Qualitative answers can often be given on the basis of general knowledge by those familiar with the details of manufacturing performance but such answers tend to be inaccurate or biased. What is often wanted is some statistical index based on quantitative data, a figure which balances the favorable features against the unfavorable to give an overall picture of quality *on the average.*

To get such a picture does not in general require special data. The detailed data obtained in the course of routine inspection, while often used only for the immediate purpose of determining the satisfactoriness of individual lots of product, are just what is needed for the present purposes. These inspections are critical examinations of the features that are essential to proper operation of the product in service. Hence the results are a measure of quality. There are of course many possible ways of classifying and combining this quantitative information, some of which are more efficient than others. The problem is to set up a method of handling the data in a way which will paint as clear a picture as possible of the overall quality.

The rate here described has been found very useful for measuring the quality of communication equipment and materials entering the plant of the Bell System.[1] It recognizes and takes account of the relative seriousness of different types of defects found in the course of

[1] This method of rating is being used extensively by the Manufacturing Department of the Western Electric Company where some of the features outlined in this paper originated.

inspection. For convenience, the rate is made a relative figure which incorporates the features of index numbers used by the economist. Just as the index numbers of cost of living, wages, corn production, etc., indicate current conditions relative to some reference condition as

$$\text{Index Number} = 100\,\frac{\text{Current Cost of Living}}{\text{1914 Cost of Living}},$$

so does the rate reflect current quality relative to that of a selected standard of reference. One of the features of the rate is its assistance in controlling quality, its provision of means for discriminating between chance and non-chance variations from the quality level which should currently be expected.

Character of Inspection

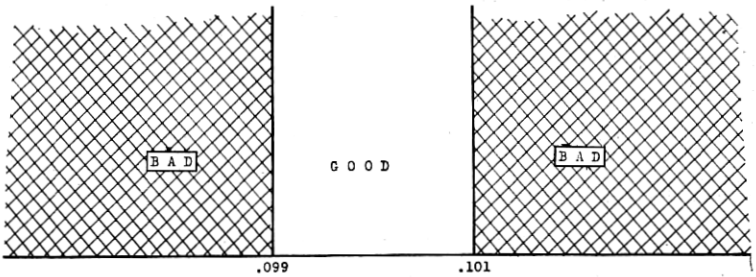
As in other fields much of the inspection work on telephone products consists of critical examinations of essential features to determine whether or not the units of product conform with specification requirements. This is done:

(1) By visual examinations in which obvious defects of material or workmanship are discovered by eye.
(2) By using "Go" and "No Go" gauges or their equivalent, which determine whether a unit does or does not conform with a requirement, or
(3) By using measuring instruments which reveal the numerical magnitude of the characteristic for each unit tested.

To illustrate the last two kinds of inspection, the specification requirement for the capacity of a type of condenser is "not less than .099 microfarad and not more than .101 microfarad." Inspection may be done by the "Method of Attributes," using a test set which shows merely whether the capacity of a condenser is inside or outside of the limits, or by the "Method of Variables," using an indicating or recording meter to show the numerical value of capacity for each test. In these two cases the data, if tabulated, would appear as in Fig. 1. Inspection data used for rating come in both varieties.

Items which Enter the Rate

Commercial measurement of quality by inspection usually consists in a comparison with stated requirements. Starting with the design and a knowledge of what can be accomplished in the shop, allowances for variations in materials, dimensions and salient properties are established in specifications. The aggregate of specification requirements constitutes a standard of quality which the manufacturer holds before him as an upper limit of attainment. To him, perfect performance is 100 per cent conformance with requirements and the resulting product he regards as of "perfect quality." The rate encompasses this narrow viewpoint of quality and measures the success to the manufacturer in living up to this adopted standard.



Capacity (in Microfarads)

Inspection by Method of Attributes		Inspection by Method of Variables	
Condenser No.	Observation	Condenser No.	Observation
1	Good	1	.0991
2	Good	2	.1006
3	Bad	3	.0985
4	Good	4	.0995
—		—	
—		—	
—		—	
121	Good	121	.0999
122	Bad	122	.1013
123	Good	123	.0994

Fig. 1—Two methods of measuring the quality of condensers in respect to capacity

The only items which enter the rate are the "defects," i.e. failures to meet requirements, found in the course of inspection. Experience has shown that percentage non-defective, the ratio of perfect parts to the total parts, while useful for certain classes of investigation, is not a very satisfactory yardstick for measuring quality of complex products. This factor fails to take into account two important things:

(1) Defects of different kinds are not equally serious.
(2) Defects of the same kind vary in seriousness according to the degree of departure from specified limits.

Thus a failure to meet a major requirement should have greater weight than a failure to meet a minor one and in like manner the degree of imperfection of a given kind should be taken into consideration.

The rating method recognizes such gradations in seriousness by making use of a system of weighting defects.

Method of Weighting Defects

The seriousness of a defect is judged from the standpoint of the consumer. A defect, if allowed to get into service, means trouble in one form or another, and trouble costs money. Seriousness depends fundamentally upon the evaluation of the loss or expense that would be incurred by using the defective unit. The determination of exact costs of trouble is generally not possible but these costs or, better, the relative costs can be estimated. Such estimates may be based on past experience, judgment, engineering knowledge of service requirements, complaints received from consumers and available information on costs associated with past troubles in service.

A standard set of classes is adopted for defects associated with a given kind of product, the classes being ordered in seriousness and each sufficiently well defined to make the business of classification a fairly simple and uniform process. The following four-fold classification has been found satisfactory for many kinds of telephone products.

Class "A" Defects—Very serious.

Will render unit totally unfit for service.

Will surely cause operating failure of the unit in service which cannot be readily corrected on the job, e.g. open induction coil, transmitter without carbon, etc.

Liable to cause personal injury or property damage.

Class "B" Defects—Serious.

Will probably, but not surely, cause Class "A" operating failure of the unit in service.

Will surely cause trouble of a nature less serious than Class "A" operating failure, e.g. adjustment failure, operation below standard, etc.

Will surely cause increased maintenance or decreased life.

Class "C" Defects—Moderately serious.

Will possibly cause operating failure of the unit in service.

Likely to cause trouble of a nature less serious than operating failure.

Likely to cause increased maintenance or decreased life.

Major defects of appearance, finish or workmanship.

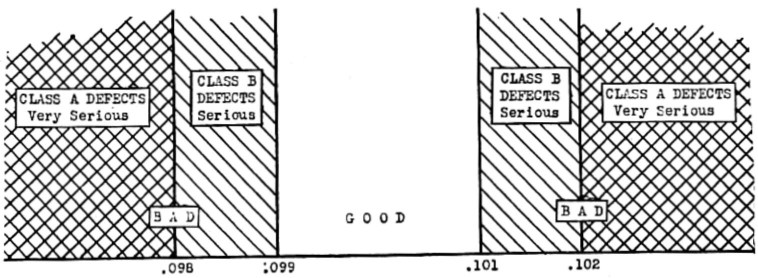
Class "D" Defects—Not serious.

Will not cause operating failure of the unit in service.

Minor defects of appearance, finish or workmanship.

It should be pointed out that the number of classes to be used is arbitrary. Two classes, major and minor, may be sufficient for some relatively simple products. The number of classes that can logically be used in any case depends upon the accuracy which can be attained in making estimates of relative seriousness.

Before proceeding further it may be well to indicate how the defects for features which are inspected as "variables" are weighted. Take the illustration accompanying Fig. 1. Any failure to meet the commercial limits of .099 and .101 microfarad will result in irregularities in transmission such as the distortion of the words spoken over a telephone line. The greater the departure from these limits the greater is the seriousness from a service standpoint. Strictly the weight for a defect should depend upon the degree of its departure from a limit but the desired result can be approximated to a satisfactory degree of accuracy by classifying the defects into two or more classes. To illustrate, assume two classes as indicated in Fig. 2. Defects falling within the



CAPACITY (in microfarads)

Fig. 2—Classification of defects for variable characteristics

ranges .098 to .099 and .101 to .102 are serious and can be considered as Class "B" defects in a four-fold classification while defects outside of the two outer limits .098 and .102 are Class "A" defects and can be weighted as such.

COMPUTATION OF THE RATE

A defect is weighted by assigning to it a number of "demerits." For a given kind of product each class of defects has a specified weight. Since the *relative* weights are alone of importance, the scale of demerits may be chosen arbitrarily.

The unit of measurement in the rating plan is "demerits per unit."[2] This factor is the simple sum of the demerits per unit contributed by the different types of defects found in inspection.

[2] The "unit" is commonly a physical unit of product such as a piece part, a partial assembly or a finished unit of apparatus or equipment. Exceptions to this rule have been found desirable for certain complicated types of product, such as switchboard sections ori nstalled central office equipment, in which cases the unit may be a natural element ol a physical unit such as a soldered connection, a circuit, etc.

$$\text{Demerits per unit} = \frac{w_1 d_1}{n_1} + \frac{w_2 d_2}{n_2} + \cdot \cdot \cdot \quad (1)$$

for all types of defects, where

w_1, w_2, etc. = weight (demerits per defect) for defects of type 1, 2, etc.

d_1, d_2, etc. = number of type 1, 2, etc., defects, and

n_1, n_2, etc. = number of units inspected for type 1, 2, etc., defects.

Instead of using equation (1) directly for indicating quality, it has seemed desirable to establish a rate which by its numerical magnitude gives an immediate indication of whether the quality is better or worse than that of some easily recognized reference condition. Resorting to methods commonly used in constructing index numbers, we therefore select a base period during which the manufacturing conditions and inspection methods were known to be essentially the same as at the present time, determine the demerits per unit for the representative data of that period, and set up the following index:

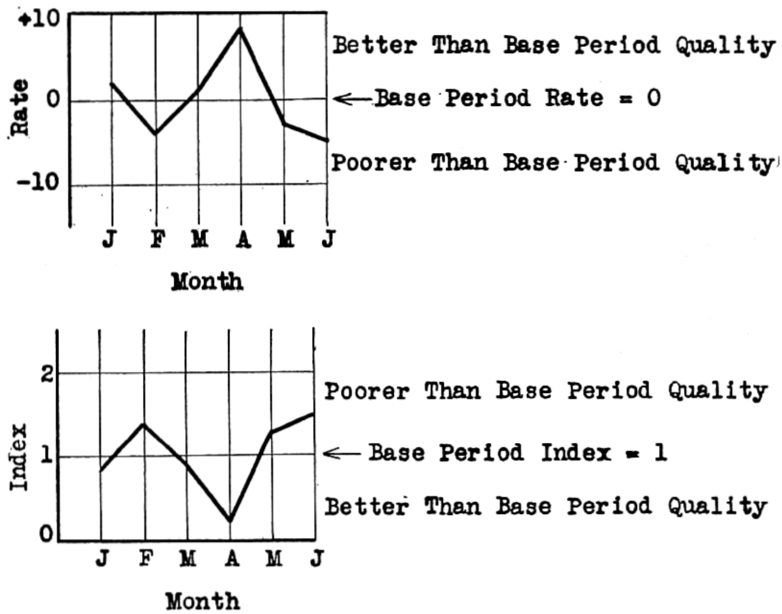


Fig. 3—Relation between index and rate for a given product and period of time

$$\text{Index} = \frac{\text{Current Demerits per Unit}}{\text{Base Period Demerits per Unit}}. \quad (2)$$

Since the demerit is an element of badness, this index increases in magnitude as the quality grows worse as shown in the upper chart on Fig. 3. It is preferable to have the rate high when the quality is good and low when the quality is bad. This has been taken care of by using the factor (1 − Index) in the rate equation,

$$\text{Rate} = 10\,(1 - \text{Index}), \tag{3}$$

where the factor 10 is introduced merely to make a convenient scale. This gives a rate of +10 for a product of perfect quality (i.e. no defects found in the material inspected), a rate of zero when current quality is the same as the average for the base period, and a negative rate when current quality is poorer than that of the base period. This equation, as portrayed by the lower chart of Fig. 3, is merely a numerical way of saying "better than" or "poorer than" base period quality and it also tells how much better or poorer.

The choice of base period rests on judgment and knowledge of conditions and must be made with the eyes open. To take care of evolutionary changes in manufacturing conditions for telephone products it has been found desirable to use a *moving* base period [3] of not longer than five years. The use of a somewhat extended period where possible has the advantage of stability in that it tends to smooth out the high and low spots resulting from temporarily abnormal conditions of production such as are liable to recur in the future. The magnitude of the *base period demerits per unit* thus establishes a reference level for quality under *average* conditions.[4]

Quality-Control Feature of the Rate

Rates obtained from week to week or from month to month are used to indicate whether quality has been controlled. If manufacturing conditions are steady and everything is running smoothly, some definite value of rate can be *expected*. But even with a perfectly controlled process, there will be fluctuations above and below the expected rate value, fluctuations resulting from the effects of a large number of causes over which the manufacturer has no control.

[3] By a moving base period of 3 years is meant the three years just preceding the current year. With a moving base period the standard of reference (the denominator of the index) will change slightly at the beginning of each year as one year is dropped and a new one, the preceding, is added to the base.

[4] The average of past experience is sometimes a suitable estimate of *expected* quality but its indiscriminate use for this purpose is to be avoided. For products which are reasonably well controlled this estimate will often serve satisfactorily. Primarily the denominator of the index is a magnitude chosen to represent some standard of reference. The numerical rate obtained at any time reflects quality relative to the standard. It is not essential to the rate that the expectancy feature be stressed in this connection. Expectancy is, however, of importance to the control limits discussed in the subsequent paragraphs.

How low does a rate have to fall before lack of control is indicated? Does a rate of -10 signify that something abnormal has happened? The following discussion gives a method which can be used to detect lack of control.

First of all we must determine the value of rate to be expected, i.e. establish a norm for expected quality. Past experience can usually be used as a guide for this purpose. If the average quality during the base period is considered satisfactory as an estimate of expected quality under current conditions, then the expected rate is 0. If only a portion of past data is judged suitable for this purpose, then the rate figure corresponding to the selected data is the expected value.

The method of establishing limits of expected variation for the rate makes use of statistical methods which have been described elsewhere,[5] but will be briefly reviewed. If the current rate deviates from the expected rate by an amount which is greater than can be attributed to chance, this will be taken as an indication of lack of control.

Just how chance enters the discussion will perhaps be better understood from the following. Each unit of product is the physical result of fashioning and combining various materials by a large number of manual and mechanical operations and processes. Every element in the production process which contributes to the final detailed character of a unit can be considered as a cause. Now the ideal state of affairs, purely conceptual to be sure but nevertheless one which is the goal in all attempts to secure greater uniformity of quality, is one in which each of the elemental causes or groups of causes (affecting a particular trait of the product) functions continuously in the same manner to produce a given elemental effect in the direction of defective quality. Considering overall quality, one group of manufacturing causes is responsible for one type of defect, another group for a second type, etc. The aggregate of these many causes which cooperate to mould the product may be considered as a system of causes. When the concept of constancy-with-time is associated with all of the causes, the system is spoken of as a "constant system of causes," [6] i.e. one whose tendency toward defective quality does not change with time. Product turned out by such a system will be referred to as "uniform product."

For product which is uniform in this sense the rates obtained week

[5] "Quality Control Charts," by W. A. Shewhart, *Bell Sys. Tech. Jour.*, Vol. V, pp. 593–603, October, 1926.

[6] "Application of Statistics as an Aid in Maintaining Quality of a Manufactured Product," by W. A. Shewhart, *Jour. Am. Stat. Ass'n*, Vol. XX, pp. 546–548, December, 1925. It should be noted that the system of causes associated with the *data* used in rating is all-inclusive, encompassing the causes which are responsible for inaccuracies of measurement introduced by inspection as well as the manufacturing causes which affect actual quality.

by week or month by month will fluctuate around some average value according to the laws of chance. For example, in the manufacture of selectors assume conditions are such as to give uniform quality with an expected rate of 0. The rates for batches of selectors turned out weekly will fluctuate about Rate $=0$, the range of variation depending on the number produced each week. A week's output can be regarded merely as a sample of the product which this system of causes would turn out if it were allowed to function in the same manner for an

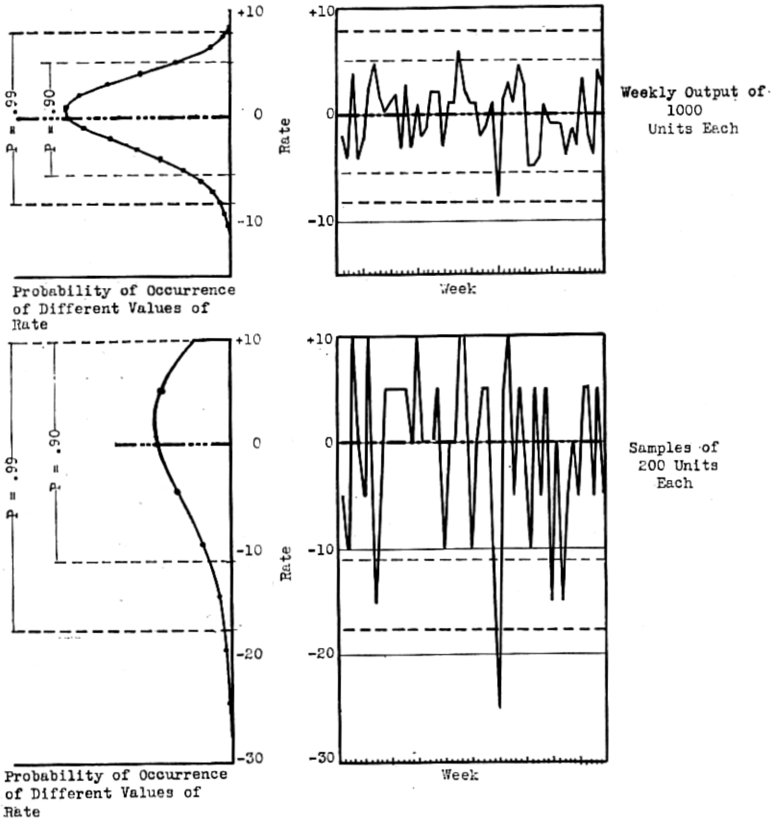


Fig. 4—Typical fluctuations of rates for uniform product whose expected rate = 0

indefinite length of time. The distribution of weekly rates is the same as would be obtained in an ordinary sampling experiment by drawing samples from an infinite warehouse of thoroughly mixed selectors having an average quality represented by Rate $=0$. If inspection consists in examining only a percentage of the selectors manufactured, this will be merely equivalent to taking smaller quantities of selectors from the warehouse and the resulting rates will be spread out more widely around 0 than when the entire product is inspected. These results are exemplified by the two diagrams in Fig. 4.

The probability curves of Fig. 4 represent the basis for setting control limits. The area under the curve between any two limits divided by the total area represents the probability that a single rate will fall between these limits. For a probability of .99 we can say that if the product is controlled at a level corresponding to the expected rate (zero in the illustration) then the chances are 99 in 100 that the current rate will fall within the limits thus established and only 1 in 100 that it will fall outside the limits.

For any product the spread between the limits is governed by two factors, the number of pieces inspected and the value of the above probability. It is necessary therefore to make an arbitrary choice of probability, a choice which will depend on the use to be made of the rate.

The control lines are used primarily to distinguish between those variations which may be attributed to chance causes and those which are more probably the result of some significant change in manufacturing conditions, either production or inspection, and therefore worthy of investigation. The criterion of the suitability of the limits chosen is the percentage of cases falling outside of the limits which on investigation are found to have resulted from some significant departures from current standards of performance.

In setting limits for rates the manufacturer has one point of view and the purchaser another. The manufacturer wishes to detect lack of control as early as possible and is willing to follow up false scents occasionally in his endeavor to prevent the persistence of costly irregularities. The purchaser is more interested in major swings or trends in quality, is not so much concerned with the use of limits for actual control and hence does not desire to instigate fruitless investigations frequently. For many telephone products, experience has indicated that a probability value between .90 and .95 is economical for shop control work while higher values such as .99 or above are better suited for quality reports issued for purposes of general information.

Inasmuch as the rate measures overall quality as determined by a number of different characteristics, its control feature relates particularly to final or partial assemblies of product. This control work should, of course, be preceded by control activities based on the same principles applied to the process inspection data for each of the essential characteristics of the parts which make up the whole.

PARTIAL SUMMARY OF INDIVIDUAL DEFECTS

Type of Defect	Demerit Weight	Base Period 1922	1923	1924	1925	1926 Jan.	1926 Feb.	1926 Mar.	1926 Dec.	1926 Total	Current Year 1927 Jan.	1927 Feb.	1927 Mar.	1927 Aug.
Electrical														
Type 1	100	0	0	0	1	0	0	0	16	24	18	5	14	0
Type 2	100	30	147	168	135	3	20	12	9	171	0	19	0	1
Mechanical														
Type 3	100	1	0	0	0	0	0	0	0	0	0	0	0	5
Type 4	100	0	0	9	9	0	3	1	0	10	1	1	0	0
Mounting and Assembly														
Type 5	35	5	0	1	1	2	11	0	1	44	0	0	0	0
Type 6	35	0	0	0	1	0	1	0	0	7	0	0	0	0
Type 7	20	6	11	8	8	0	0	0	1	5	0	0	1	1
Type 8	20	2	0	21	5	0	0	0	1	13	0	0	0	0
Wiring and Soldering														
Type 9	50	0	0	0	6	0	0	0	0	0	0	0	0	0
Type 10	35	0	0	6	1	0	0	0	0	0	0	0	0	0
Marking														
Type 11	50	3	4	8	8	0	0	0	1	5	0	0	0	2
Packing														
Type 12	50	13	0	0	0	0	0	0	0	0	0	0	0	0
Summary of Total Defects by Classes														
Class A	100	44	162	193	176	3	24	16	27	255	19	28	33	27
Class B	60	21	29	42	21	0	0	5	0	57	1	0	0	9
Class C	25	12	25	55	31	7	25	6	1	131	2	0	2	12
Class D	5	14	36	59	16	1	7	1	6	48	0	0	2	1
No. Inspected*		7,478	24,871	24,524	23,093	1,230	2,184	3,201	3,160	31,385	2,579	2,657	3,424	2,475
Rate		+1.05	+1.64	−0.63	+0.52	+4.9	−5.5	+2.9	+0.3	−1.29	+1.4	−1.7	−0.9	−5.9

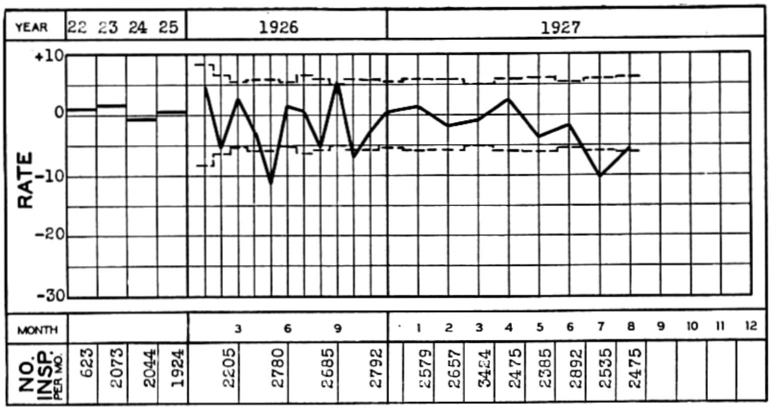
* Same for all types of defects.

Fig. 5—A typical classified summary of the defects found in inspection

Illustrative Examples

Example I—Monthly Rate for General Information Purposes Showing Quality of Finished Product

This example presents a monthly quality report for a kind of telephone apparatus which is manufactured in large quantities for use in



Defect	Wt. w	Base Period			March 1927		
		No. of Defects d	No. Insp. n	Demerits per Unit $\left(\frac{D}{n}\right)=\frac{wd}{n}$	Expected Dem. per Unit $\left(\frac{D}{n}\right)_e$	No. Insp. n	$\frac{w}{n}\left(\frac{D}{n}\right)_e$
Class A......	100	830	111,351	.7454	.7454	3,424	.02177
Class B......	60	170	111,351	.0916	.0916	3.424	.00160
Class C......	25	254	111,351	.0570	.0570	3,424	.00042
Class D......	5	173	111,351	.0078	.0078	3,424	.00001
Total				Σ.9018			Σ.02380

Computation of Control Limits.—

$$k = \frac{1}{\text{Base Period Demerits per Unit}} = 1.109, \qquad R_e = 0, \qquad K = 3,$$

$$\sigma_{R_e} = 10k\sqrt{\Sigma\left[\frac{w}{n}\left(\frac{D}{n}\right)_e\right]} = 10(1.109)\sqrt{.02380} = 1.711.$$

$$\begin{aligned}\text{Limits } R_L &= R_e \pm K\sigma_{R_e}.\\ &= 0 \pm 3(1.711)\\ &= +5.133 \text{ and } -5.133.\end{aligned}$$

Fig. 6—A typical rating chart with variable control limits

central office exchanges. The data given in Fig. 5 represent a portion of the summarized results of inspection. The detailed information given in tabulations such as this is the basic material used in investigation work relating to control of quality. The composite summary at the bottom of the figure is used directly for computing rates.

The quality report for this product is based on the following:

(1) The data are obtained by the check inspection of representative samples of the total product.
(2) The average quality for the base period is taken as the "standard expected" quality for current product. Control lines are thus placed above and below the base period rate of 0.
(3) The limits are computed each month on the basis of the number inspected during that month, using a probability value of .997.

The computations shown below the rating chart of Fig. 6 indicate the work necessary to the determination of the control limits for a given month. The basis of these computations is given in the Appendix. The limit lines on the chart are broken lines merely because the number inspected varies from month to month.

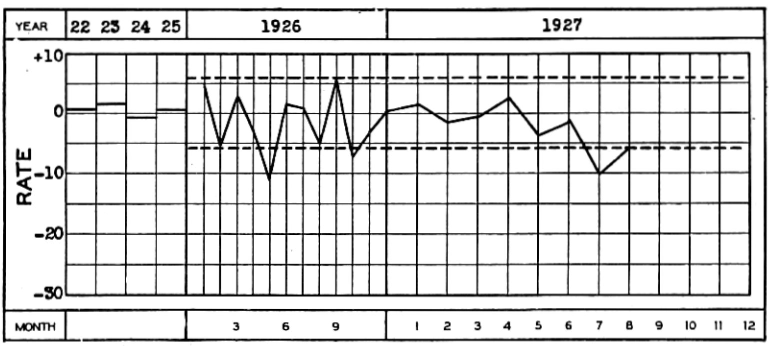


Fig. 7—Rating chart with constant control limits

When production and inspection schedules are fairly uniform so that the number inspected per month remains substantially constant, the limits may be computed once a year using some estimated size of monthly sample and drawn as parallel lines across the chart as in Fig. 7. This approximation is justified when the loss of accuracy thereby introduced is outweighed by the extra charting costs associated with monthly computations of limits.

Example II—Monthly Rates Showing Quality of a Product Before and After a Screening Inspection

Assume the following procedure to be in force.

(1) The shop product is inspected 100 per cent by an inspection group which serves as a screening medium for eliminating defective units.

(2) The product which passes this inspection is subsequently examined on a sampling basis by a check inspector who looks for the same defects.

The data obtained by the screening inspection provide a measure of the quality submitted by the Operating Department. The check inspection data give a picture of the quality of the finished product placed in stock. The rating chart is shown in Fig. 8. In a case of this sort where identical product is handled by two successive inspection groups, it is advantageous to show both rates on the same scale. In this particular instance a rate of 0 corresponds to the base period

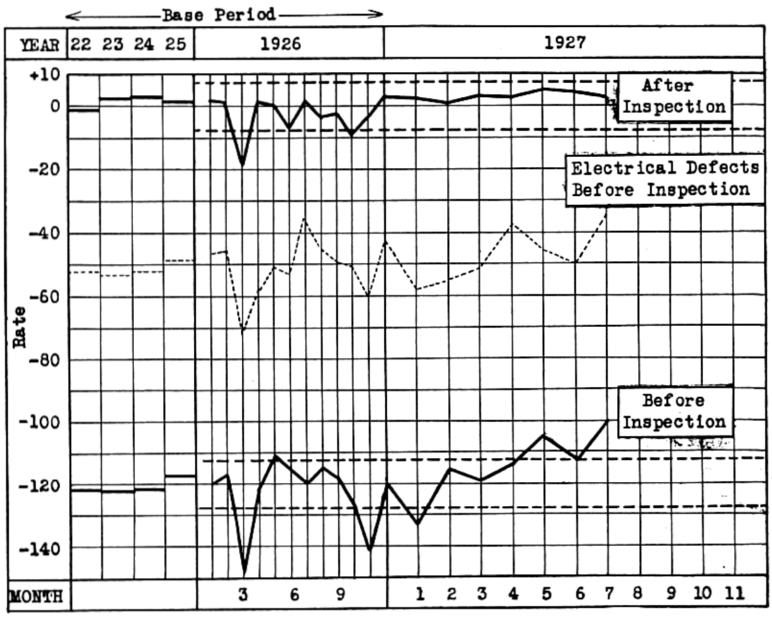


Fig. 8—Rate showing quality of a product before and after a screening inspection

quality of the product submitted to the check inspector. The control limits for the lower rate are drawn above and below the expected level of quality for product submitted to the first group of inspectors and both sets of limits are based on a probability of .95.

The results of the screening inspection can be used directly for controlling the work of the Operating Department. For this purpose it has been found valuable to prepare weekly rates with control limits based on a slightly lower probability value than that used for monthly rates. When the defects can be readily classified into two or more major groups, such as defects for electrical requirements, defects for mechanical requirements, etc., it has often been found useful to compute sub-rates with respect to such classifications of trouble. A typical sub-rate is indicated by the fine dotted line of Fig. 9. Ex-

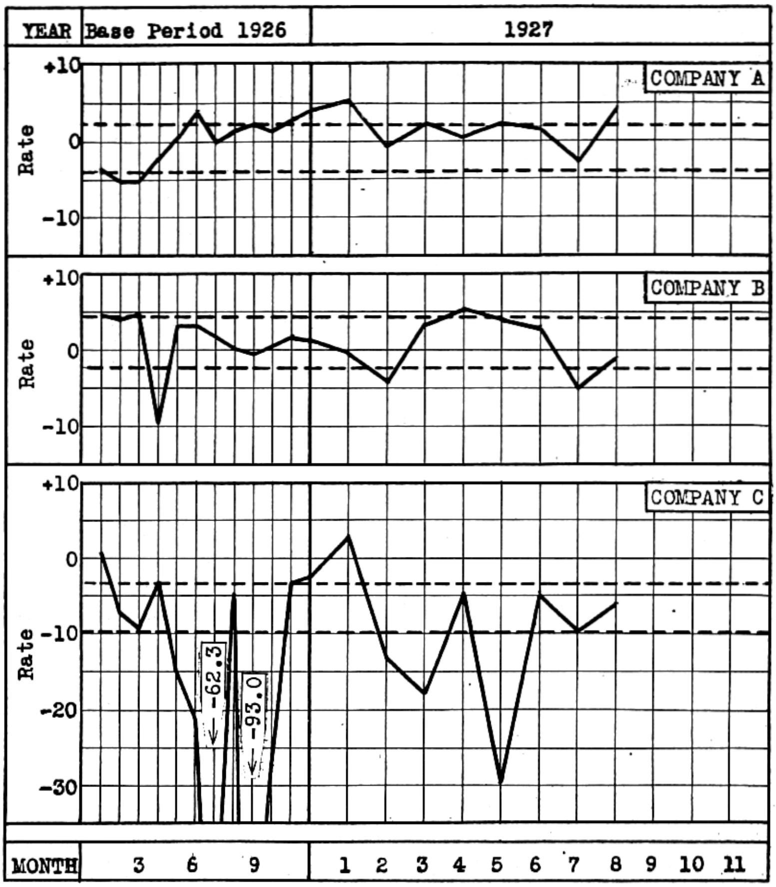


Fig. 9—Rates showing the quality of similar product supplied by three independent companies

perience has also shown the practicability of rating the quality of output of the individual operators and gangs in some classes of work for the purpose of comparing workmanship.

The principal advantage of these latter steps is to provide a ready means for tracing causes of trouble.

Example III—Rates for Comparing the Quality of Similar Product Supplied by Different Organizations

The rate can be used in like manner for comparing the quality of similar product produced by different factories of one company or by different companies when the several manufacturing units are governed by the same set of specifications.

As an example take the case of similar material supplied by three independent companies. There are several factors to be considered in constructing the rates. First of all, some standard of reference must be chosen to represent zero on the rate scale. This might correspond to the average performance of all companies, of the best company, the average of the two best, etc. Secondly, to show the degree of control for each company, the limits should be constructed independently for each company above and below the individual expectancy levels of quality.

The rating chart of Fig. 9 gives a quality picture for a particular kind of material supplied by three independent concerns, two of which are doing consistently better work than the third. Zero rate has been chosen arbitrarily to represent the average base period performance of the two best companies. The rate obtained monthly for any one company thus reflects, by its numerical magnitude, the relation between this company's current quality and that chosen as the standard of reference.

Graphical quality reports of this sort are of value to purchasing organizations in their relations with competing suppliers of similar materials.

APPENDIX

Computation of Control Limits

Assume

(1) $R = 0$ corresponds to base period demerits per unit.
(2) Control limits are to be set above and below some rate figure, R_e, corresponding to expected demerits per unit. (If expected quality is the same as base period quality, then $R_e = 0$.)
(3) Control limits are to be computed for monthly rates. (The procedure is similar for any other period of time.)

The following symbols will be used:

w = weight (demerits per defect) for a given type of defect.

d = expected number of defects per month, for a given type.

$D = wd$ = demerits for d defects.

n = number inspected for a given type of defect during the month.

$\left(\frac{D}{n}\right)_e$ = expected demerits per unit.

I_e = index for Expected Quality = $\frac{\text{Expected Demerits per Unit}}{\text{Base Period Demerits per Unit}}$.

R_e = rate for Expected Quality.

R_L = control limit value of rate.

$k = \frac{1}{\text{Base Period Demerits per Unit}}$.

σ = standard (root mean square) deviation.

The rate for Expected Quality is

$$R_e = 10(1 - I_e). \tag{1}$$

To find the control limits for the rate, first determine its standard deviation, σ_{R_e}.

$$\sigma_{I_e} = 10\sigma_{I_e}. \tag{2}$$

The index for Expected Quality is given by

$$I_e = k\left(\frac{w_1 d_1}{n_1} + \frac{w_2 d_2}{n_2} + \text{etc. for all types of defects}\right), \tag{3}$$

where d_1, d_2, etc. = expected number of defects per month for defects of type 1, 2, etc., and the subscripts 1, 2, etc., refer generally to the several types of defects.[7]

To find σ_{I_e}, the standard deviation of the index, the d's are considered as independent variables subject to sampling variations. I_e is then a linear function of d_1, d_2, etc., with the constant coefficients $\frac{kw_1}{n_1}$, $\frac{kw_2}{n_2}$, etc. Hence

$$\sigma_{I_e} = \sqrt{\left(\frac{kw_1}{n_1}\right)^2 \sigma_{d_1}^2 + \left(\frac{kw_2}{n_2}\right)^2 \sigma_{d_2}^2 + \cdots}. \tag{4}$$

The values of σ_{d_1}, σ_{d_2}, etc., are evaluated by the following consideration.

Assume that a sample of size N is drawn from a source for which the probability of occurrence of a defect is p. The expected number of defects in the sample is pN, and the standard deviation of the expected number is $\sqrt{p(1 - p)N}$. If p is small, the factor $(1 - p)$

[7] In carrying out the computations of rates and control limits, it is convenient to group together all defects having the same "weight" (w) and the same "number inspected" (n), and to let the subscripts 1, 2, etc., of the equations refer to these groups.

can be neglected,[8] which gives $\sqrt{pN}$. Considering the practical case, if the ratio of the number of defects is small compared with the possible number of defects, then the standard deviation of the expected number, d, is equal to $\sqrt{d}$. For many telephone products certain types of defects may occur several times on a single unit of product; for example, when inspection is made for the tension requirement of springs on a relay or for character of soldered connections on a switchboard, then N refers to the number of springs or the number of soldered connections, respectively, inspected during the month. Likewise p refers to the probability of occurrence of a defective spring or of a defective soldered connection. Fortunately, in the determination of the standard deviation it is not necessary to know exactly what p is nor what N is, so long as it is known that p is small (less than .10 for ordinary engineering purposes). This condition is usually satisfied in practice, hence the above result can be used.

Equation (4) then becomes

$$\sigma_{I_e} = \sqrt{\frac{k^2 w_1^2 d_1}{n_1^2} + \frac{k^2 w_2^2 d_2}{n_2^2} + \cdots}. \tag{5}$$

To simplify routine computations, this can be changed in form by removing the factor $\frac{wd}{n}$ from each term (this is merely the Expected Demerits per Unit $\left(\frac{D}{n}\right)_e$ for a given type of defect) which gives

$$\sigma_{I_e} = k\sqrt{\frac{w_1}{n_1}\left(\frac{D}{n}\right)_{e_1} + \frac{w_2}{n_2}\left(\frac{D}{n}\right)_{e_2} + \cdots}. \tag{6}$$

and the $\left(\frac{D}{n}\right)_e$ factors may be computed directly from the totality of data available for establishing the Expected Demerits per unit.

In shorter notation, equation (6) can be expressed as

$$\sigma_{I_e} = k\sqrt{\Sigma\left[\frac{w}{n}\left(\frac{D}{n}\right)_e\right]}. \tag{7}$$

If the number of units inspected is the same for all types of defects, i.e. $n_1 = n_2 =$ etc. $= n$, equation (7) becomes

$$\sigma_{I_e} = k\sqrt{\frac{1}{n}}\sqrt{\Sigma\left[w\left(\frac{D}{n}\right)_e\right]}. \tag{8}$$

[8] This follows directly from the Law of Small Numbers. Theoretically this result is obtained if p is small, N infinite and pN finite. See any standard text on the subject. Practically this law can be used as an approximation if p is less than .10 and N is greater than 16.

The control limits for the rate are obtained from the equation

$$R_L = R_e \pm K\sigma_{R_e}.$$

(assuming the Normal Law to be a satisfactory approximation for determining probabilities), where σ_{R_e} is given by equations (2) and (6), and where K is a constant whose value depends on the choice of probability.

The following table gives values of K for different values of probability.

Probability	K
.997	3.00
.990	2.33
.955	2.00
.900	1.65
.800	1.28
.683	1.00
.500	.675