Phase Distortion and Phase Distortion Correction

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SYNOPSIS: The importance of the rôle played by the steady state phase characteristics of long cable circuits has recently been emphasized in telephone and telegraph transmission. In this paper an analytical exposition of the theory of phase distortion is followed by a consideration of various methods of phase distortion correction with particular reference to terminal phase compensating networks and to the application of the lattice network to the loaded line as a terminal phase corrector.

1. INTRODUCTION

FROM the standpoint of ideal quality, a transmission system must be so designed that the received currents, which represent the transmitted signal, shall be a faithful copy of the corresponding currents which enter the transmission system at the sending end; that is, the transmission system must be distortionless. For relatively short distances the deleterious effects of phase distortion are not appreciable but as the range of transmission is increased a point may be reached where the impairment of quality becomes so serious as to reduce the commercial efficiency of the circuits. This fact was first recognized on long submarine telegraph circuits. With regard to telephony, the importance of the question of the quality of received speech was initially emphasized by the advent of the efficient telephone repeater,[1] making possible the increased length of the modern telephone system. This increased length, involving the necessity for circuits of higher quality, led to the development of improved loading designs.[2]

It has long been recognized as a principle of good telephone or telegraph transmission that the variation of attenuation over the range of speech or signal frequencies should be minimized. With the increased length of circuits, however, this requirement alone was found insufficient to insure good quality and *phase distortion* over the range of essential frequencies was found to play an important rôle. Thus steady state phase characteristics have attained prominence in the engineering of long cable circuits and in the application of this technique to telegraph, telephone and picture transmission.

Distortion in the variation with frequency of the phase difference between the current at the receiving end and that at the sending end of the transmission line, as well as distortion of the amplitude characteristic of the received as compared to the initial current, gives rise

[1] See reference 1.
[2] See reference 2.

to so-called transient effects. That is, the currents, after arriving at the distant end of the communication circuit, require an appreciable time, varying with the impressed frequency, to build up and, under certain conditions, may never build up to anything remotely resembling the transmitted currents in the short interval during which the latter exist. This effect, moreover, may produce serious impairment in the quality of the received speech or signal even when the line is so designed that the steady state attenuation of all currents in the essential range is substantially constant.

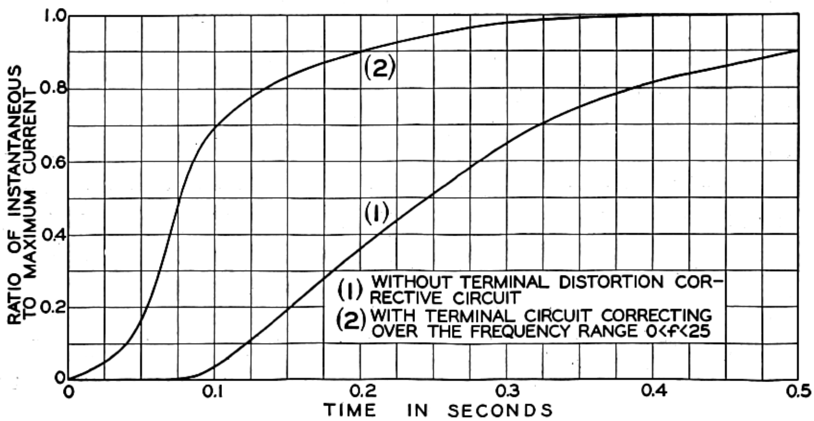

Fig. 1—Building-up of current on 1500 mile telegraph cable

As an illustration of the transient distortion on a transmission line, consider the indicial admittance, $A(t)$, of the cable of length l miles, and of resistance R and capacity C per mile; that is, the received current in response to a unit d.c. voltage applied at the sending end at time $t = 0$. This is given by [3]

$$A(t) = \frac{2}{Rl}\frac{e^{-1/y}}{\sqrt{y}},$$

where

$$y = \frac{4t}{RCl^2}.$$

Curve (1) of Fig. 1 represents relative values of $A(t)$ on a cable 1,500 miles long. (This is the same cable whose phase characteristic is shown in Fig. 5, the inductance being ignorable in determining the indicial admittance.) The departure of the received current from the abrupt wave front of the impressed d.c. voltage is clear.

[3] See reference 3.

The distortion on the loaded cable from the transient point of view is represented in Fig. 2, which shows the envelope of the building-up of current of frequency $\omega/2\pi$ in the Nth section of a periodically loaded line.[4]

The oscillograms of Fig. 3 show the received current in response to sinusoidal waves on a 600-mile medium heavy loaded (H-174) line.[5]

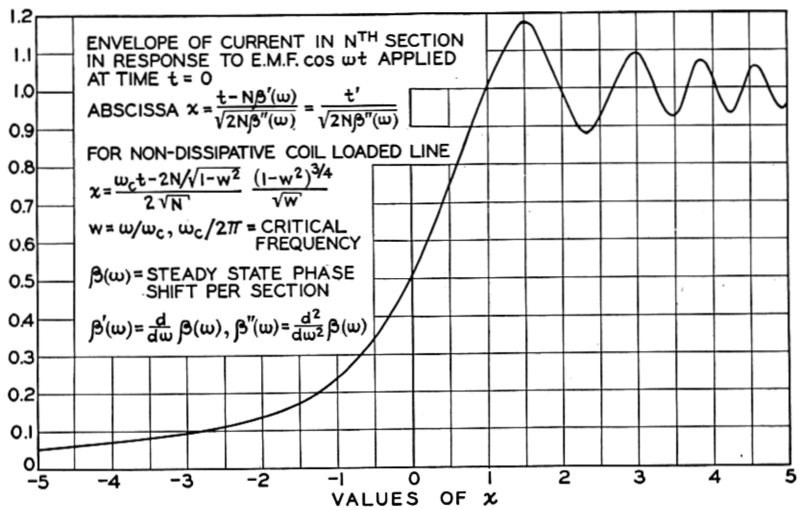

Fig. 2—Building-up of alternating currents in long periodically loaded line

Figs. 3*a* and 3*b* are for frequencies of 1,000 cycles and 1,500 cycles respectively and Fig. 3*c* is for a compound wave made up of 800 and 1,600 cycles. The last oscillogram shows clearly the greater delay of the higher frequency. Due to the relative weakness of this component the building-up transients while quite apparent are not pronounced. It will be observed that an appreciable time has elapsed in each case before the received wave has built up to the amplitude or frequency of the steady state.

The detrimental effects of transient distortion were first studied in the long submarine cable. Early in 1918 a theory of distortion correction was developed by John R. Carson from the transient point of view. The principle was arrived at that the *modified arrival curve* of prescribed form (a square-topped wave in the case of long line telegraphy) may be obtained by combining derivatives with respect to time of the *datum arrival curve* (the current obtained at the end of

[4] See reference 4.
[5] In this symbolism "H" refers to coil spacing of 6,000 ft., and the following number gives the inductance in millihenrys.

the cable itself) in the proper proportions to insure the steepness of building-up of the arrival curve and the maintaining of the tail of the wave. Terminal corrective networks[6] in combination with vacuum tubes were designed to obtain the requisite derivatives. Such a terminal corrective network is shown in Fig. 4, where the

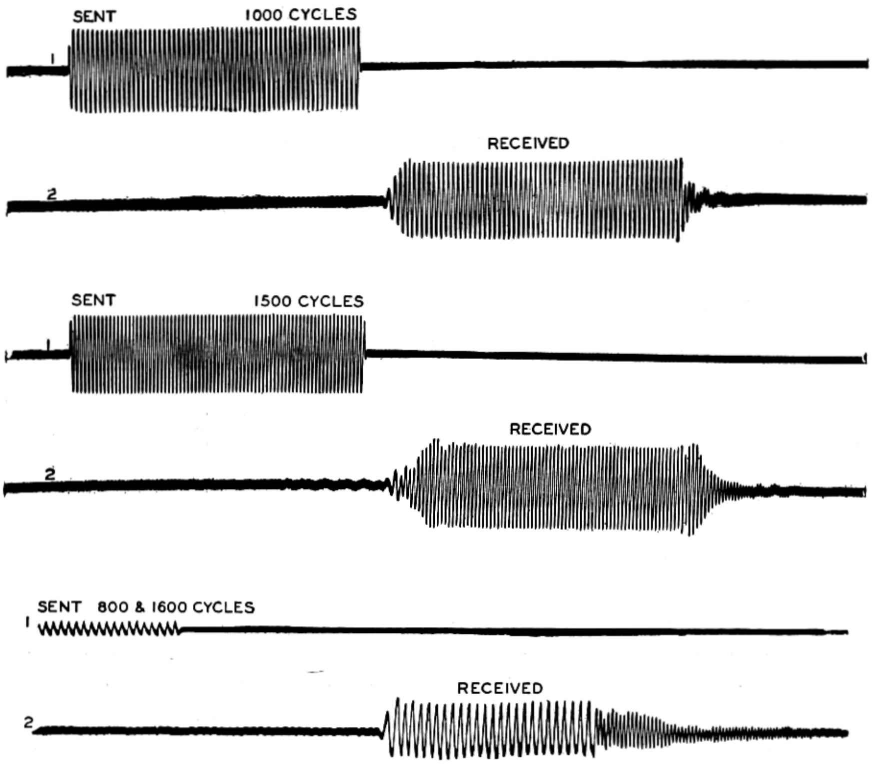

Fig. 3—Transient distortion in a 600 mile length of H-174 loaded cable

resistance R_1 is a distortionless one-way thermionic tube and $V(\omega)$ the output voltage. Examination of this network in the light of the more recent study of phase distortion correction from the steady state point of view has shown that it does also correct the phase distortion of the cable and provide some attenuation equalization as well.

Although the design of corrective networks on the basis of the steady state phase has usually been found more simple than on the transient basis, a knowledge of the arrival current or voltage as an

[6] See references 5 and 6. Also, for the development of distortion corrective circuits with vacuum tube amplifiers, or "signal shaping vacuum tube amplifiers" as they are called, in connection with their application to the new permalloy cables of the North Atlantic, see references 7 and 8.

explicit time function is sometimes essential, in the last analysis, to indicate the degree of correction afforded. This information may be supplied to sufficient precision by the first and succeeding derivatives with respect to frequency of the steady state phase, which, as explained

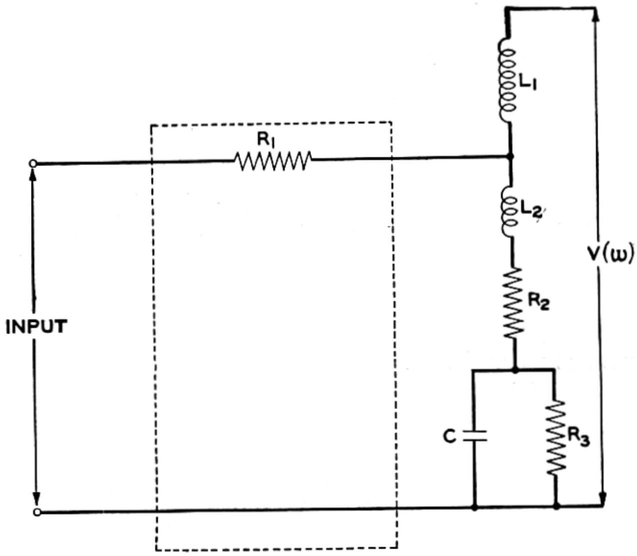

Fig. 4—Distortion corrective circuit for long telegraph cable

below, are extremely useful criteria of the improvement in the time and steepness, respectively, of building-up over a finite range of frequencies. Nevertheless, to visualize the actual effect of the applied phase compensation upon the arrival current or voltage due to an impressed e.m.f. of a specific frequency requires the evaluation of the explicit time function.

It is the object of this paper, following an analytical exposition of the theory of phase distortion, to consider various methods of phase distortion correction with particular reference to terminal phase compensating networks and the application of the lattice network to the loaded line as a terminal phase corrector.

II. Phase Distortion

1. *Steady State Theory of Phase Distortion*

The mathematical theory of phase distortion in signaling systems may be explained briefly from the steady state point of view. We suppose that it is required to transmit a signal $f(t)$, which can be represented by the Fourier integral,

$$f(t) = \frac{1}{\pi} \int_0^\infty F(\omega) \cos [\omega t + \theta(\omega)] d\omega, \tag{1}$$

and that the transfer impedance of the transmission system is given by

$$Z(i\omega) = |Z(i\omega)| e^{iB(\omega)},$$

where $\omega/2\pi$ is the frequency. The received current is, then,

$$I(t) = \frac{1}{\pi} \int_0^\infty \frac{F(\omega)}{|Z(i\omega)|} \cos [\omega t + \theta(\omega) - B(\omega)] d\omega. \tag{2}$$

$F(\omega)$ exists for all values of ω from zero to infinity but, practically, $F(\omega)$ is negligible except over a finite range which is determined by the nature of the signal. For program transmission, for example, the essential frequencies are now considered to lie in a band from about 100 to 5,000 cycles, while for slow speed telegraphy they lie in a band between zero and 10 or 20 cycles per second. If we suppose, then, that the essential frequency band extends from $\omega_1/2\pi$ to $\omega_2/2\pi$, we may replace equations (1) and (2) by

$$f(t) = \frac{1}{\pi} \int_{\omega_1}^{\omega_2} F(\omega) \cos [\omega t + \theta(\omega)] d\omega \tag{3}$$

and

$$I(t) = \frac{1}{\pi} \int_{\omega_1}^{\omega_2} \frac{F(\omega)}{|Z(i\omega)|} \cos [\omega t + \theta(\omega) - B(\omega)] d\omega. \tag{4}$$

Now suppose that within the band of essential frequencies, $\omega_1 < \omega < \omega_2$, we have

$$|Z(i\omega)| = R \tag{5}$$

and

$$B(\omega) = \omega\tau \pm n\pi,$$

where R and τ are constants and $n = 0, 1, 2, \cdots$. Then we may write

$$I(t) = \pm \frac{1}{\pi R} \int_{\omega_1}^{\omega_2} F(\omega) \cos [\omega(t - \tau) + \theta(\omega)] d\omega, \tag{6}$$

$$= \pm \frac{1}{R} f(t - \tau),$$

$I(t)$ being positive or negative according to whether n is even or odd. Whence the received current is proportional in amplitude to the applied signal and merely delayed in time by the 'transmission time' τ. Thus the received current has the same wave form as the applied signal or the transmission is distortionless. Accordingly, we have the following proposition.[7] *The necessary and sufficient condition for the practically distortionless transmission of signals in communication systems is that, over the essential range of frequencies contained in the transmitted signal, the transfer impedance of the transmission circuit be equalized both as regards amplitude and phase; that is, the amplitude must be constant and the phase angle linear in the frequency, with a value, when the frequency is zero, of* $\pm n\pi$, *where* $n = 0, 1, 2, \cdots$.

For many years the variation of the phase angle with frequency was ignored. Research in distortion correction was directed to devising networks [8] so designed that $|Z(i\omega)|$ would be a constant, R, over the range of essential frequencies. Assuming that this condition is fulfilled by the transducer but that

$$B(\omega) = \omega\tau + \sigma(\omega) \pm n\pi,$$

where $\sigma(\omega)$ is non-linear in the frequency, we may write (4) as

$$I(t) = \pm \frac{1}{\pi R} \int_{\omega_1}^{\omega_2} F(\omega) \cos\left[\omega(t - \tau) + \theta(\omega) - \sigma(\omega)\right] d\omega. \tag{7}$$

In formula (7) the *amplitudes* of the component frequencies of the arrival curve are, within a constant, the same as those in the impressed signal $f(t)$. The *wave form* of the arrival curve, owing to the presence of the phase $\sigma(\omega)$, may, however, be widely different from that of the impressed signal.[9]

2. *Examples of Phase Distortion in Transmission Systems*

Let us consider the frequency-phase angle characteristic of the two important transmission systems, the submarine telegraph cable and the loaded line.

The cable of characteristic impedance $k = \sqrt{(R + i\omega L)/i\omega C}$ and propagation constant $\gamma = \sqrt{(R + i\omega L)i\omega C}$ (with negligible leakage) is assumed terminated in its characteristic impedance at $x = l$ so that reflection is suppressed. The transfer impedance $Z(i\omega)$ is then

$$\begin{aligned} Z(i\omega) &= ke^{\gamma l} \\ &= |Z(i\omega)|e^{iB(\omega)}, \end{aligned} \tag{8}$$

[7] See reference 9.
[8] See reference 10.
[9] In telephone transmission it is not at all certain that preservation of wave form is essential. It is essential, however, that the components of different frequencies build up at approximately the same time. It is further demonstrated in the section on 'Loading Systems' below that $\sigma(\omega) = 0$ is the necessary and sufficient condition to fulfill the latter requirement.

where $B(\omega) = \omega\tau + \sigma(\omega) \pm n\pi$ and is the phase angle of the transfer impedance, provided, as we shall assume, that k is approximately a constant. Whether k is a constant or not, $B(\omega)$ represents the difference in phase between the currents at the sending and receiving ends. $B(\omega)$ is given by

$$B(\omega) = l\omega\sqrt{LC}\sqrt{\tfrac{1}{2}[1 + \sqrt{1 + (R/\omega L)^2}]}. \tag{9}$$

Fig. 5 shows $B(\omega)$ and $\sigma(\omega)$ in radians for a 500-mile length of cable whose constants are:

resistance $R = 2.74$ ohms per mile,
inductance $L = 0.001$ henry per mile,
capacity $C = 0.296$ microfarad per mile,

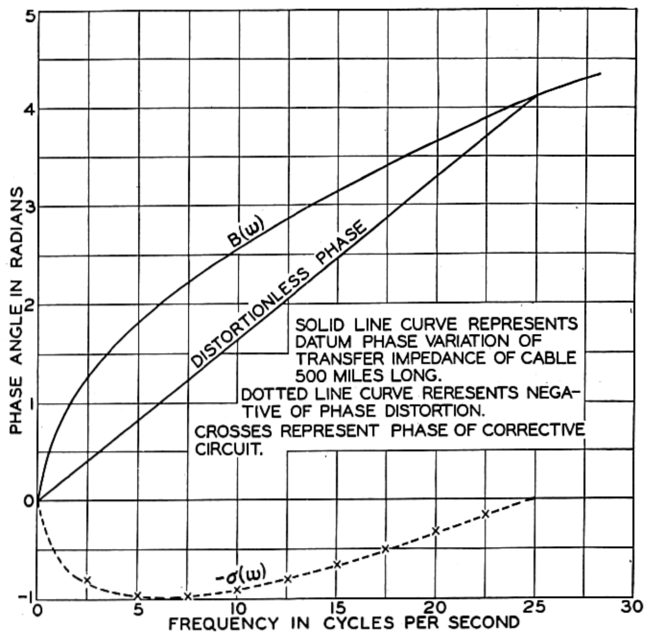

Fig. 5—Phase distortion correction on long telegraph cable

and for the frequency range, 0–25 cycles per second. The straight line, $\omega\tau \pm n\pi$, representing the distortionless phase characteristic is not fixed except that it must pass through the origin or $\pm n\pi$ at zero frequency. It is here chosen to pass through the origin and to have the same value as the cable phase itself at $f = 25$ c.p.s. The dotted curve representing $-\sigma(\omega)$ then shows the departure (with the sign reversed) of the cable phase from the distortionless characteristic.

With regard to the loaded cable, we have similarly the phase angle $B(\omega)$ of the transfer impedance of the cable of length l miles with load impedance Z and smooth line constants R, L, G and C per mile, given rigorously by

$$B(\omega) = \text{imag. comp.} \frac{l}{s} \cosh^{-1}\left[\cosh \gamma s + \frac{Z}{2k} \sinh \gamma s\right], \qquad (10)$$

where

$$s = \text{spacing of load coils in miles,}$$

$$\gamma = \sqrt{(R + i\omega L)(G + i\omega C)},$$

$$k = \sqrt{(R + i\omega L)/(G + i\omega C)}.$$

It has been found, however, that dissipation and the distributed nature of the line constants have no appreciable effect on the phase

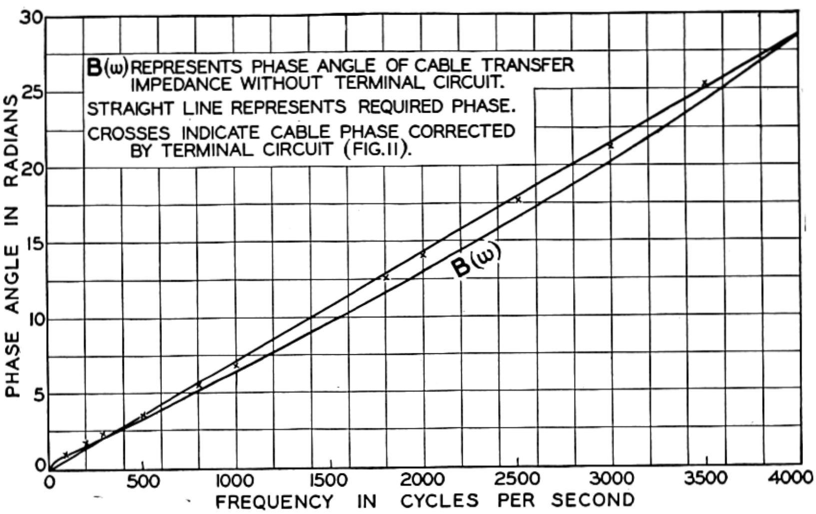

Fig. 6—Phase distortion correction on 20 mile 19 gauge H-44-25 loaded cable, $0 < f < 4000$

variation below 4,000 cycles. Hence, as a close approximation, we may take $B(\omega)$ for the non-dissipative cable without distributed inductance; i.e., simply,

$$B(\omega) = 2N \sin^{-1} f/f_c, \qquad (11)$$

where

$$f_c = \frac{1}{\pi\sqrt{L_0 C_0}},$$

$$N = \text{number of sections,}$$

L_0 is the coil inductance and C_0 the lumped line capacity per secti on

Even on the light loaded lines designed especially for good quality on long repeatered circuits, the phase distortion is appreciable. The nominal cut-off of these circuits is about 5,600. Fig. 6 shows the phase characteristic of the transfer impedance of a section of side circuit of 19 Gauge H-44 cable only 20 miles long. On Fig. 10 is represented the negative of the phase distortion, $\sigma(\omega)$, obtained by taking $n = 0$ and $\tau = B(\omega_m)/\omega_m$ where $\omega_m/2\pi$ is taken as the highest essential frequency, in this case 4,000 cycles.

In speaking of a pure sinusoidal wave of only one frequency, a phase shift of more than 2π radians or one cycle would be meaningless since every cycle is identical to the preceding and the following cycles. To consider the variation of phase shift over a range of frequencies, however, the total phase shift at any frequency as compared to that at the lowest frequency of the range is required.

III. Phase Distortion Correction

1. *Terminal Networks: Application to the Submarine Cable*

The device of a terminal network having a compensating phase distortion, that is, a network having the phase angle of transfer impedance,

$$\phi(\omega) = [\omega\tau' - \sigma(\omega) \pm n\pi], \tag{12}$$

over the frequency interval $\omega_1 < \omega < \omega_2$ (τ' being a constant), is theoretically the most simple and, in practice, is probably the most flexible and effective method of phase distortion correction. Such a distortion corrective network, in series combination with the transducer in which the attenuation has been equalized, produces an arrival curve

$$I(t) = \frac{1}{\pi R}\int_{\omega_1}^{\omega_2} F(\omega)\cos\left[\omega(t - \tau - \tau') + \theta(\omega) \pm n\pi\right]d\omega, \tag{13}$$

which is proportional to

$$f(t - \tau - \tau')$$

provided, of course, that there is no reflection at the transducer terminals. The constant phase angle $\pm n\pi$ does not affect the sinusoidal wave form but merely changes the sign of the wave if n is odd.

The terminal phase corrective network or phase compensator is applicable, at least theoretically, to any type of phase distortion correction and may be supplementary to other forms of correction such as loading, for instance. Fig. 7 is a schematic diagram of the arrangement of the given transducer of transfer impedance $Z(i\omega)$ with phase angle $B(\omega)$ and the terminal phase distortion corrective network of transfer impedance $N(i\omega)$ with phase angle $\phi(\omega)$. In

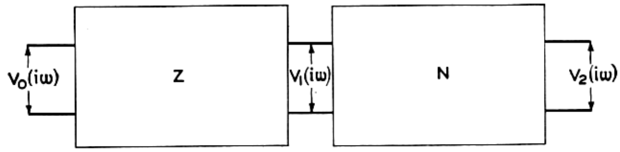

Fig. 7

response to the impressed voltage $V_0(i\omega)$, the voltage $V_1(i\omega)$ at the output terminals of the transducer, which is assumed proportional to the current, is then

$$V_1(i\omega) = \frac{1}{|Z(i\omega)|} e^{-iB(\omega)} V_0(i\omega)$$

and the final voltage $V_2(i\omega)$ is

$$V_2(i\omega) = \frac{1}{|N(i\omega)|} e^{-i\phi(\omega)} V_1(i\omega).$$

Thus

$$\frac{V_2(i\omega)}{V_0(i\omega)} = \frac{1}{|Z(i\omega)|} \frac{1}{|N(i\omega)|} e^{-i[B(\omega)+\phi(\omega)]}. \tag{14}$$

In practical applications, it is usually found advisable to take both τ' and n of equation (12) equal to zero. Then the required phase characteristic, $\phi(\omega)$, of the transfer impedance of the corrective network is

$$\phi(\omega) = -\sigma(\omega).$$

The function $-\sigma(\omega)$ is drawn in Fig. 5 for the submarine cable where $0 < \omega < \omega_m$ and $\omega_m/2\pi = 25$ cycles per second. This may be represented quite closely analytically by the expression

$$\phi(\omega) = \tan^{-1}\frac{ax}{1+bx^2}, \tag{15}$$

where

$$x = \frac{\omega}{\omega_m}\left(1 - \frac{\omega_m^2}{\omega^2}\right). \tag{16}$$

Thus

$$\text{when } \omega = 0,\ x = -\infty \text{ and } \phi = 0,$$

$$\text{when } \omega = \omega_m,\ x = 0 \text{ and } \phi = 0,$$

and

$$\text{when } 0 < \omega < \omega_m,\ -\infty < x < 0 \text{ and } \phi < 0.$$

Physically, this is realizable in the circuit of Fig. 8 consisting of a resonant element L, C, where $\omega_m = 1/\sqrt{LC}$, in parallel with a resistance R_1. The final voltage is taken across the resistance R_2 and the resistance r represents a vacuum tube. This unilateral element permits of suitable amplification and prevents reflection at the cable terminals so that the network may be designed without regard to any reaction upon the cable.

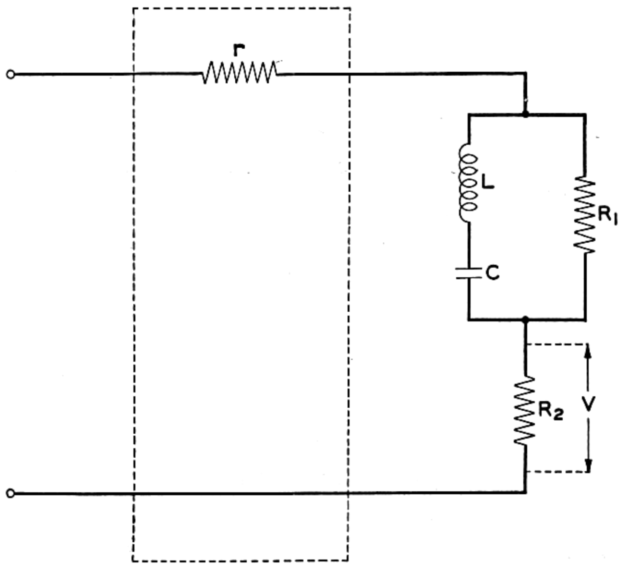

Fig. 8—Distortion corrective circuit for long telegraph cable

One section of the network of Fig. 8 with the values of the constants:

$$r + R_2 = 5{,}000 \text{ ohms}, \qquad L = 26 \text{ henrys}$$
$$R_1 = 51{,}100 \text{ ohms}, \qquad C = 1.56 \text{ microfarads}$$

is used when $l = 500$ miles. The phase of this network is shown in Fig. 5 also. Another equal network section may be added for each additional 500 miles of cable but there is no necessity, of course, for the sections to be equal. If it contains a one-way thermionic tube, each section may be added without affecting what has gone before, and the resultant phase angle will be simply the sum of all of the phase angles of the separate parts.

The improvement in the building-up of the indicial admittance accompanying the use of the phase compensator is evident from

Fig. 1 on comparing curve (2) with curve (1). Curve (2) is computed from the formula [3]

$$A(t) = \frac{2}{\pi}\int_0^\infty \frac{\alpha(\omega)}{\omega}\sin t\omega d\omega, \tag{17}$$

where $\alpha(\omega)$ is the real component of the transfer admittance of cable and network combined (equation (14)).

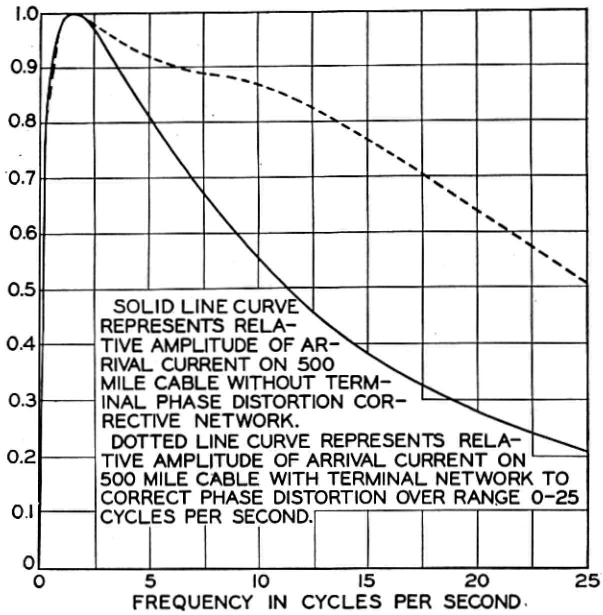

Fig. 9—Amplitude variation on long telegraph cable

The curves of Fig. 9 show that this network affords some attenuation equalization as well as very good phase correction. Amplitude and phase correction, as we have seen, are analytically independent processes. Nevertheless, some arrangements may, theoretically, be designed to correct amplitude and phase simultaneously. A method for so designing a network similar to the one under discussion at present has been developed by O. J. Zobel.[10] In such cases, however, in order to obtain physically desirable values in practical applications, it has usually been found necessary to design the network for one purpose, thereby automatically obtaining some improvement in the other respect, as in the present instance.

The maximum phase displacement obtainable with one section of

[10] This is discussed in a forthcoming paper by O. J. Zobel.

this network is $\pi/2$ radians. Thus, it becomes necessary to use more than one section on a long cable but it is also advantageous from the point of view of flexibility of design. By adopting a standard unit, for a 500-mile section of cable, for instance, the networks may be readily accommodated to different lengths of line.

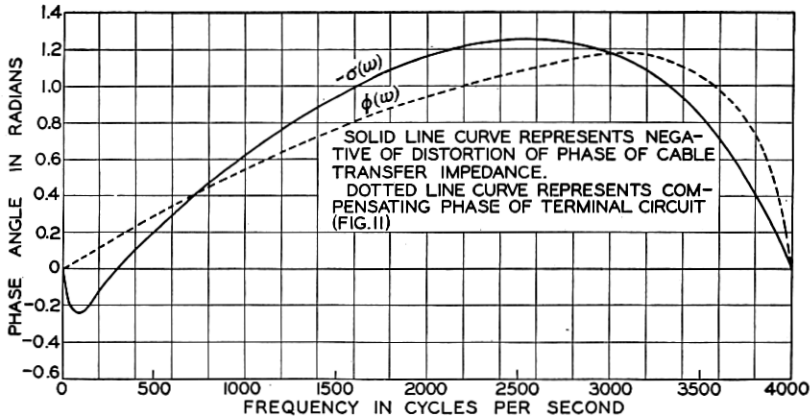

Fig. 10—Phase distortion correction on 20 mile 19 gauge H-44-25 loaded cable, $0 < f < 4000$

It is interesting to observe that the analytical expression

$$\phi(\omega) = \tan^{-1}\frac{ax}{1 + bx^2} \tag{15}$$

is positive when $0 < \omega < \omega_m$, and zero when $\omega = 0$ or ω_m, provided

$$x = \frac{\omega/\omega_m}{1 - \omega^2/\omega_m^2}. \tag{18}$$

Hence it will correct the phase distortion on the loaded cable as shown in Fig. 6. The required phase shift is obtainable in the type of network shown in Fig. 11. This network, however, has the disadvantage of tending to increase the attenuation distortion of the loaded cable rather than to equalize it.

When amplification is not required, it is undesirable to use the device of an amplifier in each section of network for the sole purpose of eliminating terminal reflection. As the characteristic impedance of a transmission line may usually be regarded as a constant resistance over the range of essential frequencies, the same purpose may be accomplished by applying networks whose characteristic impedance is also a constant resistance of the same value as the characteristic impedance of the line. A number of general recurrent networks of the so-called 'constant resistance' type[11] are known and such a

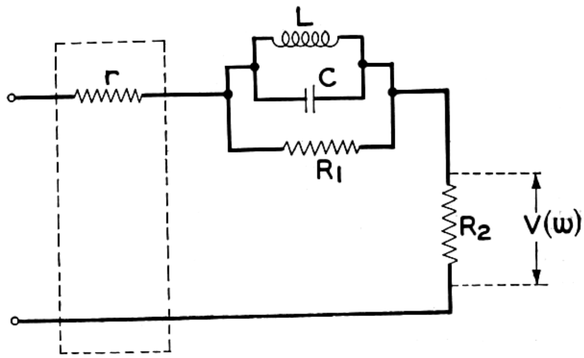

Fig. 11—Phase distortion corrective circuit for loaded cable. For 20 miles of 19 gauge H-44-25 loaded cable, r + R_2 = 5000 ohms, R_1 = 101,000 ohms, L = 0.460 henry, C = 0.0216 microfarad.

network[10] has been applied to correct the distortion on the submarine cable. The arrangement is shown in Fig. 12. It will be observed that z_{11} and z_{21} are inverse networks of impedance product R^2; that

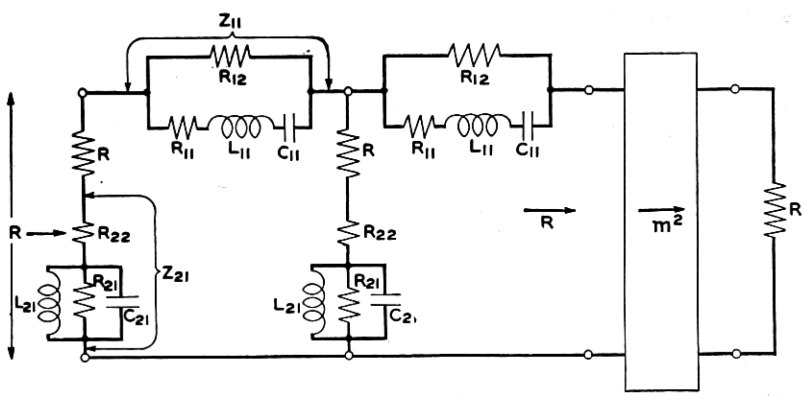

Fig. 12—Constant resistance type corrective circuit for long telegraph cable. m^2 represents distortionless amplifier

is, $z_{11}z_{21} = R^2$, which is, in general, the requisite relation among the network constants to provide a constant resistance characteristic impedance, R. In practical performance the networks of Figs. 8 and 12, each having the basic arrangement z_{11}, are equivalent. Thus the

[11] See reference 11.

function of eliminating terminal reflection, accomplished in the former by the vacuum tube, is accomplished in the latter by the additional impedance elements.

2. *Loading Systems*

The use of loaded cable circuits for the transmission of programs and for the transmission of pictures introduced some interesting problems in connection with phase correction at the higher frequencies. In this connection, a study of the building-up of sinusoidal oscillations in long loaded cables led to the establishment of two propositions which are of great value in comparing communication systems with respect to the quality of transmission.[12] These two propositions relate the variation with frequency of the steady state phase to the duration and nature of the transient distortion. They are applicable to all types of periodic loading, with or without terminal phase compensators under two restrictions; namely, (1) the line must comprise at least 100 loading sections and (2) the transducer as a whole must be approximately equalized as regards absolute steady state values of the received current in the neighborhood of the applied frequency. The successive derivatives of the total phase angle $B(\omega)$ with respect to ω will be denoted by $B'(\omega)$, $B''(\omega)$, $B'''(\omega)$. $B(\omega)$ may be understood to represent the sum of the phase differences due to transmission over the line and a terminal distortion corrective network, as well as the phase difference on the line alone. The propositions follow.

Case I: $B''(\omega) \neq 0$ and $\sqrt{B''(\omega)/2!}$ large compared with $\sqrt[3]{B'''(\omega)/3!}$.

The envelope of the oscillations in response to an e.m.f. $E \cos \omega t$ *applied at time* $t = 0$ *reaches 50 per cent. of its ultimate steady value at time* $t = B'(\omega)$ *and its rate of building-up is inversely proportional to* $\sqrt{B''(\omega)}$.

Case II: $B''(\omega) = 0$, $B'''(\omega) \neq 0$ and $\sqrt[3]{B'''(\omega)/3!}$ large compared with $\sqrt[4]{B^{\mathrm{IV}}(\omega)/4!}$.

The envelope of the oscillations in response to an e.m.f. $E \cos \omega t$ *applied at time* $t = 0$ *reaches 1/3 of its ultimate steady value at time* $t = B'(\omega)$ *and its rate of building-up is inversely proportional to* $\sqrt[3]{B'''(\omega)}$.

These propositions furnish supplementary verification of the condition with regard to phase variation already established as a requirement for distortionless transmission; namely, that the phase vary linearly with the frequency or

$$B(\omega) = \omega\tau,$$

[12] See reference 4.

where τ is a constant. It follows immediately that

$$B'(\omega) = \tau.$$

Thus, when the steady state value of phase propagation is in linear relation with the frequency, all signals of any frequency whatever reach their proximate steady state in the same interval of time τ. These propositions make it possible to calculate two important criteria of the transmission properties of the line: (1) the variation with respect to frequency of the time interval τ required for the current to build up to its proximate steady state value and (2) its rate of building-up at time $t = \tau$. The first is very important in telephony but has no significance in telegraphy since only one frequency is transmitted, whereas the second is important in both telephony and telegraphy. While the formulas underlying these propositions are approximate, they are sufficiently accurate for a study of the comparative merit of different types of transmission systems or for the design of loaded lines.

For the purpose of reducing transient distortion on a long loaded cable of N sections, the loading and critical frequency $\omega_c/2\pi$ may be designed on the basis that the two time intervals,

$$t_1 = 2N/\omega_c \tag{19}$$

and

$$t_2 = \frac{2N}{\omega_c}\left(\frac{1}{\sqrt{1 - \omega^2/\omega_c^2}} - 1\right), \tag{20}$$

should be less than given quantities determined by experience. The two expressions t_1 and t_2 represent, respectively, (1) the time of transmission of d.c. current or the nominal time of transmission and (2) the excess of the time of building-up of the frequency $\omega/2\pi$ to approximately one half the steady state value over the nominal time of transmission.[13] The right hand member of formula (20), although initially determined by Carson by a method entirely dissimilar to that in his later investigation, is, it will be noted, given by

$$t_2 = B'(\omega) - B'(0) = T - T_0 \tag{21}$$

and

$$t_1 = B'(0) = T_0. \tag{22}$$

The nominal time of transmission t_1 is significant from the standpoint of echo effects but, with regard to phase distortion correction alone, emphasis is placed upon the quantity $t_2 = T - T_0$ as the more important factor. The enormous improvement in this respect of the

[13] See reference 2.

system of light loading over medium heavy loading is shown in Fig. 19. Below about 3,200 cycles, $T - T_0 < .0005$ second on a 50 mile light loaded line. It is obvious that the higher the frequency the greater the distortion.

Another loading system proposed as a means of providing desirable phase characteristics is the lattice loaded line.[14] This consists of the use of a section of the network of Fig. 14 as the periodic loading unit instead of the ordinary coil unit. Putting $C = rC_0$, where C_0 is the capacity of the line between loading units per section, we have for this system, when non-dissipative,

$$f_c = \frac{1}{\pi\sqrt{LC_0}}, \tag{23}$$

$$B(\omega) = 2N \sin^{-1} \frac{f}{f_c}\sqrt{\frac{1+r}{1+r(f/f_c)^2}}, \tag{24}$$

$$B'(\omega) = \frac{N}{\pi f_c}\sqrt{1+r}\,\frac{1}{[1+r(f/f_c)^2]\sqrt{1-(f/f_c)^2}} \tag{25}$$

and

$$T_0 = B'(0) = \frac{N}{\pi f_c}\sqrt{1+r}. \tag{26}$$

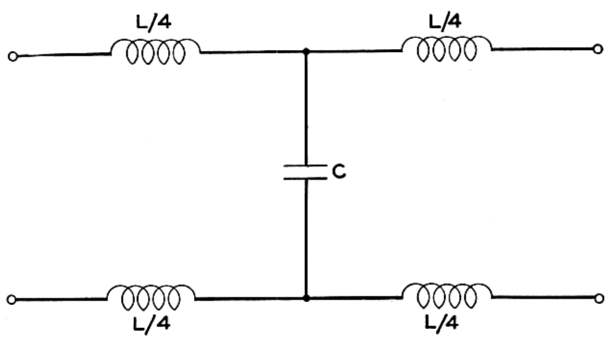

Fig. 13*a*—Section of standard non-dissipative coil loaded line. L = .044 henry, C = .0705 microfarad (for H-44 loading on 19 gauge side circuit.)

With the constants of the lattice loading system chosen to give approximately the same attenuation per mile as the standard loading, as in Figs. 13*a* and 13*b*, the time $t_2 = T - T_0$, it will be seen from Fig. 19, is less for it than that for the standard loading system for frequencies up to about 3,500 cycles but rapidly becomes greater thereafter. A combination of coil and lattice loading obtained by

[14] This was proposed by D. A. Quarles of the Bell Telephone Laboratories in unpublished memoranda.

alternating the coil and lattice loads affords some improvement in this respect. Another possible means of reducing the time interval $T - T_0$ at the higher frequencies is to reduce the spacing of the

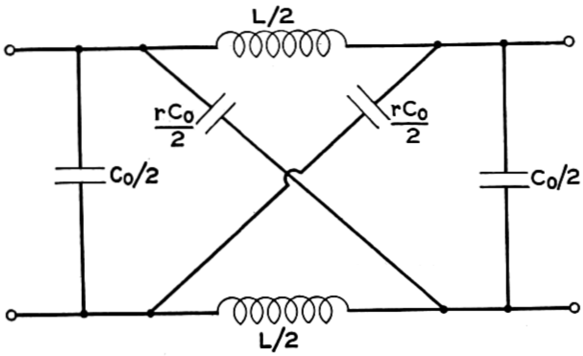

Fig. 13b—Section of non-dissipative lattice loaded line. L = .066 henry, C_0 = .0705 microfarad, r = .50 (19 gauge cable).

loading coils on the ordinary loaded line, keeping the attenuation per mile the same as before. As this change increases N and f_c in the same ratio, the effect is to reduce $T - T_0$.

3. *Lattice Type Terminal Network*

Instead of being used to replace the coil unit in the long periodically loaded line for the purpose of reducing the phase distortion, the lattice network [15] may be applied to the coil loaded line as a terminal phase compensator, a use to which it is peculiarly adapted by virtue of the nature of its characteristics.[16] These have been known for many years. The ideal non-dissipative simple lattice type recurrent network, shown in Fig. 14, with series inductance L and crossed capacity C per section, has a pure resistance characteristic impedance

$$K = \sqrt{L/C}, \tag{27}$$

a phase angle

$$\phi(\omega) = 2N \tan^{-1} (\omega\sqrt{LC}/2), \tag{28}$$

where N is the number of sections, and zero attenuation for all frequencies.[17] These relations readily follow from the general formulas (32)–(35) given below. The phase angle (Fig. 15), therefore, is

[15] The possible usefulness of the lattice network as a phase shifting device was pointed out in a general way by G. A. Campbell. See references 12 and 13.

[16] In this connection see references 12, 13 and 14 to the work of Karl Kupfmuller of the Siemens-Halske Company of Berlin, who independently applied the simple lattice network in the same way.

[17] See reference 13.

positive and varies from zero to π per section. Thus it is of a general form suitable to compensate for the distortion in the cable phase. One or more sections of the simple lattice network when properly

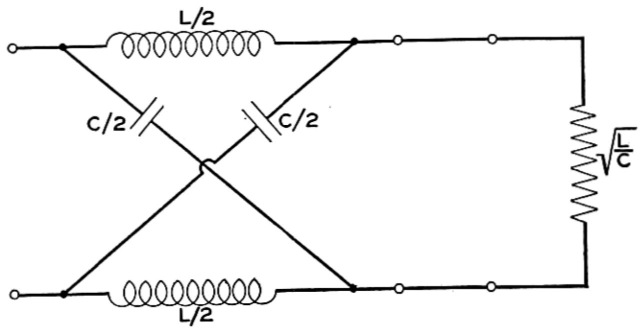

Fig. 14—Phase distortion corrective circuit for loaded cable: simple lattice type

terminated may then be connected directly to the loaded cable without appreciable reflection loss and without affecting the attenuation characteristic of the cable.

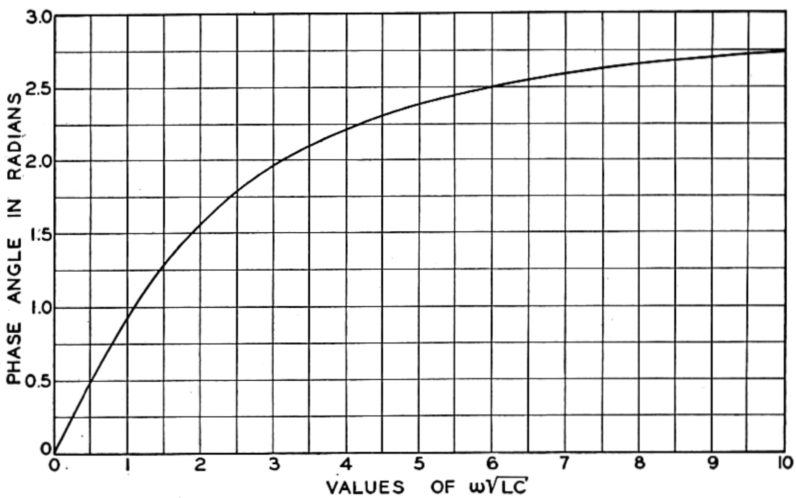

Fig. 15—Phase angle per section of simple lattice network

Proceeding to the transient aspect, the time T required for the current in response to the e.m.f. of frequency $\omega/2\pi$ to build up to 50 per cent. of its steady state value is given by

$$T = \phi'(\omega) = N\sqrt{LC}\frac{1}{1+\left(\frac{\sqrt{LC}}{2}\omega\right)^2} \tag{29}$$

and

$$T - T_0 = \phi'(\omega) - \phi'(0)$$
$$= N\sqrt{LC}\left(\frac{1}{1+\left(\frac{\sqrt{LC}}{2}\omega\right)^2} - 1\right). \tag{30}$$

The variation of $T - T_0$ in terms of the general parameter $\omega\sqrt{LC}$ is shown in Fig. 16. As T is a maximum at zero frequency and decreases with increasing frequency, its variation tends to equalize the delay on the cable. It will be demonstrated later that a more complicated type of lattice network is available to supplement the simple lattice type to improve the equalization still further.

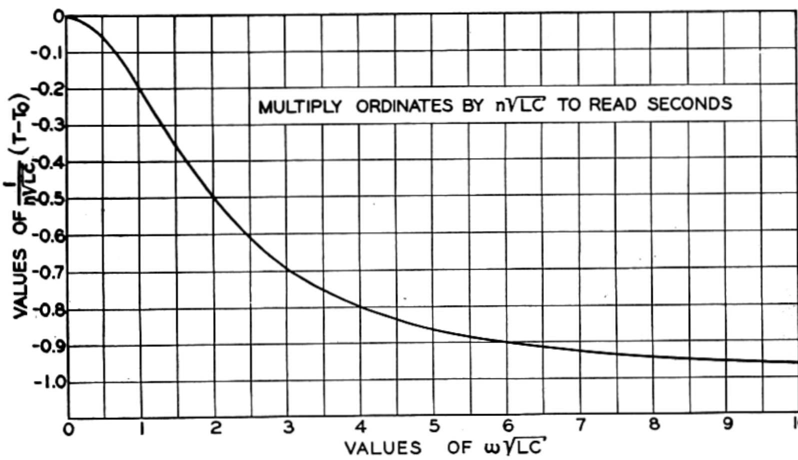

Fig. 16—Time of building-up, T − T₀, for simple lattice network of n sections

To provide partial delay equalization by means of the simple lattice network, phase angle variation alone need be considered without taking into account its derivatives. As pointed out above, the distortion $\sigma(\omega)$ of the cable phase $B(\omega)$ is given by

$$\sigma(\omega) = B(\omega) - \omega\tau \pm n\pi,$$

where τ is a constant representing the slope of the required distortionless phase. In the present case n will be zero. The constant τ also represents the time in which the current will build up to proximate steady state on the corrected cable. It is desirable, usually, that this time be as short as possible, and, moreover, the smaller the value of τ, the smaller will be the amount of distortion to be corrected, remembering, of course, that $\sigma(\omega)$ is to be negative. On the other hand, a consideration of the nature of the phase variation, $\phi(\omega)$, of the lattice network as compared to the distortion of the cable phase

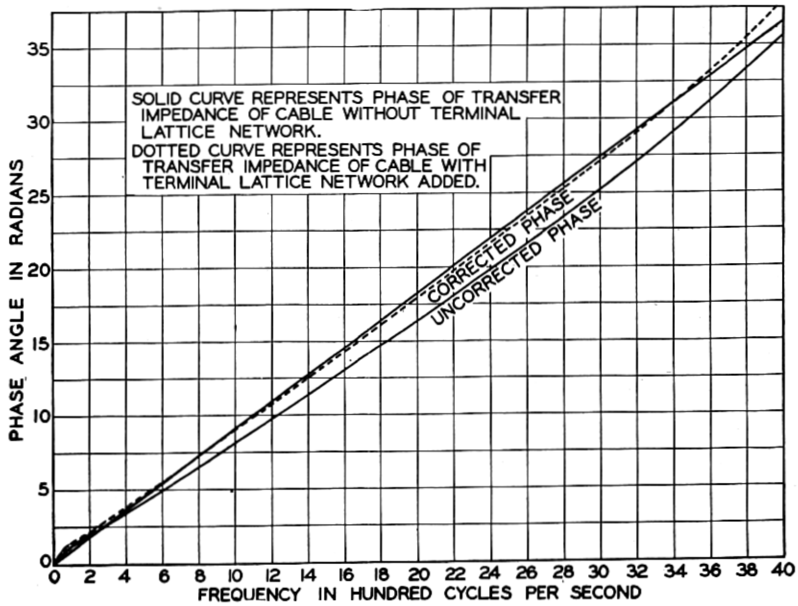

Fig. 17—Phase distortion correction on 25 miles of 19 gauge H-44-25 loaded cable

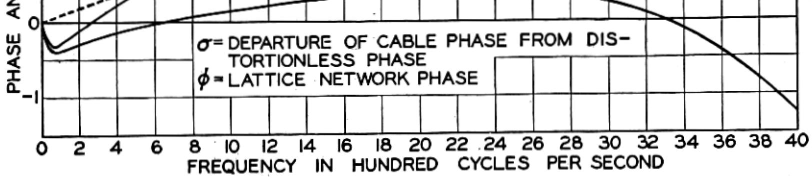

Fig. 18—Phase distortion correction on 25 mile 19-gauge H-44-25 cable

corresponding to various values of τ, leads to a choice of τ such that approximately the maximum cable distortion occurs at a frequency somewhat below the upper frequency of the essential range (Figs. 17 and 18).

While $\phi(\infty) = \pi$, it is apparent from Fig. 15 that ϕ increases very slowly for values of $\phi > 2.5$ and it is found that a distortion angle of not more than about 2.5 radians can well be compensated for with each section of lattice network. $\sigma_m/2.5$ determines roughly the number of sections required where σ_m represents the maximum distortion for the total length of line. It is only essential that the total number of sections be included where correction is desired. The corrective structure may be divided and one or more sections located at convenient points throughout the length of the cable whose phase is to be corrected. In the latter case a desirable arrangement on a repeatered cable is to insert at each repeater point a sufficient number of sections to correct the distortion on the cable length between two successive repeaters.

If the network is connected directly to the line, i.e., without the interposition of a unilateral element such as a vacuum tube, the necessary condition

$$K(\omega) = R,$$

where R is a constant representing approximately the characteristic impedance of the cable, imposes one limitation upon the constants L and C, leaving one other to be fixed by the phase. For this condition, put

$$\phi_a(\omega) = -\sigma_a,$$

where σ_a is the value of $\sigma(\omega)$ at a frequency $f_a = \omega_a/2\pi$ near the upper limiting frequency of the correction range and at which it is desirable to have exactly $\sigma(\omega) = -\phi(\omega)$. Substituting for K and ϕ_a in (27) and (28) and solving, gives

$$L = -\frac{2R}{\omega_a}\tan\frac{1}{2}\sigma_a, \qquad C = -\frac{2}{\omega_a R}\tan\frac{1}{2}\sigma_a. \tag{31}$$

An application of this method of design to the 19-gauge H-44-25 cable is shown in Figs. 17 and 18. This design requires two sections of lattice network at each repeater point of constants

$$L = .116 \text{ henry},$$
$$C = .181 \text{ microfarad}$$

per section, the distance between repeaters being 50 miles.

While the design above effects considerable improvement, it is

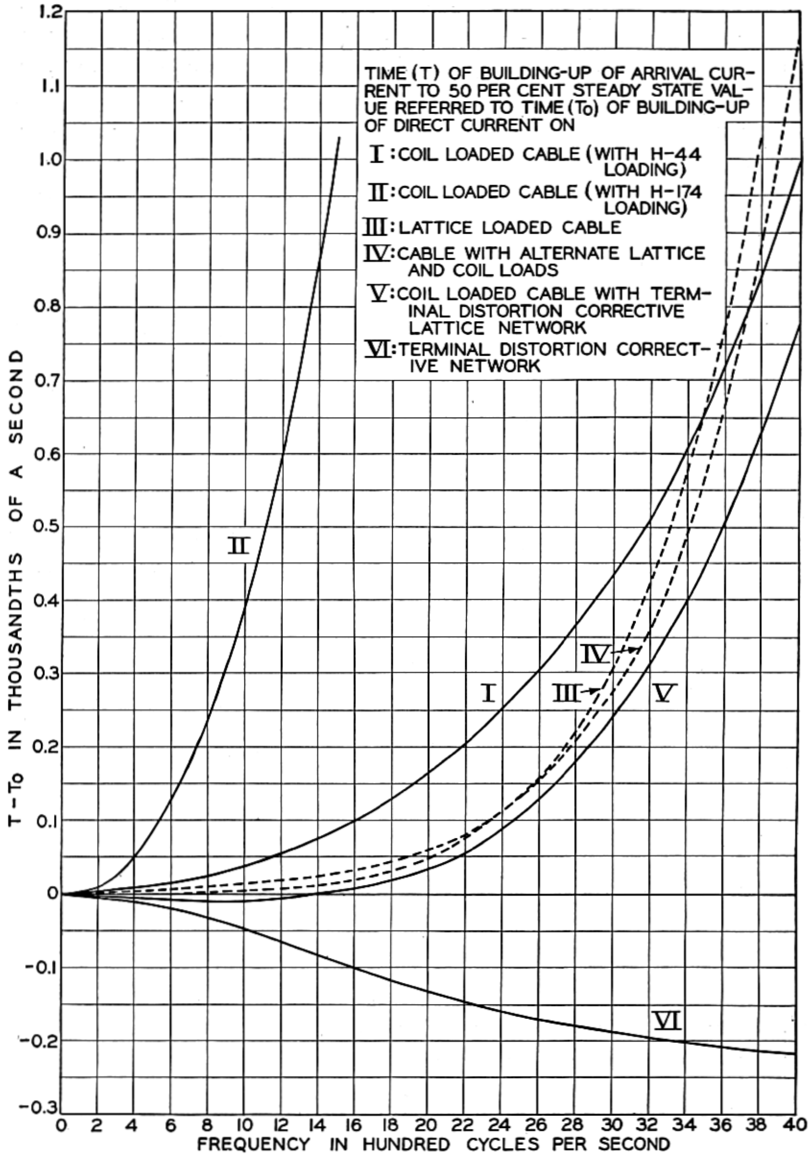

Fig. 19—Transient distortion on 50 miles of 19 gauge loaded cable

evident from a consideration of the duration of the transient distortion that the correction is not perfect, although it is appreciably better than that afforded by either lattice loading or a combination of lattice and coil loading. In Fig. 19 the delay in the time of building-up of any frequency $\omega/2\pi$ over the time of building-up of a direct current is shown for these different systems. Since the physical desideratum is a minimum constant delay for all frequencies, and the delay is quite sensitive to small deviations from the distortionless steady state phase angle, the former is probably a better basis of design and comparison than the latter.

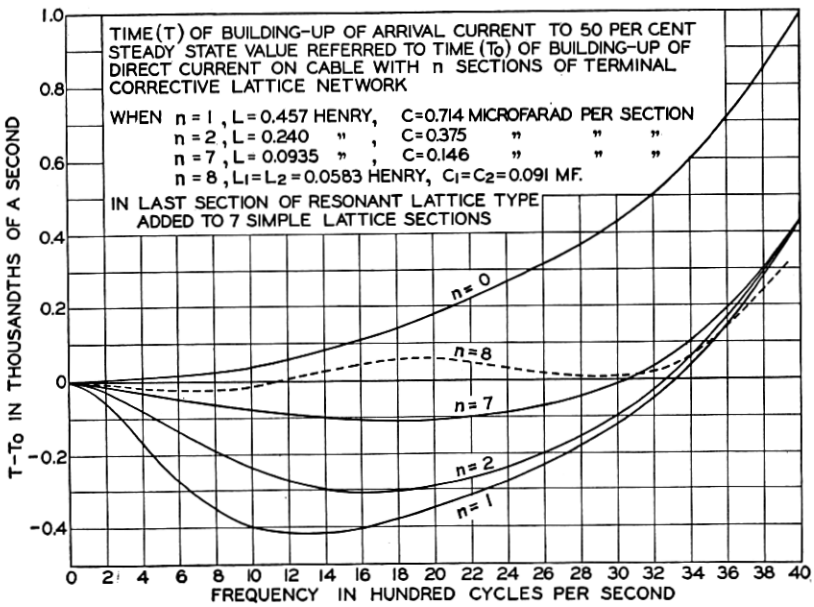

Fig. 20—Transient distortion on 50 miles of 19 gauge light loaded cable

Suppose that it is required to reduce the delay at 4,000 cycles to the value or to less than the value of the delay on the uncorrected cable at 3,000 cycles. The procedure will be to solve equation (30) for $\sqrt{LC}$ in terms of N and the required value of $T - T_0$ at the given frequency $\omega_a/2\pi = 4{,}000$, substitute these values in equation (30) and compute $T - T_0$ over the essential frequency range. It is immediately apparent from Fig. 20 that the improvement at the higher frequencies is at the expense of the intermediate frequencies where the delay is reduced too much. This disadvantage is lessened by increasing the number of sections but this method is uneconomical and only partially successful because a saturation point is soon reached in the gain obtained with more apparatus.

This difficulty may be overcome by adding networks of the more complicated lattice type shown in Fig. 21.[18] This may appro-

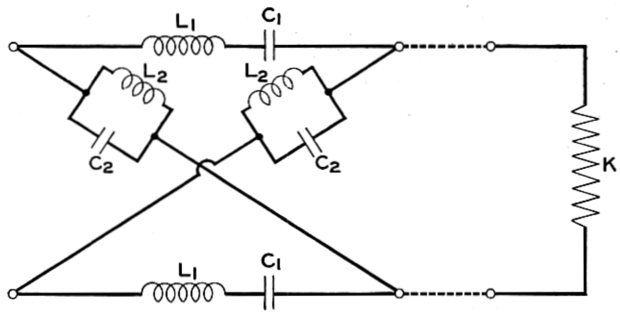

Fig. 21—"Resonant" lattice network

priately be called the 'resonant' lattice type. Its characteristics are most easily derived from the general lattice network having the impedance $z_1/2$ in each series branch and the impedance $2z_2$ in each diagonal shunt branch. Since the characteristic impedance, K, of any section of line is equal to the square root of the product of the open- and closed-circuit impedances and the propagation constant, Γ, per section, is equal to the anti-hyperbolic tangent of the square root of the quotient of the closed-circuit impedance divided by the open-circuit impedance,[19] we have, for the lattice network, in general,

$$\cosh \Gamma = 1 + \frac{2z_1}{4z_2 - z_1} \tag{32}$$

or

$$\tanh \frac{\Gamma}{2} = \frac{1}{2} \sqrt{z_1/z_2},$$

$$K = \sqrt{z_1 z_2}. \tag{33}$$

Thus the requirement that the characteristic impedance be a real constant will, in general, be fulfilled provided

$$z_2 = R^2/z_1, \tag{34}$$

where R is a real constant approximately equal to the characteristic impedance of the cable. This gives

$$\tanh \frac{\Gamma}{2} = \frac{z_1}{2R}. \tag{35}$$

[18] This suggestion was made by H. Nyquist.
[19] See reference 12.

Now if $z_1/2$ is the impedance of a series resonant circuit, we may write

$$\frac{z_1}{2R} = i\omega \frac{b}{2\omega_0} + \frac{b\omega_0}{i2\omega}, \tag{36}$$

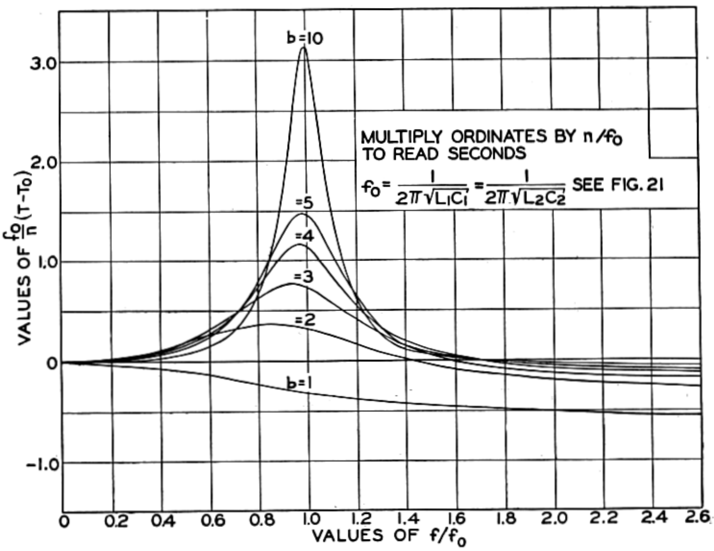

Fig. 22—Time of building-up, $T - T_0$, for resonant lattice type network of n sections

where b and ω_0 are constants. Then

$$\tanh \frac{\Gamma}{2} = \tanh i\frac{\phi}{2} = i\frac{b}{2}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right),$$

or

$$\phi = 2 \tan^{-1} \frac{b}{2}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right), \tag{37}$$

$$T = \frac{\dfrac{b}{\omega_0}\left(1 + \dfrac{\omega_0^2}{\omega^2}\right)}{1 + \dfrac{b^2}{4}\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right)^2}, \tag{38}$$

and

$$T_0 = \frac{4}{b\omega_0}. \tag{39}$$

Then, from equations (34) and (36),

$$L_1 = \frac{bR}{2\omega_0}, \qquad C_1 = \frac{2}{b\omega_0 R}, \tag{40}$$

and

$$L_2 = \frac{2R}{b\omega_0}, \qquad C_2 = \frac{b}{2R\omega_0}.$$

The expression for T or $T - T_0$ will have a maximum at $\omega = \omega_0$ if b is large enough. The value of the maximum may be increased by increasing b and its location on the frequency scale may be changed by varying ω_0. A family of curves of $f_0(T - T_0)$ for different values of the parameter b are shown in Fig. 22 with the abscissa f/f_0.

To illustrate the combination of the resonant lattice type with the simple lattice type, add one section of the former with $b = 2$, $f_0 = 2{,}190$, to the seven sections of the latter already applied to the

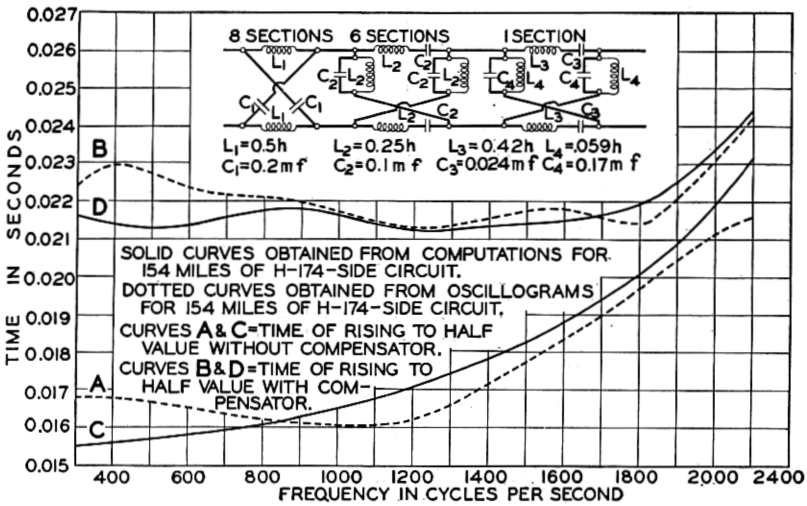

Fig. 23—Transients in loaded lines with and without phase compensator

cable (Fig. 20). The dotted curve for $T - T_0$, which results, is practically zero over most of the frequency range. The value of f_0 was determined so that the delay curve for the resonant lattice type would be complementary to the curve for the remainder of the circuit.

The combination of the two types of lattice network is effective in correcting even the large amount of phase distortion which results in transmission over the medium heavy loaded cable. Fig. 23 (which is reproduced by courtesy of Mr. H. Nyquist) shows a design for use on 154 miles of H-174 side circuit. The experimental results, obtained from oscillograms, are seen to be in close accord with the computed results.

The improvement due to correction of phase distortion by means of the phase compensator circuits employed for picture transmission is illustrated in Figs. 24*a*, 24*b* and 24*c*. Fig. 24*b* is from a negative which was transmitted over a 352-mile H-174 circuit without phase correctors. Fig. 24*c* is the same print sent over the same circuit after the circuit had been equipped with phase correctors. Fig. 24*a*, which was sent locally, is shown for comparison. Fig. 24*c* is seen to be practically as clear as Fig. 24*a* and both are substantially better than Fig. 24*b*.

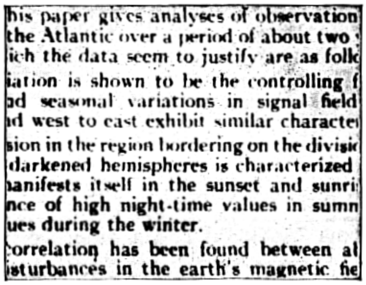
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Fig. 24*a*—Print transmitted locally

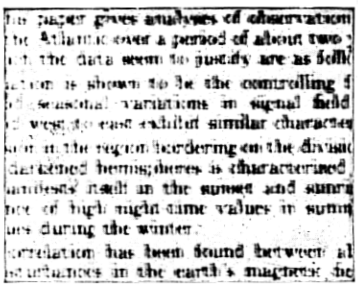
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Fig. 24*b*—Print transmitted over 352 mile H-174 loaded cable without corrector

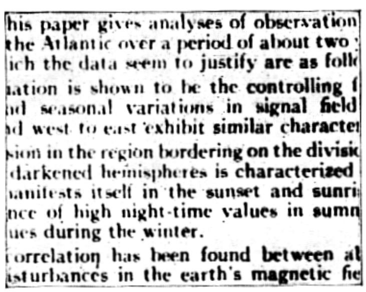
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Fig. 24*c*—Print transmitted over 352 mile H-174 loaded cable with terminal phase corrector

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