Determination of Electrical Characteristics of Loaded Telegraph Cables

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SYNOPSIS: The use of permalloy for continuous loading has introduced a number of new factors of importance in the study of transmission of signals over long submarine telegraph cables. Data to check the theoretical assumptions that are used in the design of permalloy loaded cables can be obtained by measuring on such cables the attenuation and time of propagation of sinusoidal currents of various frequencies in the telegraph range. By combining the results of these measurements with data obtained on the cable during process of manufacture, the resistance, inductance, capacity and leakance of the cables can be determined.

This paper describes the experiments that were performed on three laid cables and discusses in a general way the methods of computing the cable parameters.

WITHIN the last few years the art of telegraphing over submarine cables of transoceanic length has been revolutionized by the development of effective means of applying to such cables the principle of inductive loading. By surrounding the copper conductor of the cable with a thin layer of permalloy, a material of high magnetic permeability, the range of signal speeds attainable over cables of the order of 2,000 n.m. in length has been multiplied eight to ten times.[1] In place of the low frequency band extending from zero to about 15 c.p.s., which represents the range of frequencies which can be efficiently transmitted over the usual type of non-loaded cable, we are concerned in the case of the loaded cable with a transmission band extending from zero to about 120 c.p.s. Largely because of this comparatively high speed of operation, a number of factors, which were of negligible influence in the case of non-loaded cables, have become of primary importance in affecting the speed of signalling, and it has been found necessary, in order to establish a definite basis of estimating the performance of loaded cables, to make a thorough study of these factors by theoretical analysis supplemented by experimental work in the laboratory, and by measurements on laid cables.

PRINCIPLES OF CABLE TRANSMISSION

The theory of transmission of signals over submarine telegraph cables[2] and the principles governing the design of permalloy loaded

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[1] O. E. Buckley, *B. S. T. J.*, Vol. IV, No. 3, July 1925; Electrical Communication, Vol. 4, No. 1, July 1925; *Jour. A. I. E. E.*, Vol. XLIV, No. 8, August 1925.

[2] H. W. Malcolm, "The Theory of the Submarine Telegraph and Telephone Cable," London, 1917.

J. W. Milnor, *Jour. A. I. E. E.*, Vol. 41, p. 118, 1922.

cables [1] have been fully discussed elsewhere and only a brief summary will be given here for the purpose of indicating the importance of the measurements that will be described. On account of the fact that for a given value of sending voltage the amplitude of the signals received over a submarine cable diminishes rapidly as the speed of signalling is increased, there is a practical limit to the speed of operation of any cable. This limit depends on the electrical characteristics of the cable and the magnitude of extraneous interference encountered at the receiving terminal. The criterion for legibility of signals is, in general, that the attenuation constant of the cable at a value of frequency which may be termed the critical frequency shall not exceed a given value, the attenuation constant αs being defined by the relation

$$\frac{|V_R|}{|V_S|} = \epsilon^{-\alpha s}, \tag{1}$$

where $|V_R|$ is the amplitude of voltage arriving at one end of the cable when a sinusoidal voltage of amplitude $|V_S|$ is impressed at the other terminal. The value of this critical frequency depends mainly upon the method of operation, and it usually lies somewhere between the signal frequency and one and one half times the signal frequency.

Given the values of the four fundamental parameters of the cable, resistance (R), inductance (L), capacity (C) and leakance (G), the attenuation constant at the frequency $p/2\pi$ can be computed by means of the formula

$$2\alpha^2 = \sqrt{(R^2 + p^2L^2)(G^2 + p^2C^2)} + RG - p^2LC, \tag{2}$$

which to a close approximation reduces to the form

$$\alpha = \sqrt{\pi f CR} \tag{3}$$

in the case of a non-loaded cable, where R is large compared with $2\pi fL$, and to the form

$$\alpha = \frac{1}{2}\left(R + \frac{G}{C}L\right)\sqrt{\frac{C}{L}} \tag{4}$$

in the case of the loaded cable, where R is small compared with $2\pi fL$ at the critical frequency. In all cases it is assumed that G is very small compared with $2\pi fC$, which is strictly true for the insulating materials employed on submarine cables.

The manner in which the attenuation constant varies with frequency for typical loaded and non-loaded cables is shown in Fig. 1, the signal frequencies at which they are designed to operate being as indicated. In the case of the non-loaded cable the resistance and capacity are practically constant over the frequency range and the attenuation

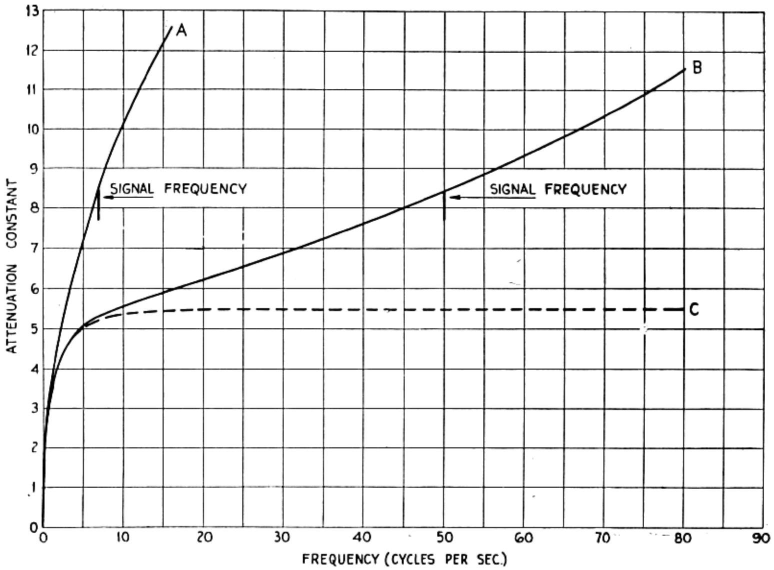

Fig. 1—*A*—non-loaded cable; *B*—actual loaded cable; *C*—ideal loaded cable

curve is approximately a parabola as indicated by formula (3). The curve for the loaded cable for small values of frequency is similar to the curve for the non-loaded cable, since for such frequencies the loading inductance has very little effect upon transmission. As soon as $2\pi fL$ becomes appreciable compared with R the beneficial effect of the inductance becomes apparent and the attenuation constant increases at a less rapid rate. If the cable parameters were constant throughout the frequency range, as in the case of the ideal cable, the attenuation constant would, at a value of frequency considerably below the signal frequency, attain a constant value, as represented by the dotted curve. On account of the fact, however, that R and G increase rather rapidly with frequency, the attenuation-frequency characteristic of an actual cable merely inflects, then increases, and at some frequency will actually cross the attenuation curve of the non-loaded cable.

To insure that legible signals will be obtained at the desired signal frequency the amplitude of the extraneous interference must be accurately determined. If, for example, the interference in the case of the cable having the attenuation-frequency characteristic shown in curve B were found to be twice as great as had been anticipated, the amplitude of received signal would likewise have to be doubled, which would mean a reduction of 0.7 in the allowable attenuation constant. This, as can be seen from curve B, would correspond to a reduction in speed of 8 to 10 c.p.s. Also since the value of attenuation constant is considerably affected by variations of the electrical parameters, it is desirable that the values of these parameters in the laid cable be capable of predetermination to a degree of accuracy comparable with that obtained in the case of non-loaded cables. Methods of estimating the value of extraneous interference to be expected at the terminals of a projected cable have been described in a previous paper.[3] The present paper will be devoted to a discussion of methods of predetermining the electrical parameters of cables.

MEASUREMENTS DURING MANUFACTURE

In the case of a non-loaded cable the attenuation constant, as indicated by formula (3), is determined solely by the dielectric capacity and the conductor resistance. For the values of frequency involved in the operation of such cables, the latter consists almost entirely of the direct current resistance of the copper conductor. The values of capacity and copper resistance of a considerable part of the cable can be measured during the process of manufacture, and, by reducing these values to sea bottom conditions, an accurate estimate of the resistance and capacity of the laid cable is obtained.

In the case of the loaded cable the problem of predetermining the electrical parameters of the laid cable is much more difficult, since a number of the quantities involved in computing the attenuation are influenced by conditions which are not entirely known and which are difficult to simulate in laboratory experiments. The dielectric leakance, for example, is affected by pressure as well as by temperature, and since the hydrostatic pressure to which the cable is subjected may be as high as 10,000 pounds per square inch, it is evident that measurements of this characteristic of the cable, on any but a very small scale basis, will be very difficult and costly. The permeability of the loading material and consequently the inductance of the cable may be affected by mechanical strain and by superposed magnetic fields. An estimate of the average inductance of the laid cable can be obtained by bridge measurements in the factory on pieces of core about 1

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[3] J. J. Gilbert, *B. S. T. J.*, Vol. 5, p. 404, and *Electrician*, Vol. 97, August 6, 1926.

nautical mile in length, selected at intervals during manufacture, the effect of strains and of superposed fields being estimated by means of experiments on short lengths of cable. However, there are ordinarily small unavoidable variations in electrical characteristics from point to point along the cable and it is not entirely certain that the average inductance obtained from measurements on a fraction of the core lengths entering into the cable structure will represent the average inductance of the entire cable. The resistance of the laid cable is likewise difficult to estimate. This parameter comprises, in addition to the copper resistance, the resistance of the return conductor consisting of the armor wires and sea water in parallel, components resulting from eddy current and hysteresis losses in the loading material and other components of lesser importance, the nature of which will be discussed later. The losses in the loading material depend upon the average permeability obtained in the laid cable, and their predetermination from factory measurements may be uncertain for reasons that have been pointed out. As regards the sea return resistance, rigorous methods of computation are available,[4] but there is some uncertainty regarding the conditions that should be assumed as existing at the ocean bottom.

MEASUREMENTS ON LAID CABLES

For the purpose of placing the design of loaded cables upon a definite basis, it has appeared desirable to measure the parameters of a number of cables of this type that have been laid, and to compare the values so obtained with the estimates based on analytical methods and upon factory measurements. In order to simplify the problem, attention will be devoted mainly to determining the values of the parameters corresponding to a very small value of current in the cable conductor. Under these conditions the hysteresis component of resistance is negligible and the inductance and eddy current resistance can be considered constant at any frequency. This is entirely consistent with the method employed in the design of loaded cables, in which the attenuation constant is computed, first on the assumption that the current is very small throughout the cable, and then corrected for "head end losses" due to the effect of hysteresis losses which are present under actual conditions of operation.

The usual method of determining the parameters of a transmission system consists in measuring the propagation constant, Γ, per unit

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[4] J. R. Carson and J. J. Gilbert, *Jour. Franklin Institute*, Vol. 192, p. 705, 1921; *Electrician*, Vol. 88, p. 499, 1922; *B. S. T. J.*, Vol. 1, No. 1, p. 88.

length and the characteristic impedance, K, which quantities are defined at the frequency $p/2\pi$ by the formulas

$$\Gamma = \sqrt{(R + jpL)(G + jpC)}, \tag{5}$$

$$K = \sqrt{\frac{R + jpL}{G + jpC}}. \tag{6}$$

Knowing these two quantities at any frequency, the values of the four parameters can be readily computed.

The propagation constant and the characteristic impedance of *telephone* cables 100 miles or less in length have been determined by measuring the input impedance of the cable with the distant end in turn insulated and grounded. These two impedances are determined for a cable of length s by the formulas

$$Z_I = K \coth \Gamma s$$
$$Z_G = K \tanh \Gamma s,$$

and given the values of Z_I and Z_G it is an easy matter to compute the corresponding values of propagation constant and characteristic impedance, the accuracy of this determination depending upon the difference between Z_I and Z_G. In the case of a submarine telegraph cable of the order of 2000 miles in length, the value of Γs is so large that Z_I and Z_G differ by less than one part in 10,000 in the frequency range in which we are interested. This means physically that the remote parts of the cable have little effect upon the terminal impedance of the cable and the values of input impedance are determined almost entirely by the parameters of the 400 or 500 miles of cable adjacent to the terminal. It is true that by going to extremely low frequencies, perhaps fractional cycles per second, the method above described could be used to determine the characteristic impedance and the propagation constant of long cables, but at such frequencies these quantities are determined almost entirely by the d.c. resistance and capacity of the cable and no information regarding the quantities in which we are particularly interested would be obtained.

The method that has actually been employed to determine the parameters of several of the continuously loaded cables which have recently been laid is to measure separately at a number of frequencies the real and imaginary parts of the propagation constant, the capacity of the cable at various frequencies being determined by correlating the results of laboratory tests with d.c. measurements of capacity made on the laid cable.

As can be seen from formula (4), the real part of the propagation constant, αs, the attenuation constant of the cable, involves all four of the cable parameters, but on account of the fact that the inductance, leakance and the various components of the effective resistance predominate in influence at different points in the frequency range it is possible, by methods of successive approximations, to obtain a reasonably good set of values of these quantities.

The imaginary part of the propagation constant, βs, is to a close approximation, given by

$$\beta s = sp\sqrt{CL}. \tag{7}$$

From this it follows that the time of propagation of a sinusoidal wave of voltage or current over the cable is given by

$$T = s\sqrt{CL}, \tag{8}$$

and knowing the time of propagation and the capacity at any frequency the inductance of the cable at this frequency can be easily computed. Since the resistance and leakance have only a slight effect upon the time of propagation, this is the most direct method of determining the average inductance of the cable.

MEASUREMENT OF ATTENUATION

The attenuation constant of the cable is determined by measuring the values of voltage received at one end of the cable, due to various values of voltage of constant frequency impressed at the other end. The impressed voltage may be either sinusoidal or square top in shape, the latter being preferable for the reason that, at the low frequencies and high voltages required, it is difficult to obtain a wave form from an oscillator sufficiently free from harmonics to enable an accurate determination of the fundamental component to be made. Square top reversals of any frequency and amplitude can be easily obtained by means of a relay actuated by an oscillator, and the amplitude of the fundamental component can be accurately computed.

At the receiving end, for the frequencies of particular interest, the arriving voltage is practically sinusoidal, since the harmonic components are eliminated by the higher attenuation of the cable for such frequencies. This voltage is measured by terminating the cable in an impedance which is very large compared to the characteristic impedance of the cable, and measuring the potential drop across all or part of this impedance by means of a vacuum tube amplifier in the output of which is a thermocouple and meter. The advantages of the high impedance termination are, first, that by reflection it doubles the amplitude of the arriving voltage, thus giving larger quantities to work with, and second, that it eliminates the necessity of taking into account the characteristic impedance of the cable and the impedance of the balanced type of sea earth which is usually employed as the earth connection of the amplifier. By means of a string oscillograph in the output of the amplifier, the wave shape of the received voltage and the nature of the extraneous interference can be determined. The amplifier is calibrated by impressing on it a measured voltage of the same frequency as that of the received voltage.

Knowing the values of received voltage and the corresponding transmitted voltage, the values of attenuation constant can be readily computed. By plotting the values of attenuation constant corresponding to various values of frequency and transmitted voltage as functions of the latter quantity and extending these curves to the axis of zero transmitted voltage, the values of attenuation constant corresponding to a very small current in the conductor can be obtained for various frequencies.

Assuming that all the parameters have been accurately predetermined, there are three sources of error which might possibly cause a difference between the measured value of attenuation constant and that computed from the average values of the cable parameters by means of formula (2). In the first place, the parameters are not uniform throughout the cable as is assumed in deriving this formula. In particular, the inductance may vary from point to point. At each point where the capacity or inductance changes value reflections of voltage and current will take place and the effect of these reflections should be to increase the attenuation constant of the cable. For variations of the parameters of the order that is to be expected in loaded cables, the increase in attenuation constant is quite small, and the magnitude of this increase can be computed approximately by a method due to Carson.[5] Another source of error is the presence of extraneous interference superposed on the received voltage. This factor is usually troublesome only at the highest frequencies and lowest voltages employed, and in this case measurements of the oscillograms of received voltage and of calibrating voltage will give a value of the received voltage independent of interference. The third source of error is due to the presence in the transmitted voltage of harmonics of the fundamental frequency. These harmonics are attenuated in transmission over the cable to a much greater degree than is the fundamental, so that they constitute only a small per-

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[5] *Electrician*, Vol. 86, p. 272, 1921.

centage of the received voltage and are practically negligible in their effect upon the thermocouple.

MEASUREMENT OF THE TIME OF PROPAGATION

The time of propagation of a steady state sinusoidal voltage over a loaded cable of transatlantic length is of the order of 0.3 second. It is measured by means of the circuit shown in Fig. 2, which is operated simultaneously at both ends of the cable. At each end a perforated tape is prepared which when inserted in the high speed transmitter *T* will cause a train of about ten reversals to be sent out over the cable. The potentiometer *P* is adjusted so that a measurable record of either transmitted or arriving trains, depending upon the position of the key *k*, will be obtained on the string oscillograph *O* after amplification by the vacuum tube amplifier *A*. The condenser *C* is inserted between the cable and transmitter in order to remove the low frequency components of the transient part of the train, which would otherwise overwhelm the steady state component at the distant end of the cable. The oscillograph, shown in Fig. 3, gives a continuous record of the current in a fine wire, which is free to respond to the interaction between the current and the strong magnetic field in which the wire is placed. The displacement of the wire, and hence the amplitude of current in it, is recorded on a long strip of sensitized paper, which is developed and fixed within the camera by a continuous process immediately after exposure. By this means it is possible to obtain a continuous record, over a period of several minutes, of voltages transmitted and received over the cable. A second wire can be used to give simultaneously a record of any other current which

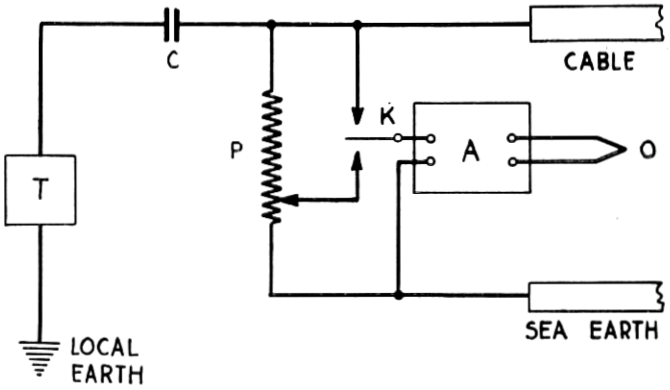

FIG. 2.

may be desired for comparison. An arrangement is provided for superposing on the records vertical timing lines at intervals of one hundredth of a second. Short pieces of record are shown in Fig. 4.

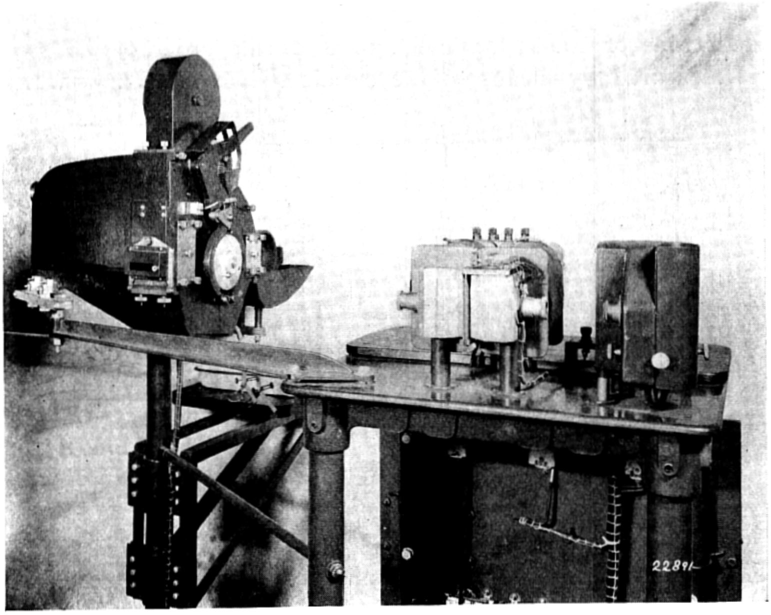

FIG. 3.

At a prearranged time the oscillographs at both ends are started. A train of reversals is transmitted from one end, a record being taken on the oscillograph at that end, and received at the distant end, where a record is also taken. Both stations quickly change potentiometer connections from send to receive or vice versa, and the distant station transmits a train of reversals, recording it on the same tape as was used for reception. Similarly at the first station the arriving train is

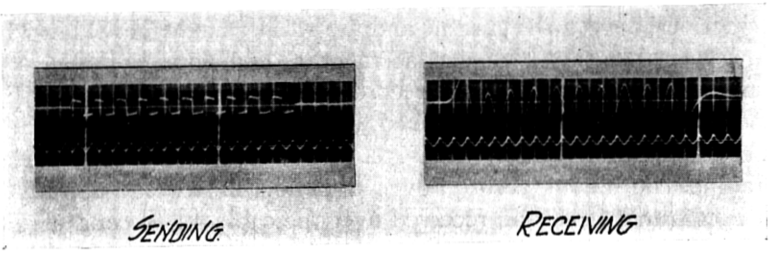

FIG. 4.

recorded on the strip containing the record of their transmitted train. Station 1 measures on its oscillogram the time elapsing between its transmitted train and its received train, and at Station 2 the time elapsing between the received train and the transmitted train is measured. After making suitable corrections, which will be described later, the difference between the interval measured at Station 1 and that measured at Station 2 will be equal to twice the time of transmission of the train of reversals over the cable.

A typical record such as would be obtained at Station 1 is shown in Fig. 4. It will be observed that in the record of received voltage the first few cycles are somewhat distorted because of the fact that the steady state has not yet been reached. Because of this fact the time of arrival or departure of a train is referred to a later cycle in the series, say the fifth. The times of departure and arrival of the various zeros following this cycle are measured, and the average of the values so obtained is defined as the time of arrival or departure of the train. In this way the possible errors due to interference or to distortion in the sent record due to improper functioning of the transmitter are eliminated.

It will be observed that, mainly on account of the presence of the condenser C, the voltage reversals impressed on the cable are not flat-topped and the zero phase of the fundamental component which we are measuring occurs somewhat ahead of the point in the transmitted voltage which we have used as the zero of reference in measuring the oscillograms. Since we are interested in the time elapsing between zero phase of the fundamental frequency in the transmitted voltage and the zero phase of the corresponding cycle in the received train, it is necessary to compute this interval, either by graphical analysis of the oscillogram or by computation from the constants of the circuit, and add the corresponding time to the time which has been measured.

Although the mechanical arrangement by which the timing lines are obtained on the oscillogram is adjusted as accurately as possible so that the interval between lines is very nearly one hundredth of a second, the very slight variations which occur in such a system are apt to introduce considerable error into the measurement of time of propagation. This is due to the fact that the time of propagation is obtained from the difference of two intervals each of which may be as much as ten times the time of propagation. An error in either interval will therefore result in a tenfold error in the final result. To guard against this condition a record is taken during the experiment of a periodic voltage obtained from a standard oscillator or fork, and the peaks of this oscillation serve as a check on the timing lines. As a final check, records similar to Fig. 4 are taken with various times elapsing between reception and transmission at the second station. If an error exists in the timing arrangement its effect on the time of propagation will be greater the greater the interval between receiving and sending, and the time of propagation corresponding to negligible error in the timing system can be easily obtained by graphical methods. The error of measurement of the time of propagation is probably less than 1 per cent.

The inductance of a loaded conductor is an increasing function of current for the range of current values used in cable practice because of the increase of permeability of the permalloy, and since with finite transmitting voltage the current at the sending end may be quite large, the inductance of this portion of the cable under such conditions will be larger than the value it would have for very small current in the conductor. Accordingly the time of propagation at a given frequency will be a function of voltage. The value of inductance corresponding to very small current in the conductor can be derived from the time of propagation corresponding to zero transmitted voltage, which is obtained by extrapolation from measurements of the time of propagation at several values of transmitted voltage.

MEASUREMENT OF CAPACITY

The dielectric capacity of submarine cables in the telegraph range of frequencies is in general comparatively insensitive to changes in temperature and hydrostatic pressure, so that it is possible to estimate this quantity rather accurately at various frequencies by means of measurements made in the factory, the factors required to reduce the results of the measurements to sea bottom conditions being relatively easy of determination. In order to check these values, however, the d.c. capacity of the laid cable is measured by the method of mixtures, employing a charging time of 10 seconds or more and a mixing time of equal duration.

COMPUTATION OF CABLE PARAMETERS

The inductance of the cable can be computed at any frequency from the measured values of capacity and time of propagation by means of equation (8), proper allowance being made for the rather small effect of resistance.

Having computed the inductance and the capacity of the cable, only the resistance and the leakance remain undetermined. The direct current resistance can be computed from factory measurements and checked by measurement on the cable. The resistance component due to eddy currents in the loading material can be computed from the resistance measurements obtained in the factory in the process of determining the inductance of sample core lengths. The eddy current resistance is proportional to the square of the product of frequency and permeability, and corresponding reduction factors must be employed in computing the eddy current resistance of the laid cable from the factory measurements. Since we are dealing with values of the parameters corresponding to very small current in the cable conductor, the hysteresis resistance is zero. In addition to the losses in the loading material there are other losses peculiar to continuously loaded cables due to currents induced in the cable structure. The loading material is ordinarily applied to the conductor in the form of a tape or wire of finite width, so that it has a definite lay, and since the magnetic flux in the loading material tends to follow the convolutions of the latter there is a component of this flux parallel to the axis of the central conductor. Consequently as the flux changes with signal current, electromotive forces are induced in those portions of the cable structure which link with it—the teredo tape and armor wires, for example. The resulting energy loss has in most practical cases comparatively small effect on the performance of the cable, and the magnitude of the corresponding resistance component can be estimated by theoretical methods and by measurements in the factory. The various components of resistance having been estimated, the total resistance at any frequency can be computed. Likewise the value of dielectric leakance of the laid cable at any frequency can be estimated from tests made during manufacture. These values of resistance and dielectric leakance should be considered merely as first approximations, since they are based in part on assumptions that cannot be directly verified.

Formula (2) is then employed to determine the effect upon the attenuation constant of departures from the approximate values of resistance and leakance, and by comparing these results with the measured values of attenuation constant, mutually consistent sets of values of resistance and of dielectric leakance can be computed at various frequencies. A choice of the best sets of values can then be made, due weight being given to the evidence available from computations and laboratory measurements regarding the manner in which these quantities vary with frequency.

From the curves relating the values of measured attenuation constant and the transmitted voltage, a check can be made of the method of computing the increase in attenuation due to hysteresis and to variation of inductance with current.[6] Since this method employs

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[6] See Buckley, loc. cit., and U. Meyer, *E. N. T.*, Vol. 3, No. 1, 1926.

the inductance-current and resistance-current characteristics of the loaded conductor, as determined in the factory, the attenuation measurements also afford a check on these characteristics.

CONCLUSIONS

Measurements of attenuation, time of propagation and dielectric capacity of the laid cable at various frequencies, supplemented by measurements of eddy current resistance in the factory and by information regarding the manner in which sea return resistance and dielectric leakance vary with frequency are sufficient for determining the values of the four parameters of a loaded cable and for dividing the resistance into its component parts. A quantitative comparison of the results so obtained with the values of parameters that would be predicted from factory measurements alone would require a detailed discussion of the methods involved in such measurements, and is outside the scope of the present paper. A general conclusion that can be drawn from the results of measurements made on three cables of somewhat different characteristics is that the method of estimating the characteristics of laid cables from measurements made on short lengths of core during process of manufacture is capable of considerable accuracy. The values of inductance and dielectric leakance obtained from factory measurements are close enough to the actual values in the laid cable to give a value of attenuation constant within a few per cent of the actual value. The value of resistance obtained from the cable measurements appears to be about three to five per cent higher than the estimated value. This may in part be due to latent errors in measurement or in the method of allowing for the effect of reflections along the cable.

The greater part of the discrepancy between the estimated and measured values of resistance is perhaps due to erroneous assumptions involved in computing the value of sea return resistance employing the method described in the paper by Carson and Gilbert. In this work it was assumed that the cable is surrounded by a homogeneous medium, the sea water. For values of frequency higher than the telegraphic range this assumption appears to be sufficiently close to the truth, since only a comparatively small region around the cable plays any part in the phenomena. In the telegraph range, however, the return current is distributed through a comparatively large cross-section and more exact specification of the electrical characteristics of this region is required. To determine by rigorous methods the sea return impedance in the case where the cable lies in a plane separating two different media is a problem of considerable difficulty. An approximate method, which gives results which are sufficiently accurate for purposes of cable design, consists in computing the combined impedance of the three parallel conductors, namely, the armor wires, the sea water, and the earth, the impedances of the latter two conductors being determined by the methods outlined in the aforementioned paper. The physical interpretation of the sea return resistance as obtained by this method is that the high value of reactance of the sea water and earth, due to the large cross-section of the conducting area, forces the return current to flow in the armor wires even though the resistance of this path is much higher than that of the paths through the sea water and earth. It appears probable that the electrical conductivity of the earth is very much less than that of sea water which would result in a larger cross-section of conducting area external to the armor wires and larger inductance of this path. This leads to higher values of sea return resistance than are obtained on the assumption that the cable is surrounded on all sides by sea water and thus gives a result more nearly consistent with the observed facts.