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Measurement of Inductance by the Shielded Owen Bridge

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SYNOPSIS: The study described in this paper shows that the Owen bridge is well adapted to the accurate measurement of inductance and effective resistance to above 3,000 cycles. The construction of a shielded bridge for audio frequencies is described and a theoretical discussion is also given. It was found possible to measure inductances ranging from 0.1 to 3 henrys with an error of measurement less than 0.1 per cent, and for 10 henrys the accuracy is better than 0.25 per cent. As a means of measuring effective resistance the bridge shows an accuracy of about 2 per cent. The sources of error and method of eliminating or correcting them are discussed.

INTRODUCTION

THE accurate measurement of inductance and capacitance is essential to the correct design of practically all precision electrical apparatus. Particularly is this so in the field of electrical communication where the successful introduction of new circuits and equipment, such as the carrier telephone and the telephone repeater, depends largely on the accuracy with which the elements can be adjusted to the nominal values, this accuracy in turn depending on the accuracy with which the electrical measurements can be made.

Owing principally to the ease with which a telephone receiver may be used to indicate a balance at audio frequencies, bridge measurements are very generally used for the measurement of capacitance and inductance in telephone work. The simplest type of bridge and the one used most for the comparison of like impedances is the equal ratio arm bridge described by Shackelton.[1] This bridge requires standards of the same kind and magnitude as the impedances which are to be measured. The calibration of these standards is a separate problem, for which a distinct type of bridge is required.

Either capacitance or inductance may be measured by a bridge method in terms of time and resistance, both of which are fundamental quantities. However, since condensers may be obtained with very low losses and small changes with frequency, this type of measurement is usually made with capacitance,[2] inductance measurements being

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[1] W. J. Shackelton, "A Shielded Bridge for Inductive Impedance Measurements," *Bell System Technical Journal*, January, 1927.

[2] J. Clerk Maxwell, *Electricity and Magnetism*, Vol. 2, pp. 776–7.

made by comparison with capacitance and resistance or with capacitance and frequency. The resonant method is adapted to the comparison of inductance with capacitance and frequency. However, this method demands an accurate measurement of the frequency used, which is not always convenient. It is therefore evident that a bridge which furnishes a comparison of inductance with capacitance and resistance serves a very useful purpose in the calibration of standards of inductance for use in simple comparison bridges.

A bridge circuit due to Owen [3] furnishes a very good example of this type, the balance conditions being independent of frequency and the equations of balance giving a relation between inductance, capacitance, and resistance. The circuit is shown in Fig. 1. It consists of a fixed resistance r_1 in the arm BC, a fixed capacitance C_3 in the arm AB, a fixed capacitance C_4 in series with a variable resistance R in AD, and a variable resistance r_2 in series with the inductance to be measured in CD. The adjustments for balance are made with R and r_2. These two adjustments are independent of each other. The relations between the quantities at balance, as will be shown later, are such that the bridge may readily be made direct reading for

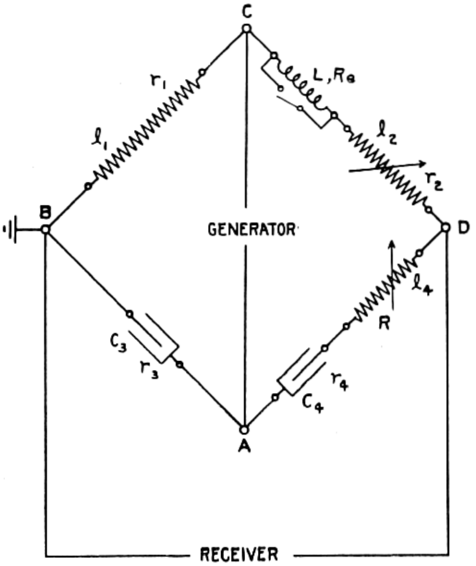

Fig. 1

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[3] D. Owen, "A Bridge for the Measurement of Self Induction," *Proceedings of Physical Society of London*, Oct. 1, 1914.

inductance, and these advantages make this bridge superior to practically all other bridges for this type of comparison.

This paper contains a discussion of the theoretical relations of this bridge circuit, its possibilities and limitations for the accurate measurement of inductance and effective resistance, and the sources of error and methods of eliminating them. A shielded bridge, constructed for use in calibrating inductance standards, is described and sufficient measurements are given to show the accuracy of which it is capable.

The maximum frequency at which measurements were given by Owen is 530 cycles. For the measurement of telephone apparatus considerably higher frequencies are used, and it is desirable that the bridge be capable of measurements up to 3,000 cycles without loss of accuracy. It is in the upper part of this range that the greatest difficulties are encountered, requiring special precautions not so necessary for the lower frequency measurements.

While in the following discussion the maximum frequency considered is 3,000 cycles, this is not meant to indicate a maximum limit to this type of bridge.

EQUATIONS OF BALANCE

Taking into consideration the phase angle of the resistances and the loss in the condensers, the complete network is shown in Fig. 1, the reactive component of the resistances being shown as series inductance, and the condenser losses as series resistance. Let

L and R_e = Inductance and effective resistance of coil to be measured,
r_1 and l_1 = Total resistance and inductance in arm BC,
r_2 and l_2 = Resistance and inductance in CD exclusive of R_e and L,
R and l_4 = Total resistance and inductance in AD including the equivalent series resistance of C_4,
r_3 = Equivalent series resistance of C_3.

The inductance in the arm AB may readily be reduced to a negligible amount and will not be considered.

We may now balance the bridge with the inductance terminals short circuited, that is, take a zero reading, and then balance again with the inductance inserted.

Writing the equations of balance in each case, subtracting one from the other, and separating reals from imaginaries, we get the following equations:

$$C_3r_1(R - R') = L + (l_2 - l_2') + C_3r_3(r_2 + R_e - r_2') + p^2C_3l_1(l_4 - l_4') \qquad (1)$$

and

$$\frac{r_2' - r_2 - R_e}{C_3} = p^2 l_1 (R - R') + p^2 r_1 (l_4 - l_4') - p^2 r_3 (L + l_2 - l_2'), \quad (2)$$

where l_2', r_2', l_4', and R' are the values of l_2, r_2, l_4, and R at balance with L short circuited, and p is 2π times the frequency. These are practically identical with Owen's equations (10) and (12).

In equation (1), each of the third and fourth terms contains two factors of second order, namely r_3 and $(r_2 + R_e - r_2')$, and l_1 and $(l_4 - l_4')$ respectively.

We may therefore write

$$C_3 r_1 (R - R') = L + (l_2 - l_2'). \quad (3)$$

In equation (2), let

$$\frac{1}{pC_3} = -X_3, \qquad pl_1 = x_1, \qquad pl_4 = x_4,$$
$$pL = X, \qquad pl_2 = x_2, \quad \text{and} \quad pl_2' = x_2'.$$

Then we may write

$$-(r_2' - r_2 - R_e)pX_3 = px_1(R - R') + pr_1(x_4 - x_4') - pr_3(X + x_2 - x_2'). \quad (4)$$

But from (3)

$$R - R' = \frac{L + l_2 - l_2'}{C_3 r_1} = \frac{-(X + x_2 - x_2')X_3}{r_1}.$$

Substituting in (4),

$$\begin{aligned} r_2' - r_2 - R_e &= \frac{(X + x_2 - x_2')x_1}{r_1} + (x_4 - x_4')\frac{(X + x_2 - x_2')}{R - R'} + \frac{r_3(X + x_2 - x_2')}{X_3} \\ &= (X + x_2 - x_2')\left[\frac{x_1}{r_1} + \frac{x_4 - x_4'}{R - R'} + \frac{r_3}{X_3}\right] \end{aligned}$$

and

$$R_e = r_2' - r_2 - (X + x_2 - x_2')\left(q_1 + q_4 + \frac{1}{Q_3}\right), \quad (5)$$

where q_1 = ratio of reactance to resistance of arm BC,
q_4 = ratio of reactance to resistance of *change* in arm AD,
Q_3 = ratio of reactance to resistance in arm AB.

From equation (3) we see that, if we take a zero reading first, the inductance is given by the expression

$$L = C_3 r_1 (R - R'), \tag{6}$$

the percentage error due to neglecting $l_2 - l_2'$ being

$$\frac{100(l_2 - l_2')}{L}. \tag{7}$$

From equation (5), the effective resistance of L is given by

$$R_e = r_2' - r_2, \tag{8}$$

the percentage error due to neglecting corrections being

$$\frac{100(X + x_2 - x_2')}{R_e}\left(q_1 + q_4 + \frac{1}{Q_3}\right). \tag{9}$$

The error in L is approximately, from equations (7) and (8),

$$\frac{x_2 - x_2'}{R_e} \cdot \frac{R_e}{X} = \frac{q_2}{Q},$$

where q_2 = ratio of reactance to resistance of change in arm CD, and
Q = ratio of reactance to resistance of the inductance being measured.

This error is usually negligible and may be approximately corrected for when appreciable. Dr. Owen has pointed out that this type of error is not peculiar to the Owen bridge, but is present in practically all methods of inductance measurement.

The error in R_e is a function of the Q of the coil measured, and of q_1, q_4 and Q_3. It is greatest for coils of high Q.

It is possible to make $q_1 = -\frac{1}{Q_3}$ for a given frequency, in which case the error reduces to approximately Qq_4 and the two errors are of the same order of magnitude for $Q = 1$,—the error in R_e becoming greater, and in L less as Q is increased.

However, in the general case we cannot cancel q_1 against $\frac{1}{Q_3}$ over any appreciable range of frequencies, and they are normally additive. Also for ordinary inductance coils Q is considerably greater than one, sometimes as large as 100. For such cases the error in R_e becomes large and difficult to determine without an accurate knowledge of the reactances of the resistances used and the losses in the condenser.

From the above relations we see that a method of this type is capable of measuring inductance with a high degree of accuracy and may be made to measure effective resistance with fair accuracy, provided that there is no coupling between any of the four arms nor any between them and the input and output circuits. This is in practice a difficult result to realize, and this difficulty in obtaining a simple but adequate system of shielding is one of the most serious limitations to the bridge.

Shielding

Since the bridge contains no inductances of appreciable magnitude, it is a comparatively simple matter to eliminate electromagnetic coupling by using input and output transformers in toroidal form, the input transformer being so designed that the core will not be saturated when using the maximum input to the bridge.

The elimination of the electrostatic coupling is not so simple, as any electrostatic shielding introduced adds capacitance which, unless due care is taken, will involve errors in the bridge. This means that such capacitances must be limited to the corners *BD* and *AC* where

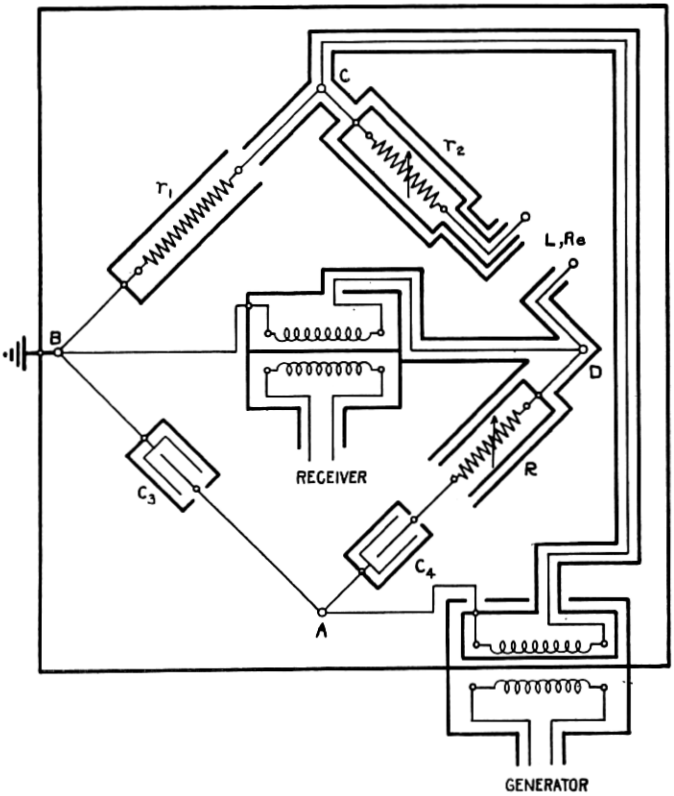

Fig. 2

they simply shunt the input and output circuits, and to *AB* where they shunt the capacitance C_3 and may be included in the assumed value of C_3. If such shielding is not used, the balance of the bridge will be affected by external conditions such as body capacitance, and the position of the bridge arms with respect to each other and to other apparatus, with the result that accurate results can be obtained only by the use of the greatest precautions.

A shielding scheme which satisfies the above requirements is shown in Fig. 2. In this system all capacitance between shields is limited to the diagonal corners of the bridge and the arm *AB*. However, this system of shielding, while about as simple as can be designed where complete shielding is required, is rather difficult to carry out in any practical bridge construction.

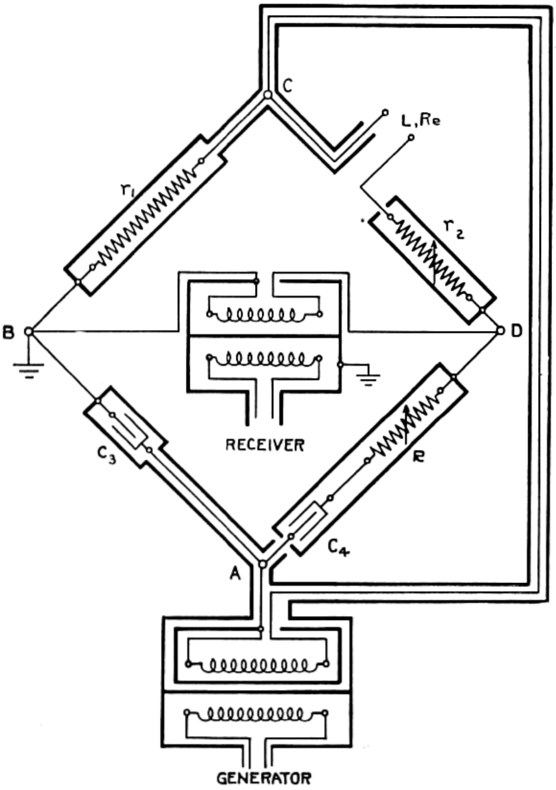

Fig. 3

The question of reducing the amount of shielding and still retaining a high degree of accuracy has been investigated and the modified scheme shown in Fig. 3 has been developed. In this circuit the shielding is complete insofar as it limits the electrostatic coupling to specific points in the bridge, and eliminates coupling between the bridge and the input and output circuits. However, in addition to capacitance across the diagonal corners and across arm AB, capacitances are introduced across r_1, across R, and across arm AD. The capacitance across AD may be made small enough to neglect since it consists of the capacitance of one condenser lead to the shield. Capacitances across r_1 and across R do not enter as first degree errors in the value of L but do directly affect the measurement of R_e. However, where the bridge is used primarily for the accurate measurement of inductance this compromise is justified. Even for the measurement of effective resistance, although the corrections may be larger due to the presence of the shielding, the bridge will give more consistent results and the corrections may be fairly well estimated.

The method of shielding shown requires one transformer having two shields between the windings and one transformer with a single shield between windings. It is essential that these shields be as perfect as possible. The other shielding shown is comparatively simple, no equipment requiring more than a single shield. The ground is shown at the point B simply because grounding at this point results in the simplest shielding. It would be desirable to have the ground at C in order that one terminal of the coil under test would be grounded, but at the time of balance the points B and D are at the same potential, and provided that r_2 is only a small fraction of the total impedance of the coil under test we may consider that one terminal of the coil is practically at ground potential. However, it should be noted that for a coil having a considerable capacitance from intermediate points in its winding to ground, a ground at B cannot be considered exactly equivalent to a ground at D. This difficulty is only appreciable in the case of very large inductances of large physical size when measured at high frequencies, and in such cases the effective inductance will be dependent on external conditions, whatever bridge circuit it is measured in. In the case of shielded coils, the ground should in all cases be connected to D rather than to B. In spite of the slight disadvantages noted, this method of shielding appears to be the most satisfactory, and a bridge has been constructed in accordance with it.

CONSTRUCTION OF THE BRIDGE

From the equation giving the value of L, it is seen that we may obtain an additional range for the inductance by having either r_1, C_3, or both, variable in steps. In the present bridge we have used two steps for C_3 and five steps for r_1. It is possible by choosing the correct values for r_1 to make the bridge direct reading for inductance. The actual values used for the capacitance were .6 mf and .06 mf. The values used for r_1 were 1,000/.6 or 1,667 ohms and multiples or submultiples of this value. In this way the bridge was made direct reading in millihenrys.

The capacitance C_4 has only one requirement to meet. It must be small enough so that the ratio of resistance to reactance of arm *AD* shall always be less than the ratio of reactance to resistance of the coil.

Taking 3,000 cycles as the maximum frequency, 10,000 ohms as the maximum resistance in arm *AD*, and 200 as a maximum value for the *Q* of the coil measured, then

$$2\pi fC < 200,$$

and

$$C < 1 \text{ mf}.$$

We have accordingly used a value of .6 mf in this arm to correspond with the value of C_3.

Resistances *R* and r_2 are dial type completely shielded resistance boxes which can be varied from 0 to 10,000 ohms in .01 ohm steps. The resistances are all of the reversed layer type, wound on impregnated wood spools and designed to give low phase angle and high stability.

The condensers are of the paraffine impregnated mica type, about ten years old, thus ensuring high stability, and having temperature coefficients less than .003 per cent per degree C., over the ordinary range of working temperatures.

The transformers are of a special type described by Shackelton.[1]

ACCURACY—MEASUREMENT OF INDUCTANCE

As previously stated the shielding, while increasing the stability of the bridge, introduces capacitances across *R* and r_1 which increase the corrections necessary in computing the effective resistance and may also require corrections in the measurement of inductance if sufficiently large. Accordingly, measurements were made on the bridge to determine the magnitude of this error. By shunting *R* and r_1 respectively, it was readily shown that capacitances as high as 200 mmf would not change the indicated inductance reading by as much as .01 per cent for all settings of r_1, for the whole range of *R*, over the whole audio frequency range. This conclusion is in accordance with equation 1. Since the shielding introduced capacitances across these points of the order of 25 to 50 mmf, this source of error may be neglected in the measurement of inductance.

Table I gives the exact values for C_3 and r_1, and the corresponding constant K by which the indicated value of R must be multipled to give the true inductance. This table shows how accurately the resistance r_1 has been adjusted to make the bridge direct reading. K is a simple number within .02 per cent in all cases when using the large condenser. The two condensers might have been made to have a ratio more nearly 10 to 1 by adding an auxiliary condenser to the larger one.

TABLE I

$K = C_3 \times r_1$ = Millihenrys per Ohm

r_1 (Ohms)	82.785	165.59	828.04	1656.1	8280.9
C_3 (mf)					
.60381.........	.049987	.099985	.49998	.99998	5.000
.06052.........	.0050103	.010022	.050113	.10023	.50117

A check was next made on a single inductance having a nominal value of .1 henry to determine the relative accuracy of different values of K at different frequencies. These values are given in Table II. It will be noticed that the value of L obtained is approximately independent of K but the highest value obtained is for the value of K corresponding to the highest value of r_1. Since the reactance of this coil is only approximately 600 ohms at 1,000 cycles and the largest value of r_1 used was 828 ohms, it is evident that the potential of the coil with respect to ground varies considerably for different values of K. This is sufficient to account for the increased inductance value obtained for values of K using r_1 = 828 ohms. Keeping this

TABLE II

Comparison of Different Values of K Using a Single Inductance

Nominal Inductance, Millihenrys	K	Frequency, Cycles	R Ohms	R' Ohms	$L = K(R - R')$ Millihenrys
100...........	.099985	1,000	1,006.64	.03	100.65
"	.49998	"	201.34	.00	100.66
"	.050113	"	2,009.4	.45	100.67
"	.049987	"	2,013.4	.09	100.64
"	.099985	3,000	1,022.0	.03	102.18
"	.49998	"	204.40	.00	102.20
"	.050113	"	2,040.3	.00	102.24
"	.049987	"	2,044.1	.09	102.17

in mind the different values of K agree with each other very closely. It has already been stated that r_1 should be small compared with X and therefore the values of K using $r_1 = 828$ ohms would not normally have been used for the measurement of this coil.

Table III gives a comparison of the inductance of several coils as measured on the Owen bridge and by a resonant method, the last column giving the difference between the two methods in per cent.

TABLE III

COMPARISON OF OWEN BRIDGE WITH RESONANCE BRIDGE

Nominal Inductance, Henrys	Frequency, Cycles	Measured Inductance		Difference, Per Cent
		Owen Bridge, Henrys	Resonance, Henrys	
.1	1,000	.10065	.10066	− .01
.1	2,000	.10124	.10118	+ .06
.15	1,000	.15072	.15082	− .04
.15	2,000	.15112	.15111	+ .01
1.0	2,000	1.0143	1.0144	− .01
2.9	1,000	2.918	2.918	.0
2.9	2,000	2.976	2.974	+ .07
10.0	2,000	11.295	11.27	+ .22

The resonant method was a highly accurate one in which frequency errors were negligible. The accuracy was probably of the same order as the measurements on the Owen bridge. The agreement between these two methods does not in itself indicate the accuracy of either method. However, the resonant measurements were made on a completely shielded equal ratio-arm bridge,[1] in terms of frequency and capacitance, using entirely different equipment from the Owen bridge in which the inductance is measured in terms of resistance and capacitance. Accordingly it is very improbable that these two methods had any errors in common and we may assume that the agreement obtained is a fair measure of the combined error of the two methods. Consequently from this table we see that for a range of .1 to 3 henrys and for frequencies up to 2,000 cycles the error in the measurement of the inductance by the shielded Owen bridge is less than .1 per cent and for 10 henrys is less than 1/4 per cent.

ACCURACY—MEASUREMENT OF RESISTANCE

The measurement of effective resistance in the case of an impedance of low reactance practically consists of the substitution of the unknown for the known resistance. In this case the accuracy of the measurement is high. However, the usual case we have to consider is the measurement of the effective resistance of coils of high Q. It is in such measurements that the greatest corrections are necessary, and it is also in such measurements that the greatest errors in effective resistance are produced by incomplete shielding in the bridge. Consequently it is in the measurement of effective resistance that shielding is most essential, and although this shielding may introduce a necessity for larger corrections due to the capacitance it introduces, these corrections may be made with a certain degree of precision and having made them the value obtained will be more reliable than in the case of a complete absence of shielding.

Table IV gives the figures for the measurement of effective resistance of three coils having a high Q. Referring to equation 5 we see that q_1 and q_4 are positive when the reactance is inductive and that Q_3 is always negative. The column headed R_e is obtained from equation 5. The column headed R_e' is obtained from a resonant method of measurement which has the same order of accuracy as the present method. Consequently the last column of differences gives the combined error in the two methods. In these measurements covering the most used range of inductance and a frequency range of 1,000 to 3,000 cycles, the largest difference between the two methods is 3 per cent. The total corrections to be made are in some cases extremely large, especially for the higher inductances and frequencies. This correction may amount to 30, or 40 per cent in some cases, and this means that an effective resistance obtained by the Owen bridge when not corrected may be in error by this amount. However, after allowing for the necessary corrections we can say that the bridge is capable of an accuracy for the measurement of effective resistance of about 2 per cent over the greater range of inductance and frequency.

TABLE IV

Inductance, Henrys	Frequency, Cycles	r_2' Ohms	r_2 Ohms	q_1	q_4	$\frac{1}{Q_3}$	$X\left(q_1 + q_4 + \frac{1}{Q_3}\right)$	R_e Ohms	R_e' Ohms	Diff., %
1	1,000	1,670.66	1,358.30	− .0004	.0000	+ .0023	− 17	329.4	326	1
1	3,000	1,670.15	1,373.14	− .0013	"	"	− 68	365	378	3
.1	1,000	1,670.66	1,641.46	− .0004	"	"	− 1.7	30.9	30.1	3
.1	3,000	1,670.15	1,643.34	− .0013	"	"	− 6.8	33.6	32.5	3
.02	1,000	1,670.66	1,659.30	− .0004	+ .0003	"	− .30	11.66	11.47	2
.02	3,000	1,670.15	1,659.15	− .0013	+ .0011	"	− .94	11.94	11.8	1