Analyzer for Complex Electric Waves

By A. G. LANDEEN

IN problems concerned with the electrical transmission of intelligence it is necessary to have means for studying complex electric waves. In certain steady state conditions these complex waves become periodic, and, although not sinusoidal as a whole, may be resolved into a number of sinusoidal components. It is particularly important to be able to measure these components individually.

In studies on systems employing carrier currents which may be transmitted over wire lines it is often necessary to measure a signal wave component which may lie anywhere in the frequency range between 100 and 100,000 cycles per second. The most important range at the present time is, however, below 40,000 cycles per second. In addition to covering a wide range of frequencies these components may also vary considerably in amplitude, both as to absolute value and as to value relative to other components in the signal wave.

For several years there has been in use in the Bell Telephone Laboratories special apparatus by means of which a single component of a complex periodic current wave may be selected from the remaining components and its amplitude determined. The sensitivity and selectivity of this apparatus are such that components of small amplitude may be accurately measured even in the presence of other components of several hundred times the amplitude and differing but little in frequency. With the latest improved form it is now possible to measure current components having amplitudes as low as 10^{-7} amperes with a possible error of 10 per cent. For such minute currents this is within the error which might be introduced by the external apparatus such as attenuators and thermocouples together with their calibration charts.

Though the apparatus was primarily designed for use in current wave analysis work, it may also be readily adapted to voltage analysis. Suitably calibrated, it can be used also as a frequency meter of extremely high precision.

INTRODUCTION

The method of analysis here described had its origin in a circuit built by J. W. Horton in 1917. This had a resistance coupled tuned circuit responsive to the component desired. Following the tuned circuit two stages of amplification were used to magnify the selected current. This current was then passed on to a third unit where it was rectified and measured by a D.C. meter. It was evaluated directly by noting the meter deflection and referring to calibrations of the analyzer which had been made with known input currents.

This elementary form of measuring circuit was developed during the World War for the analysis of the sound waves encountered in listening devices used for the detection and location of submarines and torpedoes. It covered the range of audible frequencies and had sufficient sensitivity for its original purpose.

It will be remembered that the first commercial application of multiplex transmission by means of carrier currents came almost simultaneously with the Armistice. The continued study of carrier systems found a useful tool in the current analyzer but placed considerably more rigorous requirements on its performance. These were met by the addition of a second tuned circuit and amplifier system, working from the output of the first, thus giving far greater selectivity than is obtainable in a single circuit. The presence of the multi-stage amplifier between the selective circuits facilitates tuning by avoiding interactions between the circuits. A second modification was the use of a substitution method for evaluating the amplitude of the selected components, as with the considerable increase in the ranges of amplitude and frequency covered, the calibration method for measuring the current became impracticable. To evaluate the current, the output from a sine wave oscillator, which was tuned to the same frequency as the component being measured, was substituted at the input to the analyzer and the amplitude adjusted until it gave the same meter deflection as the unknown component. Since the current from the oscillator is of the same frequency and amplitude as the original component, we can determine the magnitude of the latter by measuring the oscillator output. A convenient means for doing this is to interpose between the oscillator and the analyzer a variable attenuator. It is then possible to fix the oscillator output current at some convenient value, such as 1.0 milliampere or 10 milliamperes and to adjust the input to the analyzer by means of the attenuator. The current can then be read directly from the attenuation tables, it being only necessary to know the location of the decimal point.

The development of the analyzer in this form was carried to the limit of its practicability by F. Mohr. With an analyzer containing three units it was possible to carry through an extensive study of the modulation introduced into the Key West-Havana cable due to the non-linearity of the characteristics of the iron used for loading.

The Heterodyne Method

With the advance in carrier communication, greater refinement in measurement became necessary, calling for still higher selectivity in the analyzer. The best means for accomplishing this appeared to be to heterodyne the wave under investigation in such a manner as to move it to a lower position on the frequency scale. Then with a fixed tuned circuit which would pass only the low frequency current corresponding to the desired component, much greater selectivity might be obtained because of the relatively greater spacing.

To heterodyne the desired component there is required a separate oscillator and a modulator in which the current to be measured and the separately generated current are combined to produce a current of lower frequency. This in effect translates the current under investigation from a high frequency to one of much lower frequency; retaining, however, the relative amplitudes of the components. Since the amount of this translation is determined by the frequency of the local oscillator, a particular component can always be given a certain predetermined value by adjusting the oscillator. This permits the use of a fixed tuned circuit which is highly selective to the difference frequency in the modulator output. By choosing a low value for this frequency it is possible to make the percentage difference between this and interfering frequencies much larger than between the corresponding high frequencies. In the present analyzer 800 cycles per second has been chosen as a suitable value. If, then, the current to be measured had a frequency of 20,000 cycles per second, the local oscillator would be set at 20,800 cycles. It could of course also be set at 19,200 cycles if desired and produce the same difference frequency. If there were also present another current of say 20,500 cycles, the interval in the original wave would only be 2.5 per cent; after heterodyning, however, it would appear as a 300 cycle current, if heterodyned by the 20,800 cycle current. The interval thus becomes nearly 40 per cent of the frequency for which the tuned circuit is adjusted. If these currents are heterodyned directly in a simple modulator, there is also the possibility of modulation between components in the original complex wave. This would result, in the case chosen, in a current having a frequency of 500 cycles, but the percentage difference between the 800 and either the 500 or the 300 cycle currents is many times greater than that between the original high frequency currents so that the fixed tuned circuit would have a very high discrimination to the interfering current.

In some cases the intermodulation between components of the original wave may coincide with the component being measured, or may interfere with the measurement in some other manner. This difficulty is minimized in two ways: first, the amplitude of undesired components is reduced relative to the component under observation, by using selectivity of the type previously described, before impressing the wave on the modulator; second, the modulator is made of a balanced type which permits efficient heterodyne action between the selected component and the heterodyning current, but which gives very little intermodulation between such components as remain after the initial selection. The partial elimination of undesired components before modulation is of further advantage in preventing unnecessary loading of the modulator, which might tend to give it different efficiencies with the complex current and with the sine wave calibrating current.

Description of Apparatus

A schematic diagram showing the several functional elements is given in Fig. 1. The units have been arranged in the order in which

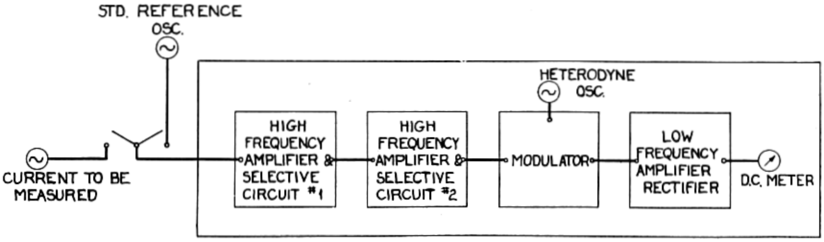

Fig. 1—Schematic diagram of heterodyne current analyzer

the measuring current would proceed, which is also the order of assembly of the completed instrument. In the following, a more detailed description will be given of the individual units.

High Frequency Amplifiers. The circuit for the first unit is shown in Fig. 2 and for the second unit in Fig. 3. These circuits differ mainly in the output terminations of the second tube. The first high frequency unit consists of a simple series tuned circuit together with two amplifier tubes. In the tuned circuit is also included the coupling resistance, R, for controlling the input to the amplifier. The sharpness of resonance will therefore depend to some extent upon the value of this resistance, but under most operating conditions it is relatively small in comparison to the total effective resistance which includes that of one tuning coil and the condensers.

In both of these circuits the A.C. input voltage to the first tubes is obtained from the drop across the condensers. If it is desired to discriminate against a component of higher frequency than the one being measured, it is advantageous to use the voltage across the condenser, whereas if the interfering component is of lower frequency, the voltage across the inductance should be used. It is therefore desirable to make provision by means of a switching arrangement whereby the voltage may be applied from either the inductance or the capacity depending upon the discrimination desired. With the coils and condensers used the voltage across the condenser at resonance is over a hundred times that across the coupling resistance.

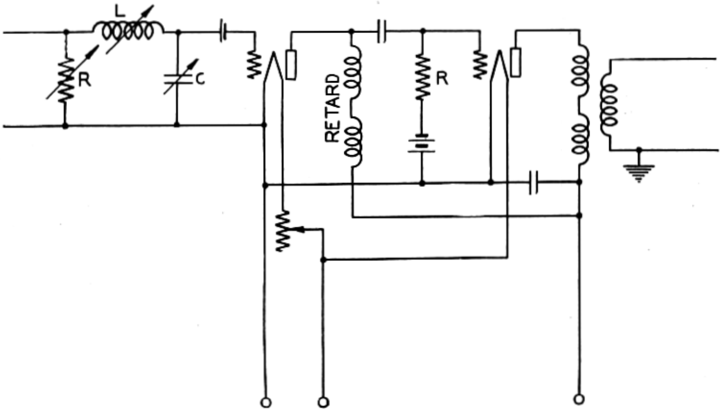

Fig. 2—High frequency amplifier unit No. 1 of heterodyne current analyzer

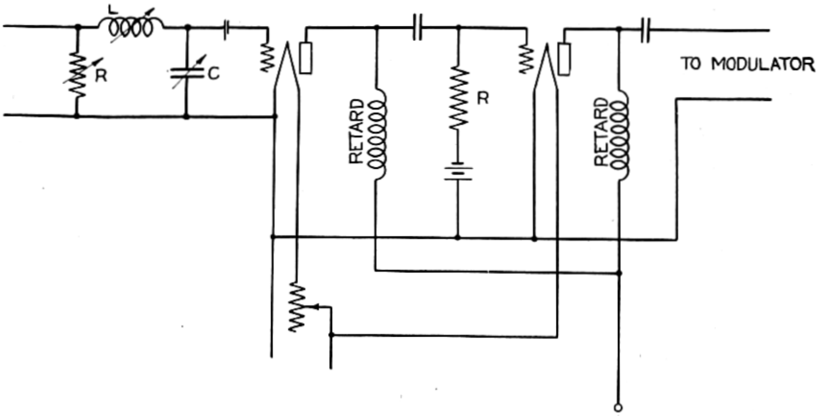

Fig. 3—High frequency amplifier unit No. 2 of heterodyne current analyzer

The first tube of each unit serves as a voltage amplifier. This works into a high resistance between the grid and filament of the power tube.

Because of the wide range of frequency over which the amplifiers will be operated, resistance, rather than transformer coupling, was chosen as the most reliable form of interstage connection. Due to the large step-down in impedance necessary between the first amplifier and the coupling resistance of the second selective circuit, a transformer having a high step-down ratio was used. The output of the second amplifier also works into a transformer through a fixed resistance and potentiometer as shown in Fig. 5. The purpose of the fixed resistance is to maintain a more uniform output impedance as the potentiometer is varied. Though the two amplifiers are almost identical in their circuits, they differ in the plate voltage. The first section is operated at

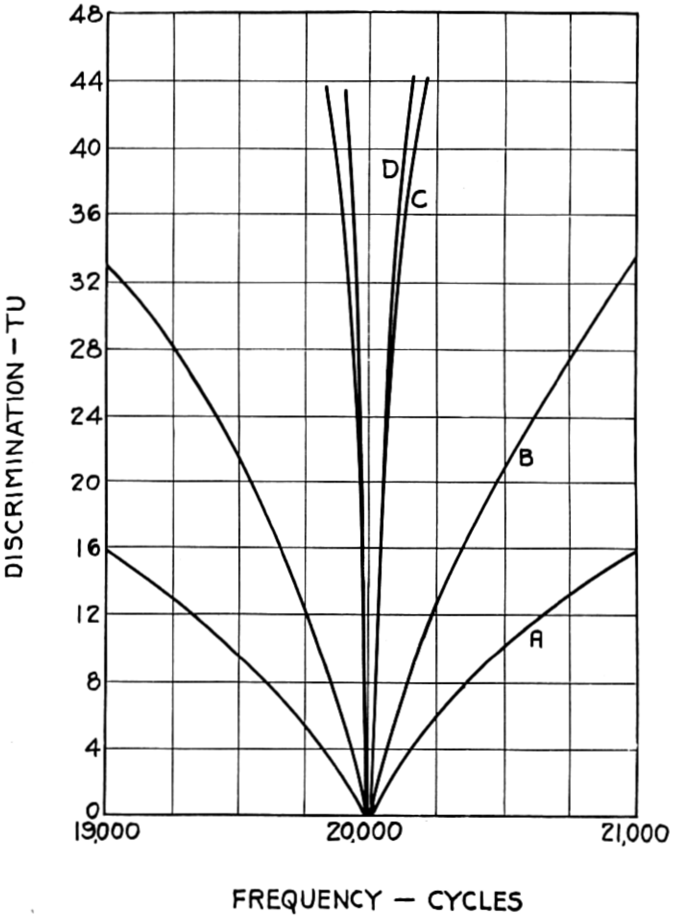

Fig. 4—Discrimination curves of heterodyne current analyzer

240 volts since its tubes are subjected to the heaviest input, being preceded by only one selective circuit which can but partially eliminate large interfering currents. This seems a most unusual arrangement until it is remembered that, although the amplitude of a particular part of the current may be increased, the total load on the first stage may well be greater than that on the final stage. This is the reverse of the situation in cascade amplification where the first tubes handle only a small current and the succeeding ones a proportionately larger current.

The discrimination of one of the tuned circuits when a coupling resistance of 1 ohm is used is given by curve *A* of Fig. 4. Curve *B* shows the effect of adding the second tuned circuit. If regenerative amplification were employed, both the selectivity and amplification could of course be greatly increased, but this has not been used because of the necessity for high stability and measurement precision.

Modulator. As previously mentioned, the second amplifier unit works into a modulator, the circuit of which is shown in Fig. 5. This is of a two tube balanced type in which modulation, or frequency transformation, takes place in the grid circuit. The heterodyning frequency is applied in the common input lead across a suitable resistance. The input from the amplifier is applied through a transformer across the grids of the two tubes in series with a high resistance in each side. No biasing potential is applied on the grids. A modulator operated in this manner has the property of giving a modulation output proportional to the smaller of the two input currents and independent of the larger. The amplitude of this output may, therefore, be determined entirely by the amplitude of the component being measured. Another desirable characteristic of this type of modulator is that its efficiency is not affected by interference, hence it will show a fixed relation between

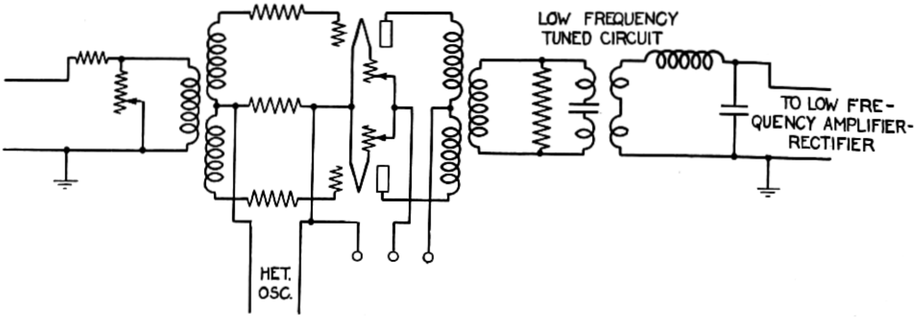

Fig. 5—Modulator for heterodyne current analyzer

the low frequency output and a given input component regardless of the presence of other interfering currents in the input side. Through the use of a balanced circuit intermodulation is reduced considerably below the limit possible with a single tube modulator.

The output of the modulator is connected directly to a double tuned circuit which selects the low frequency modulation product corresponding to the component of the complex wave being examined. The frequency to which this circuit is adjusted is, as already mentioned, 800 cycles.

Low Frequency Amplifier-Rectifier. The 800 cycle output from the modulator is generally too small to measure on a meter of the usual type without first being amplified. For this reason a low frequency amplifier has been added and the output of this rectified so that all measurements could be made on a sensitive D.C. meter, having a full scale deflection of 1 or 2 milliamperes.

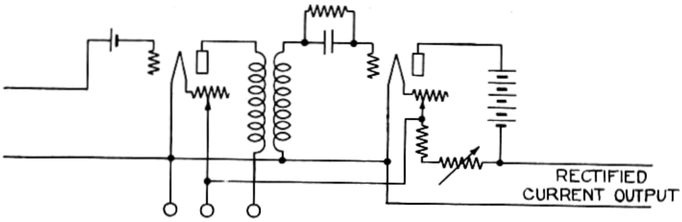

Fig. 6—Low frequency amplifier-rectifier for heterodyne current analyzer

The combined low frequency amplifier-rectifier circuit is shown in Fig. 6. A step-up transformer is used between the amplifier tube and the rectifier in order to increase the amplification so that, for a given A.C. output, a smaller input to the first amplifier might be used. Since this circuit is always to be operated at the one frequency (800 cycles), its overall frequency range characteristic is not of particular interest and its performance was studied only at the one frequency. The rectifier is of the grid leak and condenser type. Its performance differs from the usual type of rectifier since it is operated over that portion of its characteristic which gives a linear relation between input voltage and direct current output. By suppressing the space current corresponding to zero input the accuracy with which data can be taken is greatly increased. As shown by the circuit arrangement in the output side, part of the "A" battery current is used to oppose the space current through the meter. This permits using a meter of high sensitivity, having a full scale deflection of one or two milliamperes, on which 1/100th part of a milliampere can easily be read.

Heterodyne Oscillator. The major requirement to consider in the design of the heterodyne oscillator was that of frequency stability. As the only function of the oscillator was to furnish a current for heterodyning the one being measured the output requirements were moderate, 10 milliamperes into 600 ohms being ample, but it was important that the frequency remain constant during a series of measurements as even slight variations, of a fraction of a per cent, would change the attenuation of the 800 cycle tuned circuit to the sideband current. Stability in this case depended mainly upon the "A" and "B" battery voltages, since the output load consisted of a pure resistance and there was no reaction back to the oscillator due to a variable output impedance.

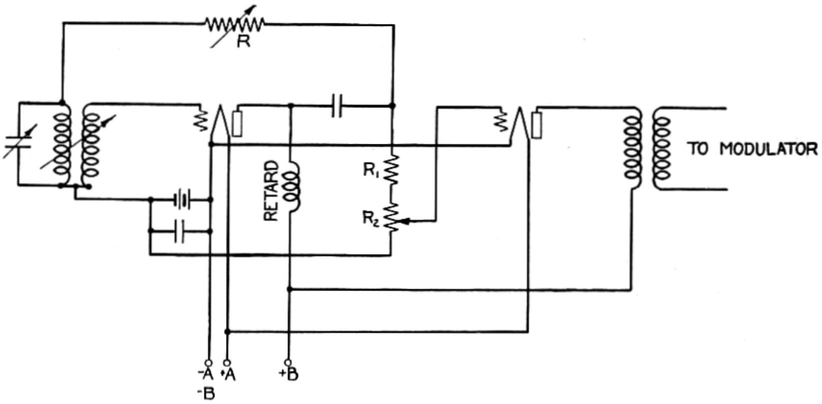

Fig. 7—Oscillator for heterodyne current analyzer

The oscillator circuit is shown in Fig. 7. This shows two tubes, one as an oscillator and one as an amplifier. The coupling consists of a 20,000-ohm resistance used as a potentiometer, which is placed in series with a 100,000-ohm resistance and the two used as the oscillator load. This makes the coupling impedance only one sixth of the total oscillator output impedance and therefore reduces the effect which the amplifier tube might have on the frequency. The change in frequency due to the "A" and "B" voltage can also be controlled by inserting a high resistance in the feed-back path between the plate and oscillation circuits. This should be several times that of the tube impedance so that any change in the latter would then be a proportionately smaller part of the total impedance and hence have a less effect upon the frequency.

The selection of tuning coils for various frequency ranges is made by keys which at the same time select the proper feed-back resistance. Only three coils are used to cover the frequency range between 3,000 and 50,000 cycles. The output of the amplifier tube works into the resistance in the mid-branch of the modulator input circuit.

MECHANICAL CONSTRUCTION

Figs. 8 and 9 show the front view of the complete analyzer as in present use and also the interior view of an individual unit (the heterodyne oscillator), to indicate the method of construction.

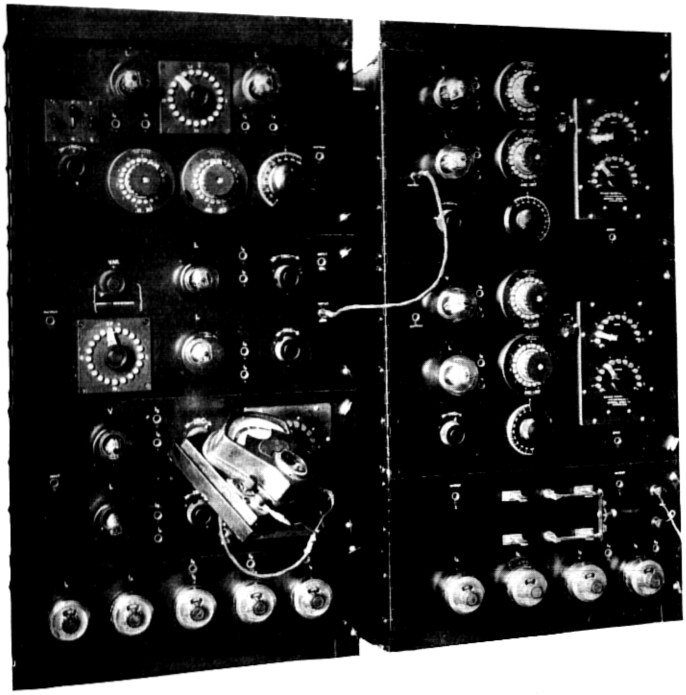

Fig. 8

Each section is completely enclosed in metal, by having shielded cases in the rear and heavy metal panels in front. Perhaps the feature of most interest in the mechanical construction is that of the hinged front panels. These were made of 1/4 in. aluminum to insure ample strength and rigidity with a minimum of strain on the supporting hinge. Each panel is provided with two thumb screws on the edge opposite the hinge so that the units may be readily inspected. This feature of accessibility is particularly desirable in making periodic inspections and in renewing the "C" batteries, which have been mounted on the panels. The heavy material such as tuning coils has been mounted inside the case. The flexible leads, as shown in the lower corner of the opened unit, connect the panel apparatus with the "A" and "B" batteries, and with the modulator. All of the panels are provided with a metal strip, such as shown along the top and bottom edge, which fits into a groove in the case and thereby provides better shielding between each of the units. Since each unit of the analyzer is provided with a separate shielding case, two sheet iron walls are interposed between any pair, forming a double shielding to electric fields occurring within the analyzer. The complete unit of two sections is very economical in space as the overall height, width and depth of each section are only 36, 19 and 12 1/2 inches respectively.

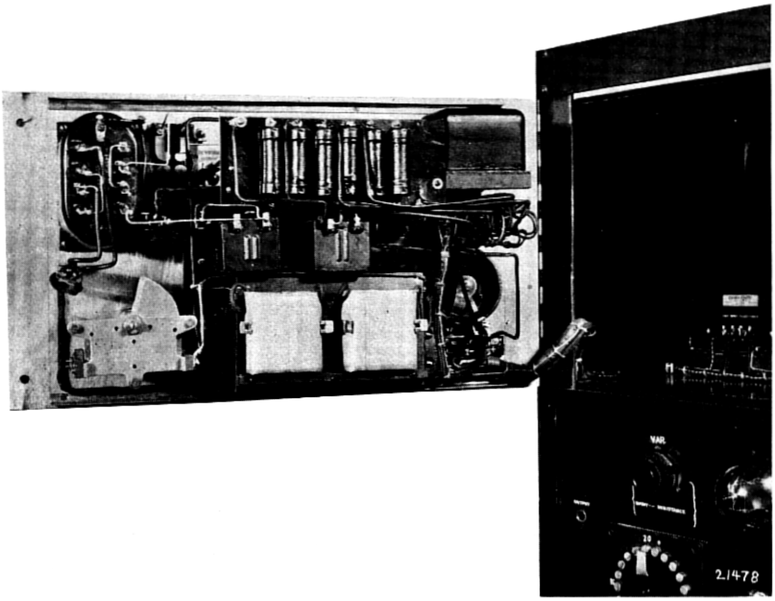

Fig. 9

CHARACTERISTICS OF COMPLETE CURRENT ANALYZER

The successful operation of the heterodyne current analyzer depends, of course, upon knowing its limitations and its reliability. Under limitations may be grouped sensitivity, selectivity and modulation in the analyzer itself; and under reliability, the limits within which readings can be repeated.

Sensitivity. The sensitivity depends upon the coupling resistance used in the two amplifier units, and increases with added resistance so long as this is only a few ohms and small as compared to the total effective resistance in the selective circuits. When these are made 1 ohm each, it requires only 1 microampere to give one milliampere of rectified current. With 10 ohms coupling resistance only 10^{-8} amperes input would be required. This, however, is a larger coupling than it is desired to use, since the analyzer becomes too sensitive and susceptible to mechanical vibrations as well as to electrical interference from outside sources.

One desirable feature is that the sensitivity characteristic of the complete current analyzer is a straight line so that doubling the input will give twice the deflection in the meter reading the rectified current. This is an advantage since, if the deflections and input amplitudes do not change by the same ratio, the presence of interfering currents is indicated.

Selectivity. The selectivity of the current analyzer depends upon the time constants of both the high and low frequency tuned circuits, and to some extent upon the coupling resistances, but the latter are usually relatively small and do not have an appreciable effect. The discrimination obtained by the use of the heterodyne method and fixed low frequency selective circuit is shown by curve C of Fig. 4. This curve may be compared with curve A, which shows what can be done with high grade elements in a single tuned circuit. The discrimination of the complete analyzer, including the initial stages and the heterodyne stage, is given by curve D. Tests with two frequencies show that if one is 250 times as large as the other the smaller may be measured without appreciable error if the difference in frequency is not less than 1 per cent; if the ratio of amplitudes is 1,000 to 1, the frequency difference need not be less than 2 per cent.

Modulation. As to modulation in the current analyzer there are two sources which contribute, the vacuum tubes and the tuning coils and transformers. Of these the tubes are the most troublesome, since they furnish both even and odd order modulation products, whereas the coils contribute only to the odd orders. Of these the third is generally the only one that is of any interest since higher odd orders are too small to produce any interference. Modulation need be considered only when measurements are made of small components in the presence of very large ones, as it is under these circumstances that conditions for modulation are the most favorable. This condition requires high sensitivity which is obtained by increasing the coupling resistance, and large resistance means greater interference voltage on the first amplifier and also less selectivity in the tuned circuits. The result is an increased load on all sections of the current analyzer, which causes modulation and may produce an error in the readings. It is, therefore, important to know the limitations which this imposes upon it as a measuring device. Then with this information data can be taken within known limits of accuracy. Measurements have been made on the combined modulation occurring in the analyzer and also in the filters which were necessarily associated with the measurements. These were made with one input current and also with two input currents of different frequencies, which were applied simultaneously to the analyzer. One or two filters were therefore necessary to suppress all components except the fundamental currents desired, but any small amount of modulation which would occur in the filters would add to that produced in the analyzer so that the results shown by the curve in Fig. 10 represent the total modulation in both the filters and the analyzer. The modulation amplitude is expressed in terms of transmission units with respect to the current into the analyzer. This curve is quite irregular and depends somewhat upon the frequency. It is the lowest at 39,000 cycles where the modulation current is shown to be 86 TU^1 down or 0.00005 as large as the current into the analyzer. Measurements can, therefore, be made at this frequency of the modulation occurring in any device when its amplitude is not less than 66 TU below the amplitude of the fundamental, with a possible maximum error of 10 per cent. At other frequencies this amplitude may be less, as for instance at 21,000 cycles, measurements may be made up to 80

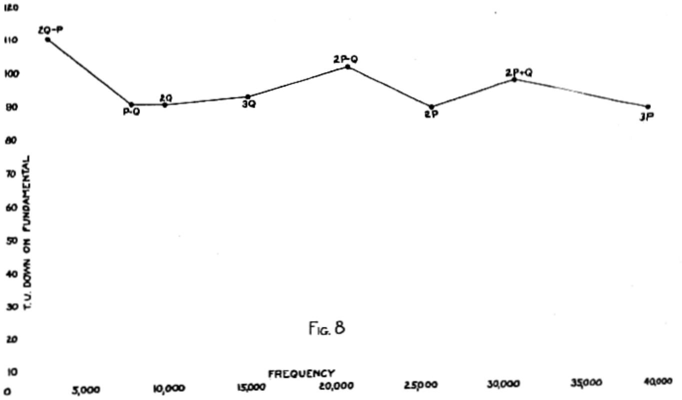

Fig. 10—Second and third order modulation in heterodyne current analyzer together with 5,000 and 13,000 cycle band-pass filters with one or two input frequencies

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[1] For a discussion of this method of expressing current ratios see "The Transmission Unit and Telephone Reference Systems" by W. H. Martin, *Journal A. I. E. E.*, June 1924, Vol. XLIII, No. 6; also *Bell System Technical Journal*, July 1924, Vol. III, No. 3; also "The Transmission Unit," R. V. L. Hartley, *Electrical Communication*, July 1924, Vol. III, No. 1.

TU without exceeding the same percentage of error, or up to 60 *TU* with 1 per cent error due to undesirable modulation in the current analyzer.

Reliability. Use of the heterodyne current analyzer over a period of two years has proven it to be one of the most reliable means for making measurements. With proper maintenance, which consists only in maintaining constant "A" and "B" battery voltages and grid voltage, and with proper precautions as to shielding and balance, readings can be taken with a precision of 2 per cent.

Vacuum Tube Curves Obtained with Heterodyne Current Analyzer. A number of curves have been added to illustrate the application of the current analyzer, though of course these represent only a small part of the field of usefulness for which it is adapted.

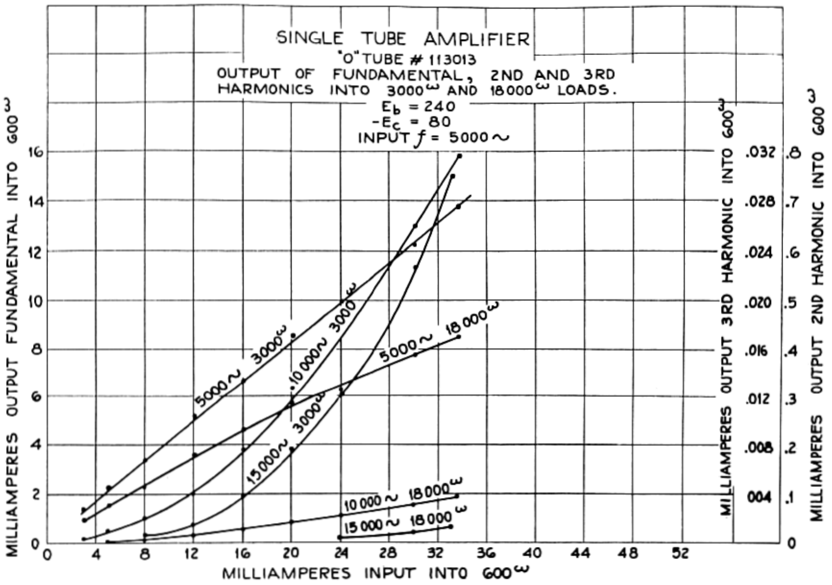

Fig. 11

The first set of curves, shown in Fig. 11, were taken on an "O" tube (104–D) to show how the fundamental current and the second and third harmonics produced in the tube changed with increase in the input amplitude of a single frequency. Two sets of curves are shown which were taken for two values of load impedance, one being equivalent to the normal tube impedance and the other being six times as large. The output currents have all been computed to show the equivalent output into 600 ohms which is a common reference standard of impedance used in telephone work.

The second set of curves, shown in Fig. 12, were taken with an "L" tube (101–D) but with two input frequencies of $Q = 5{,}000$ and $P = 13{,}000$ cycles applied simultaneously. The measurements in this case were made on the modulation product consisting of the difference frequency $(P - Q) = 8{,}000$ cycles and of the second harmonic of Q, $2Q = 10{,}000$ cycles. Such curves furnish a quick and accurate means of studying the performance of the vacuum tube and also afford a convenient check on mathematical computations which, in this particular case, with a two frequency input, have indicated that the difference frequency amplitude should be twice that of the second harmonic.

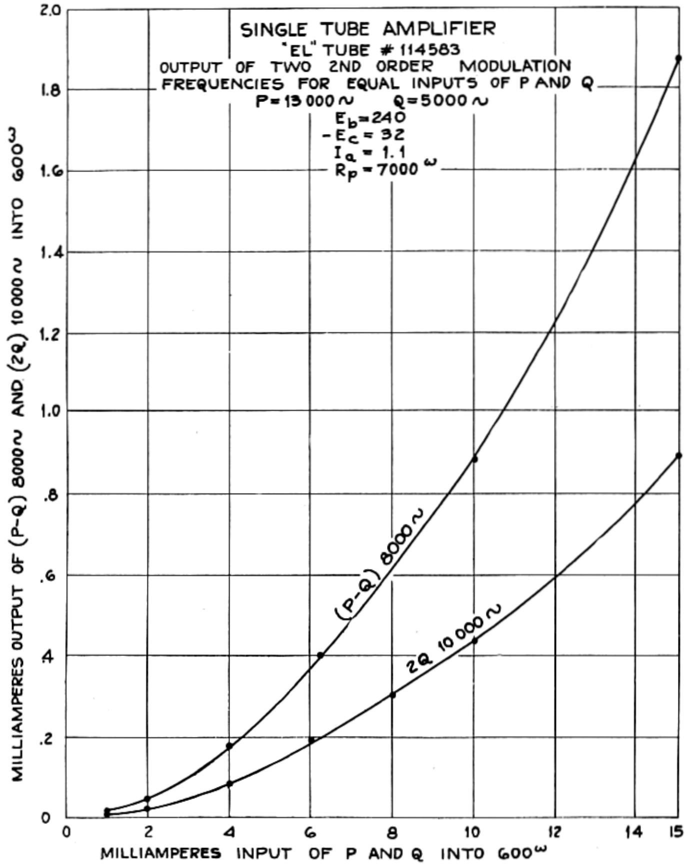

Fig. 12

In Fig. 13 are shown the third harmonic output of an "O" tube with a single input frequency of constant amplitude and with a variable load impedance. The harmonic current shown by this curve could

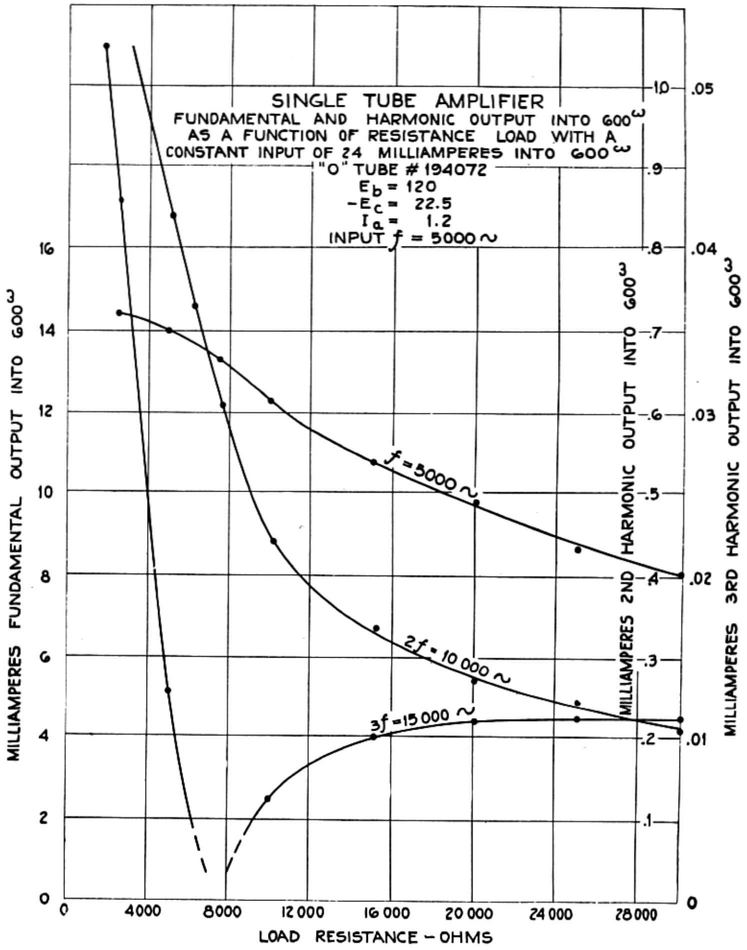

Fig. 13

not easily be predicted from computation due to the many factors involved, and can be measured only by an instrument of high sensitivity that can faithfully follow the varying amplitudes of a minute current.

Voltage Analyzer. In addition to its use as a current measuring device the current analyzer may be used, as previously mentioned, for measuring the voltage in circuits where the power dissipation is very small, or where the voltage cannot be detected except by several stages of amplification such as are obtained in the current analyzer. To adapt it for this purpose it is only necessary to precede it by a simple circuit such as shown in Fig. 14. This consists of a single vacuum tube having a large resistance across the grid and filament. This resistance should be greater than the impedance across which the voltage is to be measured. The output side works into a step-down transformer through a resistance of several times the output impedance of the tube. This tends to straighten out the characteristic and to lower the tube modulation level. The object of the step-down transformer is of course to secure greater efficiency in working into the low input impedance of the heterodyne current analyzer.

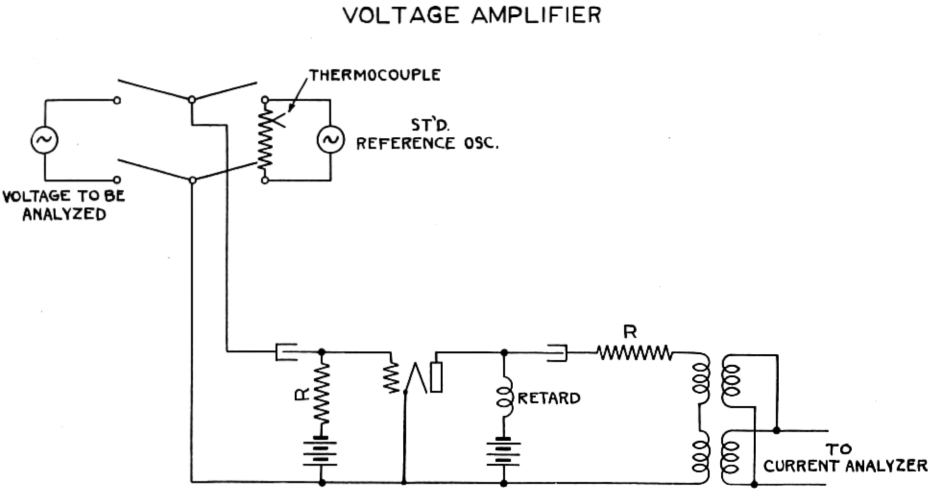

Fig. 14—Voltage amplifier

The measuring procedure would be to apply the voltage to be evaluated across the high impedance input and adjust the analyzer in the usual manner; then substitute the output from the standard oscillator of the same frequency across the amplifier and adjust the amplitude to give the same meter deflection. In order to determine the voltage applied, the oscillator may be connected across a known resistance in parallel with the input, and the current into this resistance measured. The IR drop will then be a measure of the voltage applied.

The range of voltage which can be measured of course depends upon the biasing potential on the amplifier grid as it is not desirable that grid current flow through the high resistance and increase the tube modulation. The most frequent use of the amplifier in the laboratory has been in measuring voltages around 10^{-2} volts but it can also be equally well used to measure much smaller values, of the order of 10^{-4} volts, when the frequency employed does not make the input impedance of the first tube too low.

In the choice and arrangement of the elements of this system and for many of the details of its adjustment recognition is particularly due to the extensive contributions of E. Peterson, W. A. Mueller and C. R. Keith.