Line Current Regulation in Bridge Polar Duplex Telegraph Circuits

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SYNOPSIS: A mathematical analysis of the bridge polar duplex telegraph circuit, under the condition that the bridge arms are of equal resistance, shows that there is a particular bridge arm resistance which results in maximum received current. As the bridge arm resistances are increased beyond the value giving this maximum, the received current diminishes gradually. On the other hand, as the bridge arm resistances are decreased below the value giving the maximum, the received current drops off very rapidly. It follows that when necessary to limit line current, the maximum received current is obtained by placing the regulating resistance in the bridge arms. Also when the line resistance is large enough to limit the line current to less than the maximum allowable value, a gain may be obtained by increasing the bridge arm resistance to the value which corresponds to maximum received current. Experience has shown that in many situations where difficulty is encountered in operating a duplex telegraph circuit with the regulating resistances *in the line*, a very decided improvement is obtained by transferring these resistances to the bridge arms.

FOR the operation of polar duplex telegraph circuits, line batteries of uniform voltage are generally used and it is usually desirable to maintain the line current within fairly definite limits. The most suitable line battery voltage and the desired limits for the line current depend upon the type of line and apparatus used. In order to maintain the line current within the desired limits with uniform voltage it is necessary to add resistance to the circuit in greater or less amounts depending upon the length and gauge of the line circuit used. On account of line trouble and the necessity for rerouting telegraph circuits for other reasons, it is frequently desirable to switch a duplex set from one line to another of different resistance. To facilitate line current regulation without delaying service when such changes in line assignment are made, it is of considerable operating advantage to include in the wiring of each duplex circuit an adjustable resistance in the form of a rheostat mounted in an accessible location at the duplex set so that the attendant can readily regulate the line current at the time that necessary adjustments in the balancing artificial line are made to suit the changed line condition.

This paper outlines an investigation which was made with the object of finding an arrangement of line current regulating resistance which would result in the maximum steady-state received current with the bridge duplex telegraph circuit shown by Fig. 1, where it is desired to limit the line current to about .070 ampere. The condition for maximum steady-state received current was sought as the first step toward determining the most suitable arrangement of the resistances with the viewpoint that such an arrangement would probably be the most satisfactory from a transmission standpoint if it did not adversely effect the important factor of received current wave shape. An arrangement of the resistances was found which results in the maximum steady-state received current and from oscillographic tests which were subsequently made, this arrangement fortunately appears to improve the wave shape of the received current as compared with that resulting from other possible arrangements considered. It was also found from field trials on a number of practical circuits that this arrangement

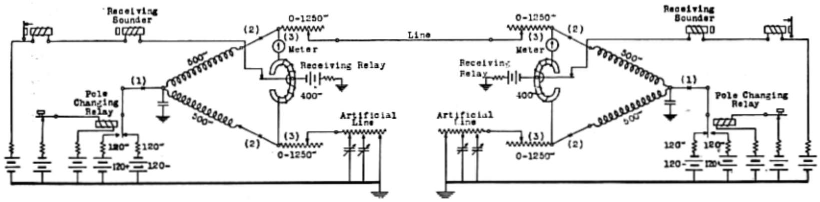

Fig. 1—Bridge Duplex Telegraph Circuit

results in improved transmission over other arrangements which have been considered for the regulating resistances.

Three different locations in the bridge duplex circuit are considered for the regulating resistances. These locations are designated (1), (2) and (3) in Fig. 1 and may be described respectively as follows:

(1) A single resistance in series with the battery branch of the circuit.

(2) Equal resistances in series with each of the bridge arms.

(3) Equal resistances in series with the line and the artificial line of the duplex set.

In considering locations (2) and (3), it is assumed that the resistances are in the form of a double rheostat with the movable arms mechanically connected to facilitate adding equal amounts of resistance simultaneously.

It will be seen from the circuit shown by Fig. 1 that of the three locations for the regulating resistance, (3) might be expected to reduce the received current most for a given line current, as that arrangement introduces resistance directly between the receiving relays. However, as that location for the resistances had been in general use, and since it was not at all obvious which of the other two arrangements would be the most favorable from the standpoint of received current, it seemed desirable to set up line current and received current equations to determine how the currents would be affected by the resistances in each location. Of the six current equations required, the one for the received current with the resistance in location (2) was found to possess a maximum within a resistance range which made that arrangement the most favorable from the standpoint of steady-state received current.

Curves i_1, i_2 and i_3 Fig. 2, show the steady-state value of received current which will be obtained on lines of 500 to 2260 ohms resistance

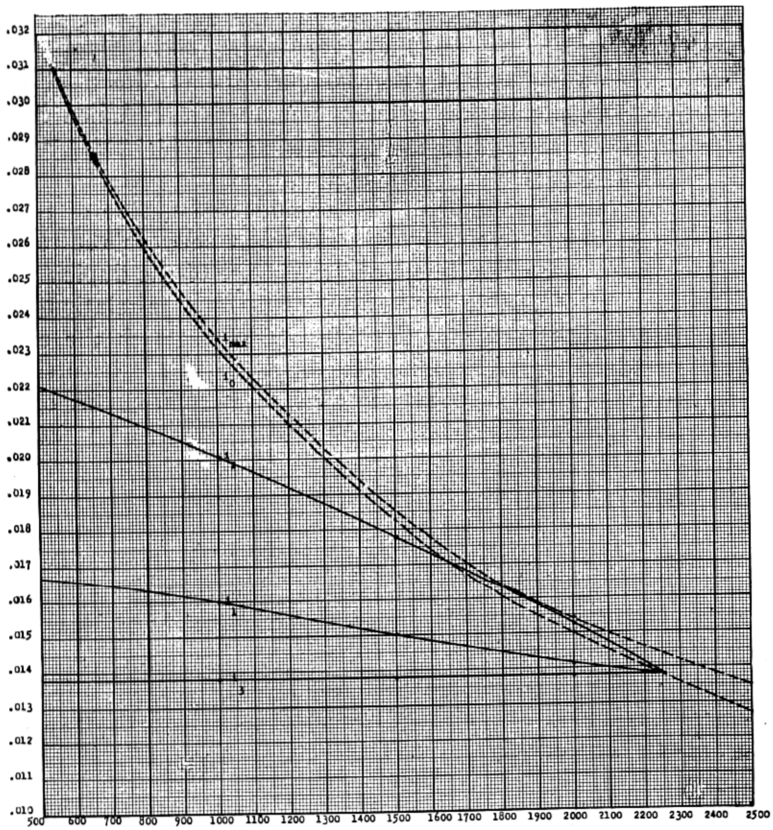

Fig. 2

with the regulating resistances located at points (1), (2) and (3), respectively. In each case just sufficient resistance is added to make the line current .070 ampere. If the resistance of the line is greater than 2260 ohms, the line current will fall below .070 ampere without the addition of resistance at either point. It will later be shown that, regardless of line current limitations, location (2) results in the maximum practicable steady-state received current in bridge duplex operation the bridge arms being of equal resistance.[1]

The method of calculating curves i_1, i_2, and i_3 Fig. 2, will be discussed presently along with certain other mathematical considerations.

In setting up the equations for the received current and line current, certain practical operating conditions of the circuit are assumed; first, that the circuit as a whole be kept symmetrical by using the same amount of regulating resistance at each station and second, that the duplex sets be maintained in a state of balance for direct currents. Line leakage will be neglected.

To express the line and received currents as direct, or explicit functions of the regulating resistances under the assumed condition of the circuit requires the use of unusually cumbersome equations which may to some extent be avoided in the early part of the solution without sacrificing accuracy. The complicated nature of the equations is due largely to the intricate relation between the regulating resistances and the overall network resistance of the duplex set from the terminal of the line to ground and, in turn, the relation between this network resistance and the two currents. With the exception of one step in the present investigation the work has been shortened by representing this network resistance by a parameter, or second independent variable, r which is itself a quadratic function of the regulating resistance, represented by R. The required values of r are then computed from the equation connecting it to R. In the expressions for the ratios of received current to line current all determinants which cannot be readily reduced to the second order cancel out so that considerable work is avoided by using these ratios rather than the explicit line current equations for calculating the line currents.

The equations expressing the relation between the received currents, i_1, i_2 and i_3 and the regulating resistance in the three locations (1), (2) and (3) respectively, are as follows:

$$i_1 = \frac{aE}{(T+R_1+r_1)\,(2a+b)+ab+a^2} \tag{1}$$

$$i_2 = \frac{R_2E}{(T+G+r_2)\,(2R_2+b)+bR_2+R_2^2} \tag{2}$$

$$i_3 = \frac{aE}{(T+R_3+G+r_3)\,(2a+b)+ab+a^2} \tag{3}$$

[1] For the case of unequal bridge arms see Heaviside's "Electrical Papers," Vol. I, p. 24.

and the expressions for the ratios of the received currents to corresponding line currents, I_1, I_2 and I_3 are:

$$\frac{i_1}{I_1}=\frac{a(1/2T+r_1)}{(T+r_1)\,(2a+b)+ab} \tag{4}$$

$$\frac{i_2}{I_2}=\frac{R_2(1/2T+r_2)}{(T+r_2)\,(2R_2+b)+R_2b} \tag{5}$$

$$\frac{i_3}{I_3}=\frac{a\,(1/2T+r_3)}{(R_3+T+r_3)\,(2a+b)+ab} \tag{6}$$

where,

a, represents the constant resistance of each bridge arm in arrangements (1) and (3);

b, the resistance of the receiving relay;

E, the voltage of the line battery which is assumed to be equal at both stations and may be either negative or positive;

G, the constant resistance in series with the line battery taps in arrangements (2) and (3);

T, the resistance of the line between the duplex sets.

R_1, R_2 and R_3 are the regulating resistances in the different locations corresponding to the subscripts. In the equations for arrangement (1), G is assumed to be contained in R_1 and in the equations for arrangement (2), a is assumed to be contained in R_2. The equations for the parameters, r_1, r_2 and r_3 are as follows:

$$r_1=\sqrt{\tfrac{1}{4}T^2+R_1T+\frac{aT(a+b)+ab(a+2R_1)}{2a+b}}-\tfrac{1}{2}T \tag{7}$$

$$r_2=\sqrt{\tfrac{1}{4}T^2+GT+\frac{R_2T(R_2+b)+bR_2(R_2+2G)}{2R_2+b}}-\tfrac{1}{2}T \tag{8}$$

$$r_3=\sqrt{\tfrac{1}{4}T^2+GT+R_3(T+R_3+2G)+\frac{2aR_3(a+b)+aT(a+b)+ab(a+2G)}{2a+b}}-\tfrac{1}{2}T \tag{9}$$

While the line current and the received current can be calculated for any values of R and T from equations (1) to (9) inclusive, explicit line current equations are needed for calculating the received current for a definite value of line current, such as shown by curves i_1, i_2 and i_3, Fig. 2. It is clear that these curves cannot be calculated from equations (1) and (9) alone, as the first step necessary is to determine the value of R which, with a given value of T, will result in the specified value of I (.070 ampere). With line current equations R can, of course, be calculated by substituting .070 for I. While the line current equations can be set up fairly readily, they are of an extremely cumbersome character. For that reason curves i_1, i_2 and i_3, Fig. 2, were calculated by the following method:

From equations (1) to (9) inclusive, the line current was calculated for the various values of T from 500 to 3000 ohms with various values of R from 0 to 2000 ohms in steps of 250 ohms. For each value of T the line current was then plotted against R and the required value of the latter read from the intersection of the curve and the .070 ordinate. The values of R thus obtained were then substituted in equations (7) to (9) for calculating r. These values of R and r in turn were substituted in equations (1) to (3). By the above method the values of R within plus or minus two or three ohms can be determined. This possible error in R will not appreciably effect the points on the curves. The point of intersection of i_1, i_2, i_3, and i_0, Fig. 2 was calculated by equating the right hand side of equation (10) to .0138.

Referring to equations (1), (2) and (3) showing the relations between the regulating resistances and the received currents, it will be noted that in the right hand member of (1) and (3), R_1 and R_3, respectively, appear only as positive terms in the denominator. This shows that the received current will inevitably be reduced for every increase in the resistance, provided r_1 and r_3 are continuously increasing functions of R_1 and R_3 and from equations (7) and (9) it will be seen that both r_1 and r_3 increase continuously for every increase in R_1 and R_3, respectively. In equation (2), however, R_2 appears in both the numerator and the denominator and in the latter it appears in both the first and second powers. It is, therefore, not so easy to determine from an inspection of the equation just how the received current will be affected by increasing the resistance. It will be seen that this difference in the received current equations offers a guide in the selection of the location for the resistances which will result in the greatest received current.

From a closer inspection of equation (2), it is seen that when $R_2=0$ the received current will be 0 and, as the denominator of the right hand member contains the second power of R_2, the received current will approach 0 if R_2 be increased indefinitely. Also, it is clear that there will be current in the receiving relay for all finite values of R_2. Thus, if R_2 be indefinitely increased from 0, the received current will rise from 0 to a maximum value and then descend again toward 0.

This suggests solving for the value of R_2 corresponding to the point where i_2 is a maximum by differentiating equation (2) with respect to R_2 and equating to 0. The nature of the equation shows also that i_2 will have but one maximum. If the value of R_2 corresponding to maximum i_2 proves to be greater than 500 ohms, it will open up the possibility of increasing the received current by adding the regulating resistances at points (2), Fig. 1.

In calculating the line and received currents for different values of R_2 it is, of course, permissible to calculate separately corresponding values of r_2 and then substitute these values as constants in equation (2). Obviously this procedure cannot be followed in finding the derivative of i_2 with respect to R_2. The expression to be dealt with in this differentiation is that which results from the substitution of the right hand member of equation (8) for r_2 in equation (2). This substitution gives the following explicit and rigorous equation for the steady-state current in the receiving relays of a balanced symmetrical bridge duplex telegraph circuit:

$$i_2 = \frac{ER_2}{(\tfrac{1}{2}T+G)(2R_2+b)+R_2(R_2+b)+\sqrt{T(\tfrac{1}{4}T+G)(2R_2+b)^2+(2R_2+b)[R_2T(R_2+b)+bR_2(R_2+2G)]}} \qquad (10)$$

Equation (10) was found useful in calculating received currents as it combines (2) and (8) and may be used instead of equations (1) and (7) by changing G to R_1 and R_2 to a, but when it is differentiated and equated to 0 the resulting equation for R_2 corresponding to maximum received current is of an extremely impractical nature as it involves various powers of R_2 up to the sixth, together with an unusually large number of terms. In this investigation, it was not necessary to solve this equation for R_2 as it was found that for values of R_2 and T within the practical ranges of 500 to 1750 ohms for R_2 and 500 to 3000 ohms for T, r_2 is very nearly equal to $1/3\ R_2+2\sqrt{T}+200$. If this expression be substituted for r_2 in equation (2) and the result differentiated and equated to 0 it leads to the following equation which gives values of R_2 corresponding fairly close to the point of maximum received current:

$$R_2=\sqrt{\tfrac{3}{4}b(T+2T^{\frac{1}{2}}+G+200)} \qquad (11)$$

With a receiving relay of 400 ohms resistance and a battery tap resistance of 120 ohms, as shown in Fig. 1, equation (11) becomes

$$R_2=10\sqrt{3(T+2T^{\frac{1}{2}}+320)}$$

From this equation, it is found that the bridge arms, each consisting of a 500 ohm bridge coil only, as shown by Fig. 1, are too small for maximum received current if the line resistance is greater than approximately 555 ohms. With line circuits ranging in resistance from 1000 to 2500 ohms, the respective values of R_2 necessary for maximum received current strength, range from approximately 624 to 935 ohms. If, then, resistance be added in the proper amounts at the points designated (2), Fig. 1, the received current will be increased thereby and at the same time, the line current will be reduced. If the line circuit resistance is approximately 1650 ohms or more the amount of resistance needed at points (2) to make the received current maximum, will be sufficient to reduce the line current to .070 ampere or less. This is illustrated by the three upper curves in Fig. 2. The lower broken curve, designated i_0, represents the received current which will be obtained with no regulating resistance in the circuit at either point. It will be seen that this curve passes below curve i_2 at a point corresponding to a line resistance of 1650 ohms. With approximately that value of line resistance and no regulating resistance, the line current is approximately .086 ampere and the received current is .0171 ampere. If approximately 410 ohms be added at points (2) the line current will be reduced to .070 ampere and the received current will remain at .0168 ampere. Curve $i_{\text{max.}}$ shows the maximum received current which can be realized by adding correct amounts of resistance at points (2). The upper curve touches curve i_2 at a point corresponding to a line resistance of 1850 ohms. That is, with a line resistance of this value, the regulating resistance required to reduce the line current to .070 ampere is just sufficient to bring the received current up to the maximum. For lines of this resistance or greater, the line current can be reduced to .070 ampere or less and at the same time the received current is increased. It will be seen from Fig. 2, that as compared to locations (1) and (3) for the regulating resistance, the advantage of location (2) from a steady-state received current standpoint, becomes greater with lines of low resistance and amounts to 32.3% and 60.1% respectively, with a line of 500 ohms resistance. On the other hand, the increase in received current due to arrangement (2), as compared to the condition of no regulating resistance, becomes greater with lines of higher resistance, as shown by the divergence of the i_2 and $i_{\text{max.}}$ curves, Fig. 2.

With line resistances in the lower range, the amount of regulating resistance needed to make the received current maximum will not be enough to bring the line current down to the desired value of .070 ampere. For example, with a line of 500 ohms resistance, the 500 ohm bridge arms are already too large by approximately 14 ohms and 1470 ohms will be required at points (2) to bring the line current down to .070. The bridge arms will then be 1484 ohms greater than needed for maximum received current. The question then arises as to why arrangement (2) results in the subtsantial received current gains with lines of low resistance, as shown by curves i_1, i_2 and i_3, Fig. 2. This part of the problem can best be solved by plotting equation (10). Fig. 3 shows this equation plotted for a 1200 ohm line. It will be seen that, from the

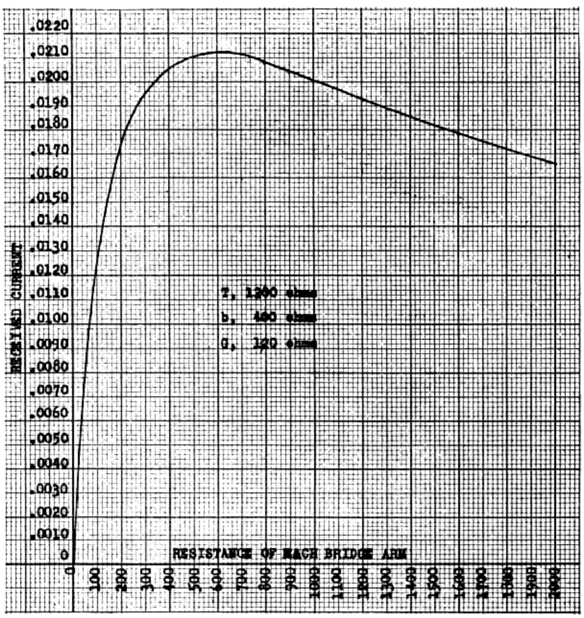

Fig. 3

standpoint of received current strength, it is better to have the bridge arm too great than too small, as the received current rises rapidly to a maximum and then descends slowly. On the other hand, if resistances be added at points (1) or (3), the operating point on the received current curve will in all cases be moved further away from the maximum, and this movement away from the maximum will take place on the side of the maximum which has the greatest effect in reducing the received current, as will be shown.

The resistance at points (1) or (3) moves the operating point on the received current curve away from the maximum due to the fact that the value of the bridge arm resistance corresponding to maximum received current is a function of both the resistance between the duplex sets and the resistance in the battery branch of the circuit, as shown by equation (11); G in this equation corresponds to R_1 in equation (1) and T represents all the resistance between the duplex sets, this being augmented by the addition of resistance at points (3). Therefore, any increase in the resistance in the battery branch of the circuit or in the resistance between the sets, such as will result

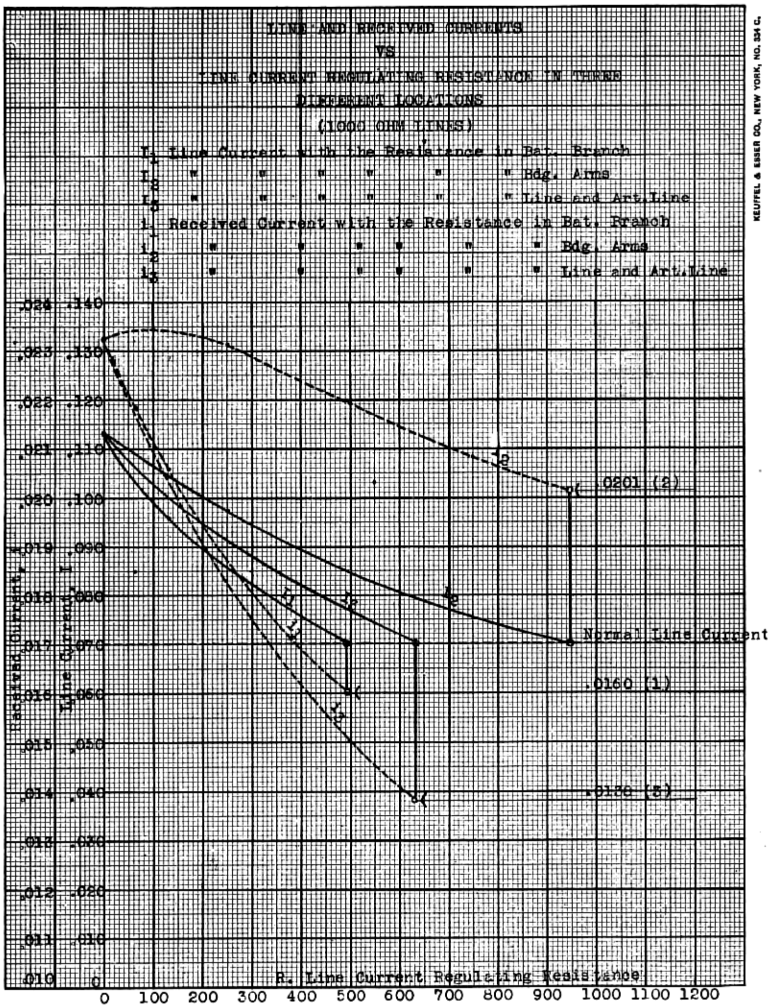

Fig. 4—Line and Received Currents vs Line Current Regulating Resistance in Three Different Locations—(1000 Ohm Line)

from adding resistance at points (1) or (3) respectively, will move the maximum point on the curve further to the right. This can best be illustrated by the following example:

With a line of 1500 ohms resistance, the resistance required in each bridge arm for maximum received current is about 750 ohms, so that the normal 500 ohm bridge arms as shown by Fig. 1 are short of the maximum by 250 ohms. If the line current be reduced to the desired value of .070 ampere by adding resistance at points (3) about 250 ohms will be required at each station. This will make the resistance between the duplex sets, corresponding to T in equation (11), $1500+500=2000$, for which the value of the bridge arms for maximum received current is about 855 ohms. The operating point on the curve is, therefore, 355 ohms on the left hand side of the maximum, as compared with 250 ohms before the resistances were added. The change in the maximum due to adding resistance at points (1) takes place in the same general way though not in exactly the same degree.

Fig. 4 shows how the line and received current are affected by the resistances in each location with a 1,000 ohm line. From these curves it will be noted that location (2) for the resistances results in a gain of about 25.6 per cent. in received current strength as against location (1) and as compared to location (3) the gain in received current amounts to about 45.6 per cent.

As the ratio of the bridge arms is not changed by adding the line current regulating resistance in equal amounts at points (2) that arrangement should introduce no difficulties in maintaining a balance between the line and artificial line. Furthermore, arrangement (2) should not increase disturbances due to small extraneous currents in the line.