Electrode Effects in the Measurement of Power Factor and Dielectric Constant of Sheet Insulating Materials

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SYNOPSIS: It is shown that, aside from the guard ring type of electrode which can only be used with certain special types of measuring circuit, the most accurate results can probably be obtained by the use of equal foil electrodes and making proper allowance for the edge effects. From the standpoint of convenience, mercury electrodes and foil electrodes of unequal size have certain advantages. It is believed that the results obtained with these two types are also sufficiently accurate for most purposes when the corresponding corrections are applied.

INTRODUCTION

WHEN it is desired to determine the dielectric constant and power factor of an insulating material, the first problem that presents itself is that of finding a suitable means of applying a potential to the material in question. In order to accomplish this, a sample of the material must, in general, be placed between two conductors or electrodes to which the desired potential is applied. The size, shape and manner of application of these electrodes affect directly the quantities to be measured from which the dielectric constant and power factor are derived. Therefore, unless proper allowance can be made for them, these features of the electrodes will affect the values obtained for the properties of the insulation.

It is the purpose of this paper to discuss only the part played by the electrodes in the measurement of power factor and dielectric constant and not to discuss complete methods for measuring these quantities. Experimental data will be presented to show the magnitude of the various effects discussed.

If any form of test is to be of general use for determining the properties of any material, there are certain fundamental requirements which must be fulfilled. First, the method should lead to exact reproducibility of results. That is, a test on a given sample of material should lead to the same result regardless of when, where or by whom the test is made. Second, the result obtained should be the correct result, that is, the absolute accuracy should be high. Third, the method should be as convenient as possible to use. If it is not, the method loses its practical value to a large extent since the tendency will be for most people to use a more convenient method even at the expense of accuracy and reproducibility.

SOURCES OF ERROR

There are certain sources of error inherent in all electrodes which affect both the reproducibility and the accuracy of the results obtained. First, we have the question of contact between the electrode and the sample. If the electrode does not make perfect contact over its entire area with the sample, the result obtained is not a true value for the material of the sample, but is a resultant, depending upon the amount and nature of the material filling the gap. Air spaces between the electrode and the sample have a marked effect on the apparent dielectric constant. An air-gap .001 in. thick in series with a sample having a dielectric constant of 5 will have the same effect on the capacitance as increasing the thickness of the sample by .005 in. If the actual thickness of the sample is .05 in. this results in an error of nearly 10% in the value of dielectric constant. The power factor will also be reduced and the loss factor or product of power factor and dielectric constant will be reduced by the factor $\left(\frac{50}{55}\right)^2$, or about 17%. Thus it is evident that the elimination of all gaps between the electrode and the sample is one of the first requirements both for reproducibility and accuracy.

A second effect inherent in all electrodes which is a source of error unless properly allowed for, is the so-called fringing of the electrostatic flux, that is, the lines of force tend to spread out and include an area of the sample greater than that of the electrode. This is illustrated in Fig. 1. So far as the flux which is confined entirely to the material of the sample is concerned, this produces an error in the dielectric constant, which involves a determination of the effective area of the sample, but not in the power factor which is independent of the area. However, there are some lines of force

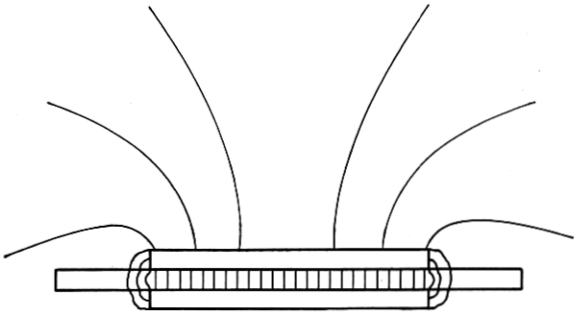

Fig. 1

which terminate on the vertical surfaces of the electrode and whose paths are partly through the sample and partly through air. These introduce a slight error into the power factor also. They also make the capacitance depend to a slight extent on the thickness of the electrodes. These edge effects vary with the thickness of the sample and also with the ratio of the perimeter to the area of the electrode and hence with its size and shape. They are also increased materially when one electrode is larger than the other.

The third inherent source of error is the capacitance from the ungrounded electrode[1] to earth due to lines of force which pass out in all directions other than through the sample. This also is illustrated in Fig. 1. This increases the measured capacitance by an amount depending somewhat on the nature and position of surrounding objects. If this capacitance is due to flux passing entirely through air it makes no difference in the dielectric loss,[2] but if the path of the flux includes other material such as the wood, brick, plaster, etc., in the walls and floor of the room as is often the case, it may add an appreciable loss.

We will now consider the above errors and also the question of convenience as applied to certain specific types of electrodes. The following types will be considered.

1. Plain metal electrodes.
2. Mercury electrodes.
    a—Confining ring of metal.
    b—Confining ring of insulating material.
3. Foil electrodes.
    a—Both same size as sample.
    b—Both same size, but smaller than the sample
    c—One materially larger than the other.
4. Conducting paint electrodes.
5. Fixed gap electrodes.

PLAIN METAL ELECTRODES

One of the simplest forms of electrode would be two similar blocks of metal between which the sample would be placed. If the surfaces of both the electrode and sample were true planes, this would be fairly satisfactory. However, as the surfaces of samples of insulating material usually available for test are not true planes, the air-gap

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[1] Assuming one electrode to be grounded as is usually the case.

[2] The apparent power factor is reduced by the increase in capacitance but the loss factor is not affected since the dielectric constant is increased to the same extent that the power factor is reduced.

error is usually prohibitive and makes this type of electrode practically useless.

Sometimes electrodes of this type are amalgamated and flooded with mercury and then pressed onto the sample, the excess mercury being brushed away.

This is an improvement but still leaves considerable uncertainty as to the degree of contact obtained.

Mercury Electrodes

Primarily on account of the ease with which a liquid will conform to the contour of an irregular surface, mercury has frequently been used as an electrode material. The usual procedure is to float the sample in a tray of mercury which serves as the lower electrode. A confining ring of some form is then placed on top of the sample into which a pool of mercury is poured which serves as the upper electrode.

When transparent samples are floated in this way it is observed that if a sample is simply laid flat upon the surface of the mercury, considerable air is trapped between the sample and the mercury. However, if the sample is lowered obliquely on to the surface of the mercury the air can usually be eliminated. When the mercury has been poured on top of the sample it is impossible to see whether the air has been eliminated from the upper surface of the sample or not. This is sometimes considered a serious objection to the use of this form of electrode especially when used with very thin samples. It may also be questioned whether or not the mercury completely fills the angle between the sample and the inside surface of the ring.

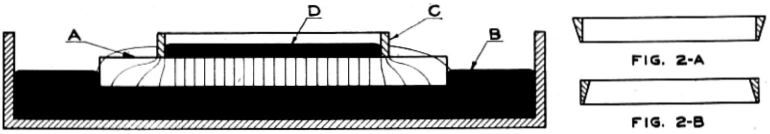

FIG. 2-A

FIG. 2-B

Fig. 2

Another point to be considered in the use of this type of electrode is whether the confining ring should be of conducting or of insulating material. The ideal condition, of course, would be to have the confining ring eliminated or, what is the same thing, to have it infinitely thin. Practically, however, something is required to confine the mercury to a definite area. Consider for a moment the arrangement shown in Fig. 2. *A* is the sample under test, *B* is the mercury constituting the lower electrode, *C* is the confining ring and *D* the mercury of the upper electrode. Consider first the ideal case where the ring is of finite height but infinitely thin. This corresponds to the condition illustrated in Fig. 1 except that the fringing is increased due to the size of the lower electrode. Suppose now that the ring is made of insulating material and of appreciable thickness. Some of the lines of force then pass through the material of the ring and the results become dependent on the nature and amount of this material. On the other hand, suppose the ring is made of metal and of appreciable thickness. This, of course, increases the effective area of the electrode but this is easily remedied by making the outside diameter of the ring equal to the size of electrode desired. However, that part of the surface which is actually covered by the ring is subject to the same error as in the case of plain metal electrodes, namely, that if the surface of the sample is not a true plane there will be a gap between the ring and the sample. One remedy for this is to make the ring thin enough so that the area affected is negligible compared with the total. This may not seem to be consistent with suitable rigidity. However, if a ring of at least 4½ inches diameter is used it can be as much as 1/16 inch thick and therefore quite rigid without serious error. In this case the area of the ring is about 5.6% of the total area and assuming a 10% error due to the air-gap as in the case of the plain electrodes the result would be a net error of .56% in the dielectric constant and a correspondingly slight error in the power factor.

Another means of reducing this error without sacrificing the rigidity of the ring is to bevel the edge of the ring either on the outside or inside as shown in Figs. 2-A and 2-B, respectively. In this way the area covered by the ring can be reduced to less than 1% of the total area and the air-gap error to less than 0.1%. However, if the ring is beveled on the outside the outer surface of the ring is no longer perpendicular to the surface of the sample and a slight increase in the stray field from this surface is produced. On the other hand, when the ring is beveled on the inside, the angle between the inner surface of the ring and the sample is less than a right angle and any tendency of the mercury not to fill this angle will be increased. However, both of these factors are probably but little affected by the slight bevel which is sufficient to reduce the thickness of the edge to a small value.

Another form of mercury electrode designed especially to eliminate all errors due to air bubbles has been used by Dye and Hartshorn for measuring the dielectric constant of mica in very thin sheets.[3]

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[3] Proceedings of the Physical Society of London, December 15, 1924.

In this arrangement the sample is clamped in a vertical position between two ebonite plates. These plates are recessed in such a way that a closed cell is formed on each side of the sample. An air vent is provided at the top of the cell and the mercury is introduced through a capillary U tube connected to the bottom of each cell. Thus the mercury is caused to rise slowly along the surface of the sample displacing the air completely. Also a slight head of mercury can be maintained in order to force it into all angles. However, this electrode is subject to the errors of the confining ring of insulating material in an exaggerated form on account of the mercury electrodes being entirely enclosed by the ebonite clamping plates. This would unquestionably lead to appreciable errors in the power factor especially in the case of thick samples of low loss material.

Foil Electrodes

Another form of electrode which has been widely used consists simply of a sheet of tinfoil applied to either side of the sample usually with a thin film of wax or petrolatum to serve as an adhesive. This has the advantage that the thickness of the electrode can be made negligible thereby practically eliminating the error due to the field from the vertical surface of the electrode passing partly through air and partly through the sample. The capacitance from the upper surface of the electrode to ground however is not eliminated.

The question of the size of the electrodes also arises. If both the electrodes are extended entirely to the edge of the sample the edge correction for the capacitance is greatly reduced since the fringing all takes place in air having a dielectric constant of 1 instead of the higher dielectric constant of the sample. On the other hand the fringing through the air does produce a small effect on the power factor which does not exist when the fringing is all through the sample. However, the biggest objection to this arrangement from the practical standpoint probably is that the samples very frequently are not uniform in thickness near the edge and this makes it difficult to determine the effective thickness of the sample.

Another possibility is to make both electrodes the same size but smaller than the sample. This should result in a comparatively small edge correction but requires careful manipulation to insure the electrodes being exactly opposite each other. The simplest arrangement from a convenience standpoint is to have one large and one smaller electrode. This however, results in a further increase in the edge correction.

CONDUCTING PAINT ELECTRODES

Another form of electrode which might be considered as a variation of the foil electrode consists of a coating of conducting paint on either side of the sample. In general the conditions are the same as with foil electrodes. A possible advantage might be more intimate contact with the sample and a possible disadvantage is that the film may have sufficient resistance to materially affect the power factor measurement. Many metallic paints are almost entirely non-conducting and are therefore entirely unsuitable for this purpose. If suitable conductivity is obtained, the discussion of foil electrodes given above may be applied to this type also.

FIXED GAP ELECTRODES

Another type of electrode occasionally referred to [4] consists essentially of a parallel plate air condenser the capacity of which is measured first alone, and then with the sample of insulating material inserted in the air-gap but not necessarily filling the gap completely. From these measurements and the known dimensions of the air condenser and sample the dielectric constant and power factor of the sample can be computed. This arrangement is capable of high accuracy if the dimensions of the sample and the thickness of the gap are known with sufficient accuracy. The computations are not as simple as for the other electrodes referred to and a slight error in determining the thickness of the sample or gap results in a much larger error in the final results. Therefore, this method does not seem to offer any marked advantages over the simpler forms.

EVALUATION OF ERRORS

Having now discussed in a general way the various types of errors and their probable effect in connection with various types of electrodes, an attempt will be made to determine by experiment the magnitude of the more important of these errors with respect to certain definite types of electrodes. From the discussion already given, it appears that some form of the mercury or foil electrode should be the most suitable for general use. Hence this investigation will be confined to these two general types.

The first question, namely, that of reproducibility, is one which cannot be determined by a single observer except as it applies to his own particular method of manipulation. Since the results obtained

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[4] A. Campbell, Proceedings of the Royal Society, Volume 78, Page 196 and Dye and Hartshorn Local Citation.

may depend considerably on the skill and patience exercised in the handling of the samples and electrodes, they may vary considerably with different observers. Hence, a comprehensive discussion of this point is beyond the scope of this paper. It has been the experience of the writer, however, that there is little choice between the two in this respect and that a decision between them rests primarily on other factors.

Since the magnitude of the ground capacitances and fringing effects both for foil and for mercury confined by a metal ring can be determined from a single series of tests, such an experiment will now be described.

Samples of insulating material 6 inches square were entirely coated on both sides with tinfoil using petrolatum as an adhesive. After the foils were in place, a 4½ inch circle was described on each foil from the center of the square and cut through so that the inner and outer portions were not in electrical contact, although the separation between them was very small. This left two 4½ inch disc electrodes L and M, on opposite sides of the sample surrounded by the annular pieces N and O respectively. When the inner and outer sections are connected, we have the condition of foil electrodes covering the entire surface of the sample. Then if N is used as a guard ring, it is possible to obtain measurements between L and M under uniform field conditions with all fringing effect eliminated. If N is removed and M and O are connected, we have the condition of one large and one smaller electrode. If a metal ring 4½ inches outside diameter is placed on the foil, we have a condition similar to that of a mercury electrode confined by a metal ring. If both N and O are removed, we have the case of two equal foil electrodes smaller than the sample. All of these variations can be obtained without any variation whatever in the conditions of contact between L and M and the sample and therefore are directly comparable.

The method of making these measurements[5] by means of a completely shielded capacitance and conductance bridge[6] is the same as that described by Campbell for the measurement of direct capacitance and will not be described here. The measurements were made at a frequency of 1,000 cycles as fewer difficulties are encountered than at radio frequencies and the general results are the same for any frequency. The capacitances were balanced to 0.1 mmf. or better, and the conductances to 0.0001 micro-mho.

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[5] G. A. Campbell, *Bell System Technical Journal*, July, 1922 and *Journal of the Optical Society of America and Review of Scientific Instruments*, August, 1922.

[6] G. A. Campbell, *Electrical World*, 43, 1904, 647–649.

The complete series of measurements made on the samples discussed above was as follows:

1. Grounded capacitance of L+N to M+O. This includes the stray capacitance of the upper electrode to ground and also any fringing effect in the air around the edges of the sample.
2. Direct capacitance of L+N to M+O. This eliminates the stray capacitance to ground but includes the above fringing, therefore, 2 minus 1 gives the stray capacitance of the 6″ square electrode to ground.
3. Direct capacitance of L+N to ground using M+O as a shield. This should check 2 minus 1 above.
4. Direct capacitance from L to M using N as a guard ring and eliminating all fringing and ground capacitance.
5. Direct capacitance from L to M+O with N removed. This includes the fringe effect from a small to a large electrode but eliminates the capacitance of L to ground. Hence, 5 minus 4 gives the fringe effect.
6. Grounded capacitance of L to M+O with N removed. This includes both fringe effect and capacitance of L to ground. Hence, 6 minus 5 gives the capacitance of L to ground.
7. Same as 6 with 4½″ metal ring on top of foil L. 7 minus 6 gives the added capacitance due to the ring.
8. Direct capacitance of L to ground using M+O as shield. This should check 6 minus 5 above.
9. Direct capacitance of L to M with N and O removed. This includes the fringe effect between equal electrodes but eliminates the capacitance of L to ground.
10. Grounded capacitance of L to M with N and O removed. This includes the fringe effect and ground capacitance for equal electrodes. 10 minus 9 equals capacitance of L to ground with equal electrodes.
11. Direct capacitance of L to ground using M as shield. This should check 10 minus 9 above.

The results of these measurements on a number of samples including several thicknesses of phenol fibre, hard rubber and glass are given in Table I. In all cases, the capacitance of the leads was measured separately and deducted. The differences between readings as indicated above, and the corresponding check readings are tabulated in Table II.

In using the results tabulated in Table II the directly measured values of stray capacitance to ground (items 2, 6, 12) should be considered much more reliable than those determined by differences (items 1, 5, 11). The degree of agreement between the two should be considered more as a check on the accuracy of the individual values from which the latter are derived than as a check on the former.

Hence, we may consider 3.3 mmf. (item 2, Table II) as the stray capacitance for the 6 inch square and about 2.3 mmf. (item 12) for the 4½ inch circle. This checks fairly well with the theoretical value of $\frac{R}{\pi}$ (CGS units). If the square is considered equivalent to a circle of equal area the value of $\frac{R}{\pi}$ is equivalent to about 3.03 mmf. For the 4½ inch circle the value of $\frac{R}{\pi}$ is 2.0 mmf. The measured values should be somewhat higher than the theoretical since the shielded bridge and other apparatus comprise a considerable mass of grounded metal at no great distance from the sample.

The fringe effect for the equal circular electrodes may be compared with values computed from Kirchhoff's formula

$$C=\frac{r}{4\pi}\left(\log_e\frac{16\pi(b+t)r}{b^2}+\frac{t}{b}\log_e\frac{b+t}{t}-3\right),$$

where C is the fringe effect and r and t are the radius and thickness respectively of the electrodes and b is their separation, all in CGS units. For very thin electrodes this reduces to

$$C=\frac{r}{4\pi}\left(\log_e\frac{16\pi r}{b}-3\right).$$

Values for this expression reduced to a percentage correction are listed as item 13 in Table II and are plotted in Fig. 3. It will be noted that the observed effect is at least a third less than that computed. It should be borne in mind, however, that Kirchhoff's formula applies primarily to electrodes in air. To completely simulate this condition with a solid dielectric would require that the electrodes be completely surrounded by a considerable thickness of the dielectric. If the difference noted above is due to lines of force which pass partly through air and partly through the sample, this difference should be greatest for a sample of high dielectric constant and diminish as the dielectric constant of the sample approaches that of the air. It is seen from Fig. 3 that this is the case, the curve for hard rubber having a dielectric constant of 3 being nearer to the computed curve than that for glass having a dielectric constant of 7.7. The anomalous shape of the curve for phenol fibre is apparently due to a non-uniformity in the samples which will be discussed later.

That the above results are materially affected by flux passing partly through air and partly through the sample was proved by an additional test. The 3/8″ phenol fibre sample with 4½″ discs on each side was first measured in air. Since the upper electrode could not be entirely covered and still obtain contact with it, the ¼″ phenol fibre plate was laid on top of it so as to cover just one-half of the sample and electrode. This caused an increase of 1.4 mmf. Using the ¼″ plate in the same way to cover one-half of the lower surface of the sample and electrode, the increase was .7 mmf. showing that

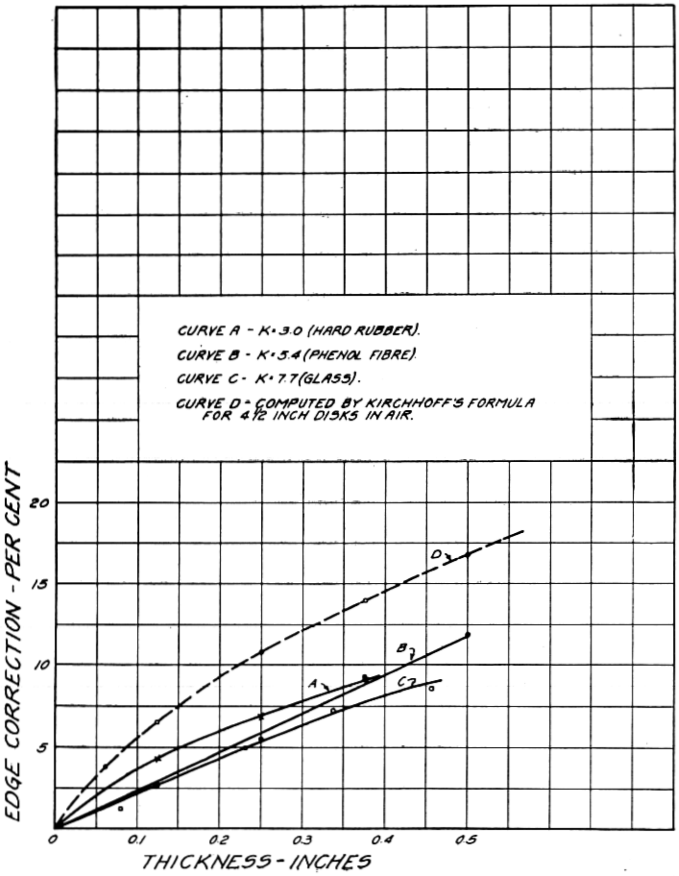

Fig. 3—Percentage edge correction for samples 6 inches square with foil electrodes both 4½ inches in diameter

when one electrode is grounded the effect is not symmetrical. Covering both sides of the sample should, therefore, cause an increase of 4.2 mmf. or 8.6% of the capacitance of the sample. Adding this to the 9.3% fringe effect already determined makes a total of 17.9% as compared with 13.9% by Kirchhoff's formula. At least part of this difference is also due to the non-uniformity referred to above which is very marked in this sample.

A similar test was made on the 1/8″ phenol fibre sample. However, this sample was somewhat warped so that there was an appreciable air-gap between the electrode and the cover plate. The total effect computed as above was 2.6%, which added to the 2.8% already determined makes a total of 5.4% or slightly under the value computed by Kirchhoff's formula. This, no doubt, is accounted for by the air-gap.

The agreement with Kirchhoff's formula is reasonably good, therefore, for disc electrodes completely surrounded by dielectric, but it is evident that the formula does not apply to the case of disc electrodes applied to sheet materials.

The fringe effect for the 4½″ upper circle and 6″ lower square electrodes is shown in Fig. 4. It is found to be about 2½ times as large as for the equal 4½″ electrodes. It also varies somewhat with the dielectric constant of the sample, being greater for the lower dielectric constant. The anomalous behavior of the phenol fibre samples is shown in this figure also.

From Item 8 of Table II, it is seen that when a shallow metal ring is placed on the upper disk to simulate the conditions of a mercury electrode, a further increase of from 2 to 4% of the true capacitance of the sample takes place. This likewise is greater the thicker the sample and the lower its dielectric constant. A similar test for the increase in capacitance due to vertical height of the metal ring was made for the more exaggerated case of a 4″ disk and a ring ¾″ high; the lower electrode being 6″ square. In this case the increase varies from 2½% for 1/8″ phenol fibre to 8% for 3/8″ hard rubber. This shows the importance of keeping the vertical dimension of the metal ring as small as possible.

INSULATING RING

In order to determine the corresponding effect when an insulating ring is used for confining mercury, a somewhat similar test was made. Several different rings were used as follows: ring No. 1 is ¾″ high cut from hard rubber tubing having a 1/8″ wall with the edges cut square. Ring No. 2 was the same as above except that the edge was beveled on the outside as in Fig. 2-A to a thickness of 1/64″. Ring No. 3 was cut from phenol fibre sheet ⅛″ thick and had a radial width of ⅜″. Since rubber tubing of the desired size was not available the rubber rings were somewhat smaller than 4½″ in diameter but in all cases the foil electrode with which they were used was cut to exactly the same size as the inside diameter of the ring and the results were reduced to the equivalent value for 4½″ diameter.

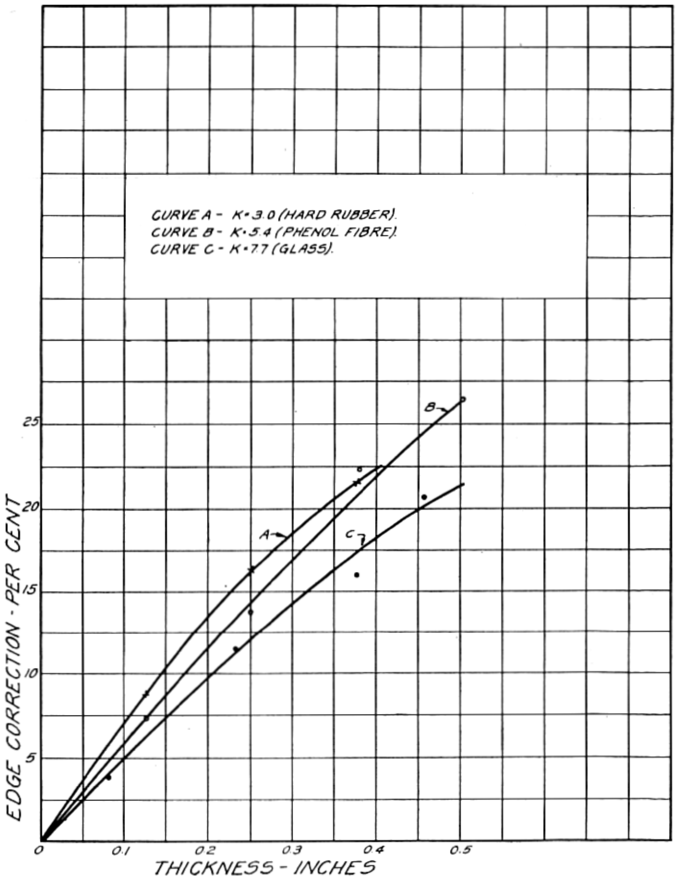

Fig. 4—Percentage edge correction for samples 6 inches square with foil electrodes upper, 4½ inches diameter, lower, 6 inches square

In this test in order to simulate the depth of the mercury without changing the electrode contacts, the lower part of the inner surface of the ring was coated with foil to a height equal to the assumed depth of the mercury. This was taken as ⅛″ in each case which is about the minimum depth of mercury that can be depended upon to cover the surface of the sample and fill the angle between the sample and the ring. This, of course, has slightly less effect than if the entire surface of the electrode were raised as is the case when mercury is poured into the ring, but the difference is probably negligible. The procedure was the same as previously described for the metal ring. The lower foil electrode covered the entire surface of the sample. The upper foil is the same diameter as the inside of the insulating ring. (In the case of the metal ring the foil was the same size as the outside diameter of the ring at the lower edge). The capacitance is measured first between the foils alone and then with the ring superimposed on the upper foil. The change should represent very closely the increased edge effect due to a depth of 1/8″ of mercury in the ring.

The results of this test are tabulated in Table III. It will be noticed that the change in capacitance due to the beveled rubber ring is about the same as for the metal ring, while the change due to the square edged ring is materially greater than for the metal ring. The flat phenol fibre ring produces a change in capacitance two or three times as great as the metal ring and the apparent power factor for the rubber sample is more than doubled. This, of course, is due to the dielectric loss in the ring itself and therefore the lower the power factor of the sample under test the greater is the proportional error. While the rubber rings had no appreciable effect on the power factor of the samples tested, it is believed that in the case of very low loss materials such as fused quartz the effect might still be appreciable. Since the insulating rings under the best conditions are no better than a metal ring and under poor conditions are very much inferior, it is believed that in general metal rings will be found more satisfactory for confining mercury electrodes.

Thus far we have considered the experimental data primarily with regard to the capacitance and dielectric constant. Table IV shows the power factors computed from the conductance readings corresponding to readings 1, 4, 6 and 10 in Table I. These values were computed from the capacitance and conductance values for the various types of electrodes and without correction for edge effects or ground capacitances. The values for hard rubber illustrate fairly well the variation for different electrodes which would be expected on the basis of the preliminary discussion. The value obtained with the 4½″ circle and guard ring should be the true value. The other values should be slightly lower on account of the additional capacitance without corresponding power less due to the flux which passes partly and entirely through air. In general these variations are small and almost beyond the limit of accuracy of the measurements. However, a careful analysis of the results (omitting the 3/8″ phenol fibre sample which will be discussed separately) seems to indicate that the power factor values obtained with the two 4½″ circles agree slightly better with those obtained with the guard ring electrode than do those obtained with any of the other electrodes.

In the case of the 3/8″ phenol fibre sample the variations with different electrodes are much greater than the probable inaccuracy of the measurements. Apparently, they can only be attributed to non-uniformity of the material in different parts of the sample. Therefore, a special test to determine this fact was made on this sample. By interchanging the connections to the guard ring and the 4½″ center electrode, it is possible to measure the capacitance and conductance of the outer part of the sample without including the center part. The sum of the values of the two parts checks well with the value for the entire 6″ square. These results show that while the inner part of the sample has a power factor of 1.97%, the power factor of the outer part is 3.17% or approximately 60% higher. The corresponding dielectric constants are 5.08 and 5.48, respectively. The reason for this difference probably is that since the material is of a laminated nature moisture penetrates more readily from the edges than from the sides of the sample and thus causes a progressive variation of the electrical characteristics from the edges to the center of the sample. It is obvious that when the two 4½″ electrodes are used without guard rings, some of the outer part of the sample is included due to the fringe effect and that when the 4½″ upper and 6″ lower electrodes are used still more of the outer part is included for the same reason, and the values obtained are increased accordingly.

As previously mentioned, it is probable that this non-uniformity is responsible for the different shape of the edge effect curve for phenol fibre as compared with those of hard rubber and glass and it is almost certainly the cause of the point representing this particular sample being exceptionally high. Since there are wide variations in the power factors of the different samples of both phenol fibre and glass it is possible that there are minor variations through the sample due to causes other than moisture and that these may account for some of the other apparent irregularities in the results.

Method of Applying Corrections

While percentage values are the most convenient for discussion of the relative importance of the various corrections involved in the use of a given type of electrode, the absolute values of these corrections in micro-microfarads are probably somewhat more convenient for actual use in making the necessary calculations.

Consider the case of mercury electrodes with the lower electrode grounded. On the basis of the foregoing discussion the total capacitance C which is measured may be considered as made up of four parts, namely, C_x the capacitance between the electrodes which would exist under uniform field conditions as when a guard ring is used. C_{e1} the edge effect which would exist if the upper electrode were a thin disk. C_{e2} the additional edge effect due to the height of the metal ring. C_g the capacitance to ground of the upper surface of the electrode. The dielectric constant $K=4.46\,\frac{C_x d}{A}$ where C_x is in micro-microfarads d and A are the thickness and area of the sample in inches and square inches, respectively. To compute K, C_x must therefore be obtained from the relation

$$C_x=C-(C_{e1}+C_{e2}+C_g).$$

Values for C_{e1}, C_{e2}, C_g are given in the tables for certain particular cases and may be determined in a similar manner for any other cases. For foil electrodes, conditions are similar except that C_{e2} is zero. C_{e1} which is the largest of the corrections is plotted in Fig. 5 for various values of K and several thicknesses of the sample as taken from the previous tables. It will be seen that in order to apply this correction the approximate value of the dielectric constant of the sample must be known in advance. This can always be obtained by making a preliminary computation neglecting the corrections entirely.

As an example of the above method suppose measurements have been made on a 1/8″ sample of material having a dielectric constant of about 4.5 using mercury electrodes with a shallow metal ring. From Curve A, Fig. 5 we get 9.2 mmf. for C_{e1}. C_{e2} is estimated by interpolation from item 7 of Table II at about 2.7 mmf. C_g is taken as the average for item 6 of Table II or 0.8 mmf. This makes a total of 12.7 mmf. to be subtracted from the measured capacitance in order to obtain C_x from which the dielectric constant is computed. If 4½″ foil electrodes were used C_{e1} would be taken from Curve B, Fig. 5 as 4.0 mmf. and C_g from the average of item 11, Table II as 2.4 mmf. making a total correction of 6.4 mmf.

The values given in Fig. 5 for C_{e1} are, of course, applicable only to a given size of electrode, namely 4½″ in diameter. If the edge effect capacitance is considered equivalent to that of an additional ring electrode surrounding the main electrode, this capacitance would be proportional to the mean radius of this ring, or to the radius of the upper electrode plus one-half of the width of the ring. If this equivalent ring is of constant width for various sizes of electrodes, the edge correction would be proportional to the radius of the electrode plus a constant. As Fig. 5 shows that the correction in micro-microfarads does not vary greatly with the thickness of the sample, the width of the above hypothetical ring must be approximately proportional to the thickness of the sample. Computations for several samples show that the width of this ring is of the order of twice the thickness of the sample. Hence it appears that as a first approximation the edge correction for various sizes of electrodes may be taken as pro-

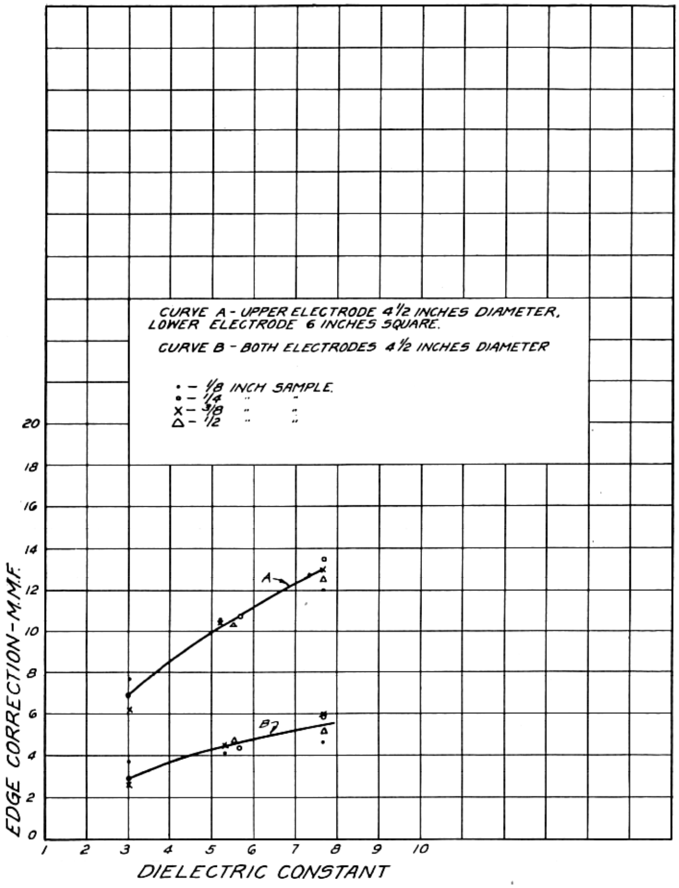

Fig. 5—Edge correction versus dielectric constant for samples 6 inches square with foil electrodes

portional to $r+t$, r being the radius of the electrode and t the thickness of the sample. Table V shows a series of measurements on one sample with electrodes of several different sizes. In this table the edge correction C has been taken as the difference between the capacitances measured with and without a guard ring. This includes the direct capacitance of the upper electrode to ground. It will be seen that while the value of $\frac{C}{r+t}$ is not exactly constant it varies much less than $\frac{C}{r}$. Hence if C_1 is the edge correction for an electrode of radius r_1 the edge correction for an electrode of radius r_2 under similar conditions would be approximately

$$C_2 = C_1 \frac{r_2+t}{r_1+t}.$$

This, of course, applies only to the case of one large and one smaller electrode. For the case of the equal electrodes t in the above expression probably should be multiplied by a constant of the order of 0.4.

Tables for the Article

By E. T. Hoch

TABLE 1

Capacitance Readings as Described under "Evaluation of Errors" Micromicrofarads

Reading No.	Phenol Fibre				Hard Rubber			Glass			
	½ in.	⅜ in.	¼ in.	⅛ in.	⅜ in.	¼ in.	⅛ in.	.455 in.	.338 in.	.232 in.	.080 in.
1	94.7	116.3	*	335.0	69.5	100.5	200.4	147.0	190.2	278.0	790.8
2	*	113.1	*	332.2	66.7	97.8	195.1	*	*	*	*
3	*	3.2	*	3.3	*	3.3	3.3	*	*	*	*
4	39.7	48.6	60.2	146.6	28.9	42.3	86.4	61.4	82.0	118.4	344.6
5	(50.1)	59.1	91.0	157.2	35.1	49.1	94.1	(74.0)	(95.0)	(131.9)	(356.6)
6	50.8	59.6	92.4	157.7	35.7	49.4	94.4	74.7	95.7	132.6	357.3
7	*	60.9	*	160.8	36.9	51.3	96.5	*	*	*	*
8	*	0.8	1.3	1.0	0.7	0.6	0.7	*	*	*	*
9	(44.7)	53.1	84.6	150.7	31.5	45.2	90.1	(66.6)	(88.0)	(124.3)	(349.2)
10	46.7	55.7	87.9	153.1	33.6	47.2	91.2	66.9	90.3	126.6	351.5
11	*	2.6	3.0	2.3	2.3	2.3	2.1	*	*	*	*

* Not measured.

NOTE: Readings 5 and 9 given in parentheses are derived from readings 6 and 10 respectively using average values for readings 8 and 11.

TABLE 2

Values Computed from Table 1 and Comparison with Measured Values

Value Computed (mmf. unless otherwise stated)	Phenol Fibre				Hard Rubber			Glass			
	½ in.	⅜ in.	¼ in.	⅛ in.	⅜ in.	¼ in.	⅛ in.	.455 in.	.338 in.	.232 in.	.080 in.
(1) Cap. of 6 in. square electrode to ground—Reading 1–2 (Table 1)................	*	3.2	*	2.8	2.8	2.7	5.3				
(2) Ditto by direct meas. Reading 3....................	*	3.2	*	3.3	*	3.3	3.3				
(3) Fringe effect 4½ in. circle to 6 in. square—Reading 5–4...	10.4	10.5	10.8	10.6	6.2	6.8	7.7	12.6	13.0	13.5	12.0
(4) Ditto in per cent..........	26.2	22.4	13.5	7.2	21.4	16.1	8.8	20.5	15.8	11.4	3.4
(5) Stray cap. to grd. from 4½ in. upper with 6 in. lower electrode (6–5)............	*	0.5	1.4	0.5	0.6	0.3	0.3				
(6) Ditto by direct meas. Reading 8....................	*	0.8	1.3	1.0	0.7	0.6	0.7				
(7) Additional capacitance due to metal ring Reading 7–6...	*	1.3	*	3.1	1.2	1.9	2.1				
(8) Ditto in per cent..........	*	2.7	*	2.1	4.1	2.6	2.4				
(9) Fringe effect 4½ in. circle to 4½ in. circle—Reading 9–4..	4.7	4.5	4.4	4.1	2.6	2.9	3.7	5.2	6.0	5.9	4.6
(10) Ditto in per cent..........	11.8	9.3	5.5	2.8	9.0	6.8	4.3	8.4	7.3	5.0	1.3
(11) Stray cap. to grd. from 4½ in. upper with 4½ in. lower electrode (10–9)...........	*	2.6	3.3	2.4	2.1	2.0	1.1				
(12) Ditto by direct meas. Reading 11....................	*	2.6	3.0	2.3	2.3	2.3	2.1				
(13) Fringe effect by Kirchhoff's formula for 4½ in. circles in air (per cent)....					13.9	10.7	6.6				

*Not determined.

TABLE 3

Effect of Various Types of Rings for Confining Mercury Electrode

	Increase in Capacitance Per Cent		Power Factor Per Cent	
	⅛ in. Glass	⅜ in. Hard Rubber	⅛ in. Glass	⅜ in. Hard Rubber
Without ring	0.0	0.0	2.14	.43
With metal ring 3/16 in. high, outside bevel	2.1	4.1	2.11	.45
With square edged hard rubber ring	2.7	7.0	2.11	.45
With bevel edged hard rubber ring	1.8	4.9	2.13	.44
With flat phenol fibre ring	4.7	12.6	2.45	1.08

TABLE 4

Power Factors as Measured with Different Electrodes

Electrodes	Phenol Fibre				Hard Rubber			Glass			
	½ in.	⅜ in.	¼ in.	⅛ in.	⅜ in.	¼ in.	⅛ in.	.455 in.	.338 in.	.232 in.	.080 in.
6 in. squares	2.40	2.24	1.95	2.18	0.43	0.41	0.45	1.67	2.46	2.18	2.94
4½ in. circle with guard ring	2.56	1.96	2.05	2.19	0.44	0.47	0.44	1.81	2.43	2.21	2.86
4½ in. upper, 6 in. lower	2.50	2.16	2.08	2.20	0.40	0.39	0.46	2.00	2.43	2.14	2.85
Ditto with metal ring	*	2.12	*	2.18	0.39	0.39	0.46	*	*	*	*
4½ in. upper and lower	2.58	2.14	2.11	2.20	0.44	0.45	0.44	1.80	2.50	2.20	2.90

* Not measured.

TABLE 5

Measurements on ¼ In. Phenol Fibre with Electrodes of Various Sizes

Radius of Upper Electrode In.	Cap. With Guard Ring Mmf.	Cap. Without Guard Ring Mmf.	Edge Correction C Mmf.	C/r	C/r+t	Diel. Const. Computed from 2nd Col.
.51	4.25	7.8	3.55	7.0	4.7	5.7
1.00	15.9	21.9	6.0	6.0	4.8	5.58
1.50	35.4	44.1	8.7	5.8	5.0	5.50
2.00	62.6	73.2	10.4	5.2	4.6	5.50
2.26	82.1	92.9	10.8	4.8	4.3	5.60

NOTE: Sample was 6 in. square with lower electrode covering entire lower surface.