Directive Diagrams of Antenna Arrays

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SYNOPSIS: Two systematic collections of directive amplitude diagrams are shown for arrays of 2 and of 16 identical antennae spaced at equal distances along a straight line with equal phase differences introduced between the currents in adjacent antennae, assuming that each antenna radiates equally in all directions in the plane of the diagram. Three diagrams show the effect of increasing without limit the number of antennae in a given interval. Two models show the effect of distributing the antennae over an area.

INTRODUCTION

ONE of the means proposed for obtaining directive radio effects, both in sending and in receiving, is the antenna array, consisting of a system of two or more antennae situated at specified fractions of a wave-length apart and with relations imposed upon the amplitudes and phases of the currents in the several antennae. For example, consider a sending array consisting of two vertical antennae so arranged that the currents in the antennae are equal in magnitude but a half period apart in phase, the individual antennae being identical and radiating equally in all directions in the horizontal plane. If the two antennae were placed at the same point there would be zero transmission in all directions, since the effects of the two antennae would neutralize each other. If, however, the two antennae are separated by a small fraction of a wave-length, while there will still be zero transmission in the direction perpendicular to the axis of the array, there will be transmission in all other directions. If this separation is increased to exactly one-half of a wave-length, the radiation from the array along the axis will become a maximum.

This particular type of antenna array was proposed by Brown [1] in 1899. A few years later, Stone [2] proposed a similar array with the two currents exactly in phase. This array gives maximum transmission perpendicular to the axis, zero transmission along the axis. About the same time, Blondel [3] made several suggestions, among them, two antennae placed a quarter of a wave-length apart and with a phase difference of a quarter of a period. With this arrangement a unilateral effect is obtained, there being maximum transmission in one direction along the axis, zero transmission in the opposite direction.

[1] S. G. Brown, British Patent No. 14,449 (1899).

[2] J. S. Stone, United States Patent No. 716,134 (1901).

[3] A. Blondel, Belgian Patent No. 163,516 (1902), British Patent No. 11,427 (1903).

These early suggestions have been followed by a number of more complicated arrangements proposed by Braun,[4] Bellini,[5] and others.[6]

Most books on radio communication contain one or more directive diagrams showing the variation of either the amplitude or the energy for systems of two, three, or four antennae which are separated by given fractions of the wave-length and with currents which have assigned amplitude and phase relations. Bellini,[7] Koerts,[8] and Zenneck [9] each give about a dozen such diagrams. Recently, Green [10] and Friis [11] have published directive diagrams for a pair of loops; the latter gives a curve obtained experimentally which agrees very well with the theoretical curve. Of the published diagrams, one of the most extended and systematic sets appears to have been that of Walter.[12] He showed a total of 21 diagrams for arrays of two antennae, with the three separations of 1/10, 1/4, and 1/2 wave-length and the seven phase differences of 0, 1/12, 1/6, 1/4, 1/3, 5/12, and 1/2 period.

In the present paper an effort has been made to present a more systematic and comprehensive collection of directive diagrams for arrays consisting of 2 and of 16 antennae, respectively, spaced at equal distances along a straight line or axis, with currents of equal amplitude in all the antennae, and with equal phase differences introduced between the currents in adjacent antennae. These diagrams are polar diagrams showing the relative amplitude of the field of the radiation at a great distance in a plane through the array, assuming that each antenna radiates equally in all directions in this plane. The unit circle shown in each diagram represents the amplitude of the radiation if all the antennae were made coincident in space and in phase.

These directive diagrams may be used to obtain the directive diagram in any plane through an array made up of antennae which

[4] F. Braun, Electrician, 57, pages 222–224, 244–248, 1906.

[5] E. Bellini, Electrician, 74, pages 352–354, 1914.

[6] For extensive bibliographies see L. H. Walter, Directive Wireless Telegraphy, London, 1921, pages 119–121; H. H. Beverage, C. W. Rice, and E. W. Kellogg, Journal of the A. I. E. E., 42, pages 736–738, 1923; A. Koerts, Atmosphärische Störungen in der drahtlosen Nachrichtenübermittlung, Berlin, 1924, pages 149, 150; J. Zenneck and H. Rukop, Drahtlose Telegraphie, fifth edition, Stuttgart, 1925, Chapter XIII.

[7] E. Bellini, Jahrbuch der drahtlosen Telegraphie und Telephonie, 2, pages 381–396, 1909.

[8] A. Koerts, *loc. cit.*, pages 101, 102, 104, 105, 110, 111, 130, 131, 133.

[9] J. Zenneck and H. Rukop, *loc. cit.*, pages 412–415, 419, 421, 423, 428, 432.

[10] E. Green, Experimental Wireless, 2, pages 828–837, 1925.

[11] H. T. Friis, Proceedings of the I. R. E., 13, pages 685–707, 1925.

[12] L. H. Walter, Electrician, 64, pages 790–792, 1910.

do not radiate equally in all directions in this plane, but which satisfy the other conditions named above; the total directive effect is the product of the individual effect multiplied by the group effect. Thus, since the amplitude of the radiation in the horizontal plane from a single vertical loop varies as the cosine of the angle between the direction of transmission and the plane of the loop, the directive diagram in the horizontal plane of an array of loops, all loops being oriented the same, is the corresponding directive diagram of an antenna array as presented in this paper, with the radius vector multiplied by a cosine factor.

The present discussion has been stated in terms of transmission, but the directive diagrams apply equally well to the case of reception by an array from a distant source.

Each of the two sets of diagrams is presented in a rectangular arrangement so as to exhibit the effect of changes both in the separation between adjacent antennae, specified in wave-lengths, and in the phase difference introduced between the currents in adjacent antennae, specified in periods. These drawings were originally made at the suggestion of Dr. G. A. Campbell to illustrate the application of antenna arrays as a means for reducing the ratio of static to signal.

The antenna array is analogous to the optical diffraction grating. By eliminating the transmission wires connecting together the individual antennae of the array and utilizing instead re-radiation from suitably designed antennae, the radio system would correspond more closely with this optical analogue. With this arrangement, however, the phase difference cannot exceed a value in periods numerically equal to the separation in wave-lengths, a restriction to which the ordinary ruled grating is also subject. The retardation grating proposed by Rayleigh [13] and the echelon spectroscope of Michelson [14] offer more complete analogies to the antenna array in that there is no theoretical limitation on the separation and phase difference.

Two Antennae, Fig. 1

A total of 90 directive diagrams for an array of two antennae is shown by Fig. 1. The separation between antennae varies from 0 to 2 wave-lengths, in steps of 1/8 wave-length; the phase difference between antennae varies from 0 to 1/2 period, in steps of 1/8 period; an additional set of diagrams is included with a separation of 4 wave-lengths. These curves were carefully drawn with a unit circle ten

[13] Rayleigh, Collected Papers, 3, pages 106–116.
[14] A. A. Michelson, Astrophysical Journal, 8, page 37, 1898.

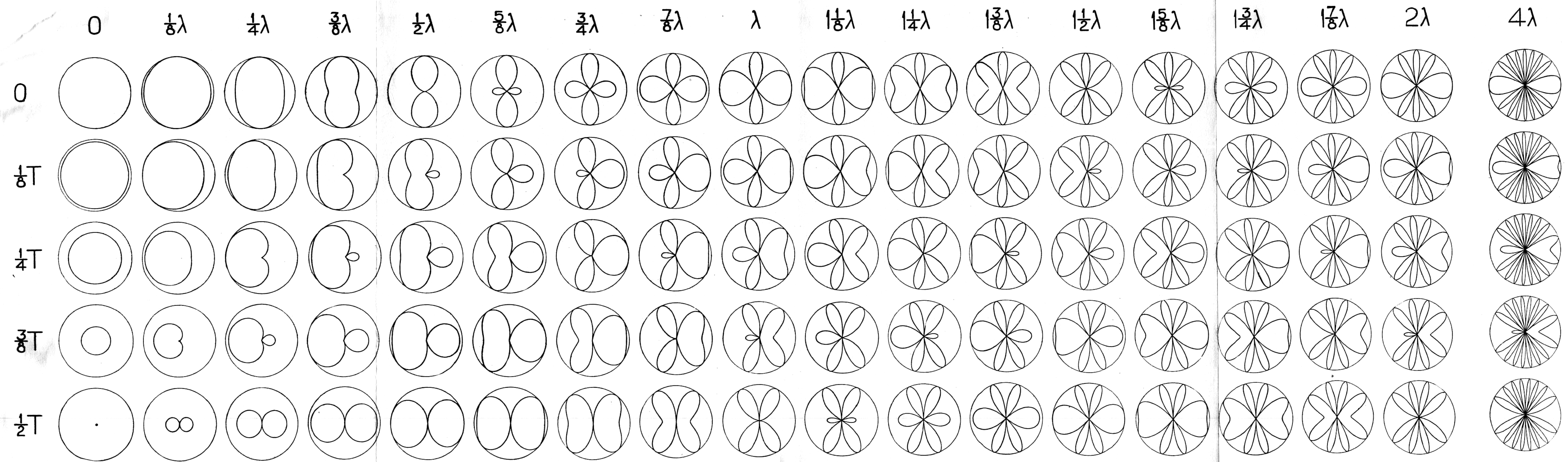

Fig. 1—Directive amplitude diagrams for an array of two antennae; separation in wave-lengths (λ) along the top, phase difference in periods (T) at the left

inches in diameter, so that, on the reduced scale of reproduction, the accuracy should leave nothing to be desired. For a sending array the specified phase difference is the lag of the current in the right-hand antenna behind the current in the left-hand antenna; for a receiving array it is the lag introduced in the current from the right-hand antenna. In each case the line of the array is parallel to the horizontal axis of the diagram.

Reversing the sign of the phase difference reflects the directive diagram about its vertical axis, that is, the right and left sides are interchanged. With increasing phase difference the diagrams repeat cyclically, those from $\frac{1}{2}T$ to $\frac{3}{2}T$ being the same as those from $-\frac{1}{2}T$ to $\frac{1}{2}T$, and so on.

In the first column, that is, for zero separation, all diagrams are circles, since the two antennae are coincident, and thus the array radiates uniformly in all directions. For zero phase difference the directive diagram is the unit circle, since the radiations from the two antennae reenforce each other without interference. As the phase difference is increased, this circle grows smaller, due to increasing interference, until, for a phase difference of a half period, the two radiations completely neutralize each other, the directive diagram shrinking down to a null circle.

The diagrams in the first row, that is, for zero phase difference, are symmetrical about the vertical axis in addition to being symmetrical about the horizontal axis. In every case the amplitude is unity along the vertical axis, that is, in a direction perpendicular to the line of the array. As the separation is increased from zero, the amplitude along the horizontal axis diminishes, until it reaches zero for a separation of $\frac{1}{2}\lambda$, it then increases to unity at λ, it diminishes to zero at $1\frac{1}{2}\lambda$, it reaches unity at 2λ, and so on.

The diagrams in the bottom row, that is, for a phase difference of a half period, are also symmetrical about the vertical axis. In every case the amplitude is zero along the vertical axis. For small separations, the directive diagram is approximately a pair of tangent circles, which increase in size as the separation is increased. When the separation reaches $\frac{1}{2}\lambda$, the amplitude along the horizontal axis reaches unity, it then falls off to zero as the separation is increased to λ, it rises to unity at $1\frac{1}{2}\lambda$, it falls to zero at 2λ, and so on.

The diagram for $(\frac{1}{4}\lambda, \frac{1}{4}T)$ is particularly interesting in that there is a single direction of unit amplitude with zero amplitude in the opposite direction. This array was proposed by Blondel, as stated above, and it is the basis of the Alexanderson barrage.[15] The diagrams

[15] E. F. W. Alexanderson, Proceedings of the I. R. E., 7, pages 363–378, 1919.

situated on the line from (0λ, $\frac{1}{2}T$) to ($\frac{1}{4}\lambda$, $\frac{1}{4}T$) have a similar property: each has a relative maximum in a single direction, with zero in the opposite direction. The diagrams on the line from (0λ, $0T$) to ($\frac{1}{2}\lambda$, $\frac{1}{2}T$) have a maximum along the horizontal axis to the left, with an amplitude to the right decreasing from unity at $0T$ to zero at $\frac{1}{4}T$, and then increasing to unity at $\frac{1}{2}T$.

The number of lobes tends to increase as the separation is increased, as shown by (a) of Fig. 2. A zigzag starting at (0λ, $\frac{1}{2}T$)

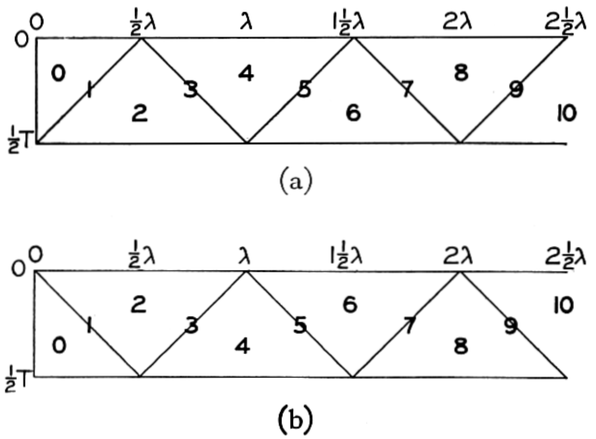

Fig. 2—(a) Number of null directions (which is also the number of lobes) for two antennae. (b) Number of unit directions (directions of absolute maximum amplitude) for any number of antennae, in terms of separation and phase difference between adjacent antennae

and made up of lines sloping up and down at an angle of 45° divides the rectangular arrangement of diagrams into sections with 0, 2, 4, 6, . . . null directions in each diagram, respectively. On these lines the number of null directions is 1, 3, 5, 7, . . . , respectively, with the intermediate numbers at the junction points. The number of lobes is, of course, equal to the number of null directions.

Part (b) of Fig. 2 is a diagram specifying the number of unit directions (directions of absolute maximum amplitude) in terms of the separation and the phase difference between adjacent antennae, and it holds regardless of the number of antennae, that is, the number and position of the main lobes are not changed by increasing the number of antennae, provided the same separation and phase difference are preserved between adjacent antennae.

It is interesting to observe the variation in the diagrams along any line in this rectangular arrangement of Fig. 1, whether horizontal, vertical, or diagonal. A lobe starts as a small bud, it grows in size until it reaches the unit circle, it then becomes dented; the two prongs of the lobe separate more and more until a division into two lobes takes place; then these lobes separate as a new lobe starts to grow

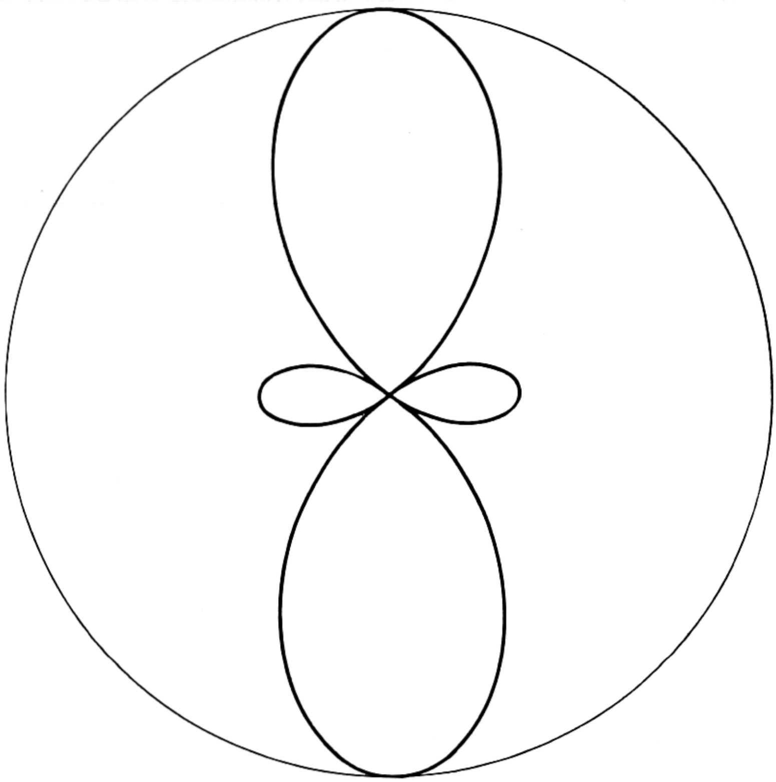

Fig. 3—Directive amplitude diagram for two antennae (0.6098λ, 0*T*) having the minimum area (0.2986) relative to the unit circle

between them. The additional column of diagrams for a separation of 4λ still further illustrates the way in which the lobes multiply and narrow as the separation between the two antennae in increased.

The area of the polar directive diagram of an array relative to the area of the unit circle is a measure of the reduction in the energy ratio of random static to signal for that array, assuming that the signal comes from a direction in which the radius vector of the diagram is unity while the static is uniformly distributed. All the diagrams for a phase difference of 1/4 period have an area of 1/2.

The area of the other diagrams oscillates about 1/2 and approaches it as a limit upon increasing the separation and keeping the phase difference constant. The minimum area (for a diagram in which the radius vector reaches its maximum of unity [16]) is 0.2986, obtained

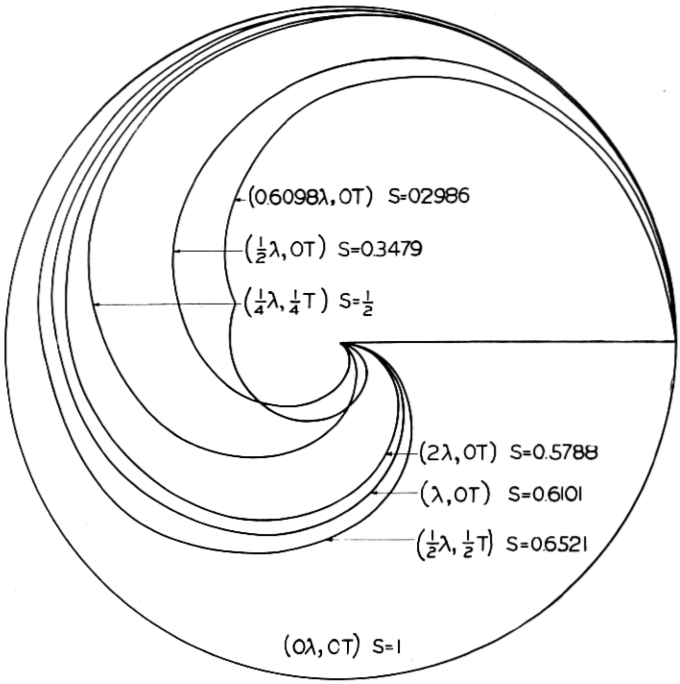

Fig. 4—Cumulative amplitude diagrams for two antennae

by the array (0.6098λ, $0T$). The directive diagram for this case is shown by Fig. 3.

Cumulative amplitude diagrams are shown by Fig. 4 for six selected arrays, including the unit circle, which is the cumulative diagram for the array (0λ, $0T$). In this figure, the angle corresponding to any value of the radius vector is equal to the total angle of the directive diagram of the array throughout which the relative amplitude

[16] A. Koerts, *loc. cit.*, pages 104, 105. In those cases in which the radius vector does not reach the unit circle, in order to obtain a measure of the reduction of the energy ratio of random static to signal, the unit circle should be replaced by the circle with a radius equal to the maximum radius vector. The absolute minimum 0.2986 is not changed upon including these cases but a relative minimum 1/3 occurs for the array ($a\lambda$, bT) upon letting a and b approach 0 and 1/2 in such a manner that $a+2b=1$. If the two antennae are loops, with planes parallel to the axis of the array, the area approaches the relative minimum 3/14 upon letting a and b approach the same limits 0 and 1/2 but in such a manner that $3a+4b=2$.

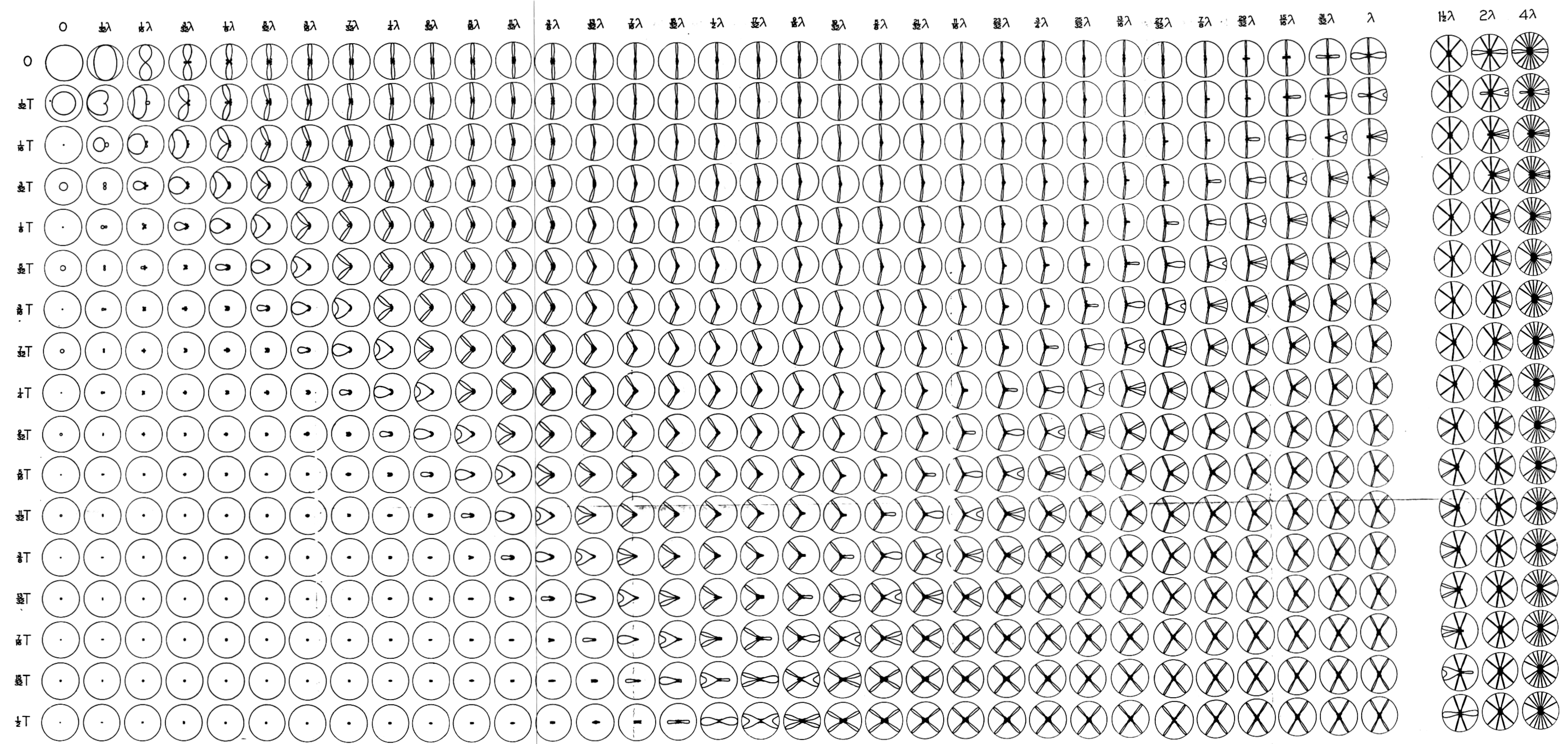

Fig. 5—Directive amplitude diagrams for an array of sixteen antennae; separation in wave-lengths (λ) along the top, phase difference in periods (T) at the left

is equal to or greater than this value. The area (S) relative to the unit circle is given in Fig. 4 for each of the cumulative diagrams, this area being equal to the area of the corresponding directive diagram.

Sixteen Antennae, Fig. 5

A total of 612 directive diagrams for an array consisting of sixteen antennae is shown by Fig. 5. The separation between adjacent

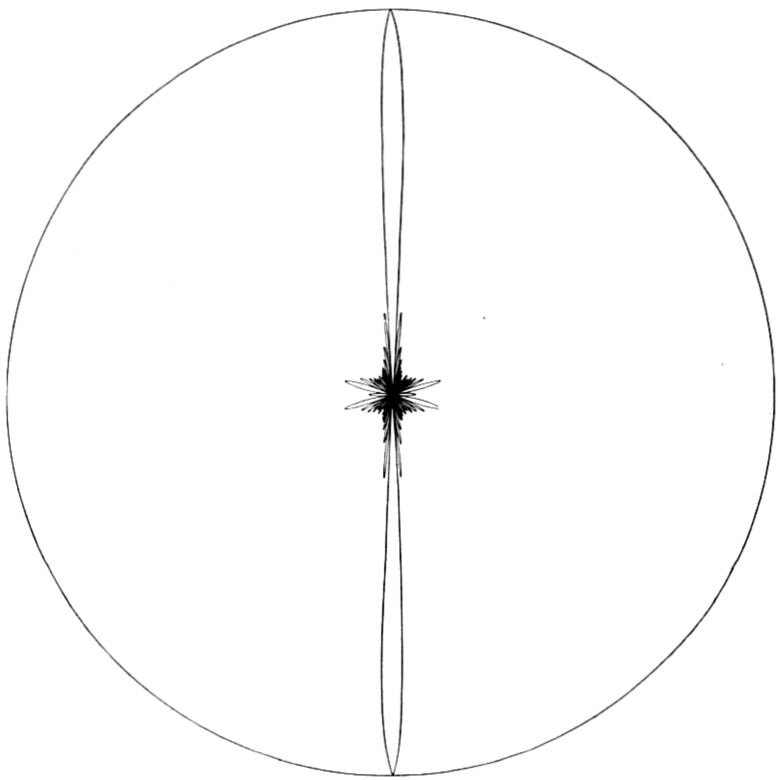

Fig. 6—Directive amplitude diagram for sixteen antennae (0.8825λ, $0T$) having the minimum area (0.0254) relative to the unit circle

antennae varies from 0 to 1 wave-length, in steps of 1/32 wave-length; the phase difference between adjacent antennae varies from 0 to 1/2 period, in steps of 1/32 period; additional sets of diagrams are included with separations of $1\frac{1}{2}$, 2, and 4 wave-lengths. The specified phase difference is the lag of the current in one antenna behind the current in its left-hand neighbor. The diagrams are reflected about the vertical axis upon changing the sign of the phase difference, and they repeat cyclically with increasing phase difference. These curves were copied from the original drawing; detailed accuracy is not claimed, but the arrangement and relative sizes of the lobes are approximately correct.

Comparison of Figs. 1 and 5 shows that, for the same set of parameters, the main features of the two diagrams are similar, that is,

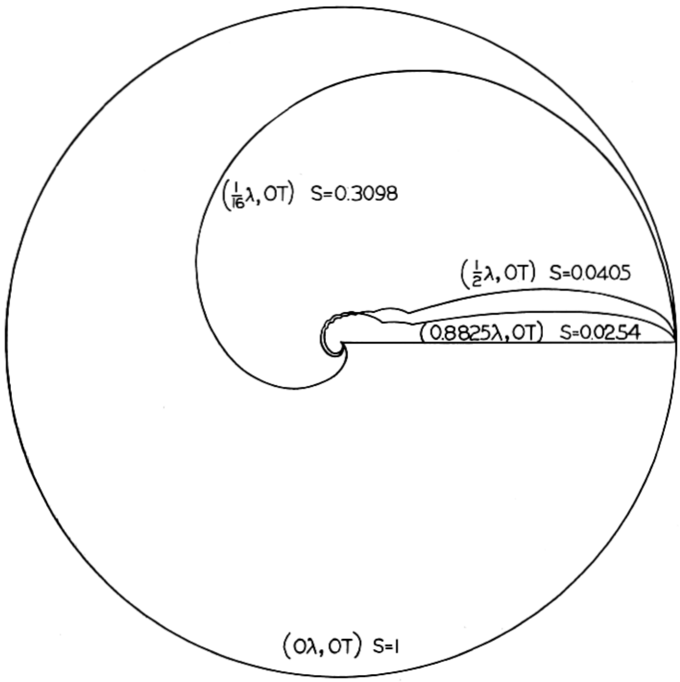

Fig. 7—Cumulative amplitude diagrams for sixteen antennae

the main lobes are located in the same positions, but with 16 antennae the main lobes are much narrower and in addition a multiplicity of small lobes occurs in many cases.

The area of the directive diagrams for 16 antennae oscillates about 1/16 and approaches it as a limit upon increasing the separation, keeping the phase difference constant. The minimum area (for a diagram in which the radius vector reaches its maximum of unity) is 0.0254, obtained by the array (0.8825λ, $0T$). The directive diagram for this case is shown by Fig. 6. Cumulative amplitude diagrams for 16 antennae are shown by Fig. 7 for three selected arrays in addition to the array (0λ, $0T$), the area of each diagram being given on the drawing.

Infinite Antenna Arrays

In view of the points of similarity between the diagrams for 2 and for 16 antennae, in particular for those pairs of diagrams for which the parameters of Fig. 1 are eight times the parameters of Fig. 5, provided the latter are relatively small, the question naturally arises as to the effect of increasing the number of antennae without limit.

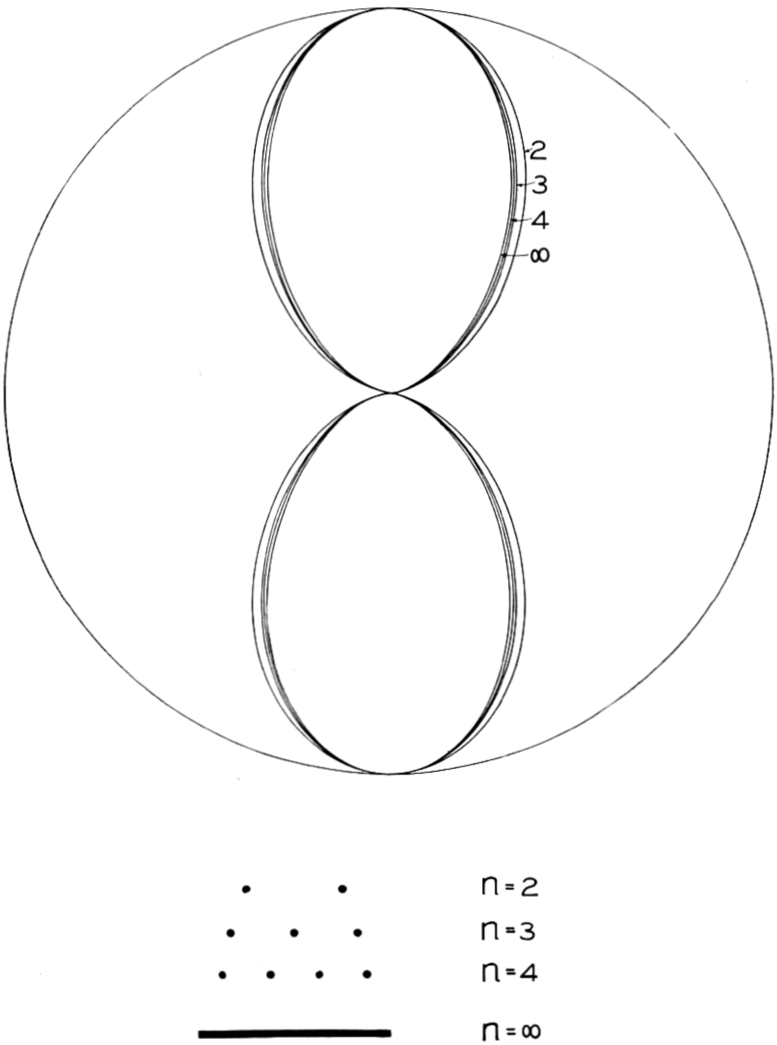

Fig. 8—Comparison of finite and infinite arrays within a total distance of one wave-length, with zero phase difference.

The similarity among the diagrams for arrays of antennae within a given fixed interval is illustrated by Figs. 8, 9, and 10.

In Fig. 8 are shown diagrams for 2, 3, and 4 antennae situated within a total distance of one wave-length with separations of 1/2, 1/3, and 1/4 wave-length, respectively, and with zero phase difference between adjacent antennae. The curve (∞) gives the limit of the family of directive diagrams for arrays of n antennae with a separation of $1/n$ wave-length, as n becomes infinite.

Fig. 9 gives similar curves for arrays of n antennae within an interval of two wave-lengths, with the parameters $\left(\frac{2}{n}\lambda,\ 0T\right)$. Fig. 10 shows a similar set of diagrams for arrays within an interval of one wave-length and within a total phase interval of one period, that is, for arrays of n antennae with the parameters $\left(\frac{1}{n}\lambda,\ \frac{1}{n}T\right)$.

For any interval $(A\lambda,\ BT)$ a similar family of curves can be obtained for arrays of n antennae with the parameters $\left(\frac{A}{n}\lambda,\ \frac{B}{n}T\right)$. As the number n is increased without limit, the directive diagram approaches a limiting curve. This limiting curve never has more than two directions of unit amplitude. There are zero, one, or two such directions, depending upon whether A is less than, equal to, or greater than B. The diagrams for the infinite case are reflected about the vertical axis upon changing the sign of the phase difference, but they do not repeat cyclically with increasing phase difference.

The rapidity with which the diagrams approach this limiting curve as n is increased is well illustrated by Figs. 8, 9, and 10. On the scale of these drawings, the curves for 16 antennae would be indistinguishable from the limiting curves for the infinite case. The upper left-hand corner of Fig. 5 may thus serve as a chart of the directive diagrams for the infinite case if the column and row headings are multiplied by the factor 16 to give the total separation and phase difference of the interval. For larger values of these parameters, however, the curves for the infinite case depart more and more from those of Fig. 5.

The diagrams with $A = B$ are of particular interest since these are unilateral, with the main lobe growing narrower as the total separation and phase difference are increased. In the case of the Beverage antenna, the ideal system [17] consists essentially of a long loop, which we may think of as the limiting case of a succession of a large number of narrow loops. The directive diagram of such an antenna system

[17] H. H. Beverage, C. W. Rice, and E. W. Kellogg, *loc. cit.*, pages 372, 373.

would, therefore, be the product of the group curve for an infinite number of antennae in the given interval multiplied by a cosine factor for the individual narrow loop.

SPACE CHARACTERISTICS

When the antennae are not confined to a straight line but are distributed over an area, a surface with its radius vector propor-

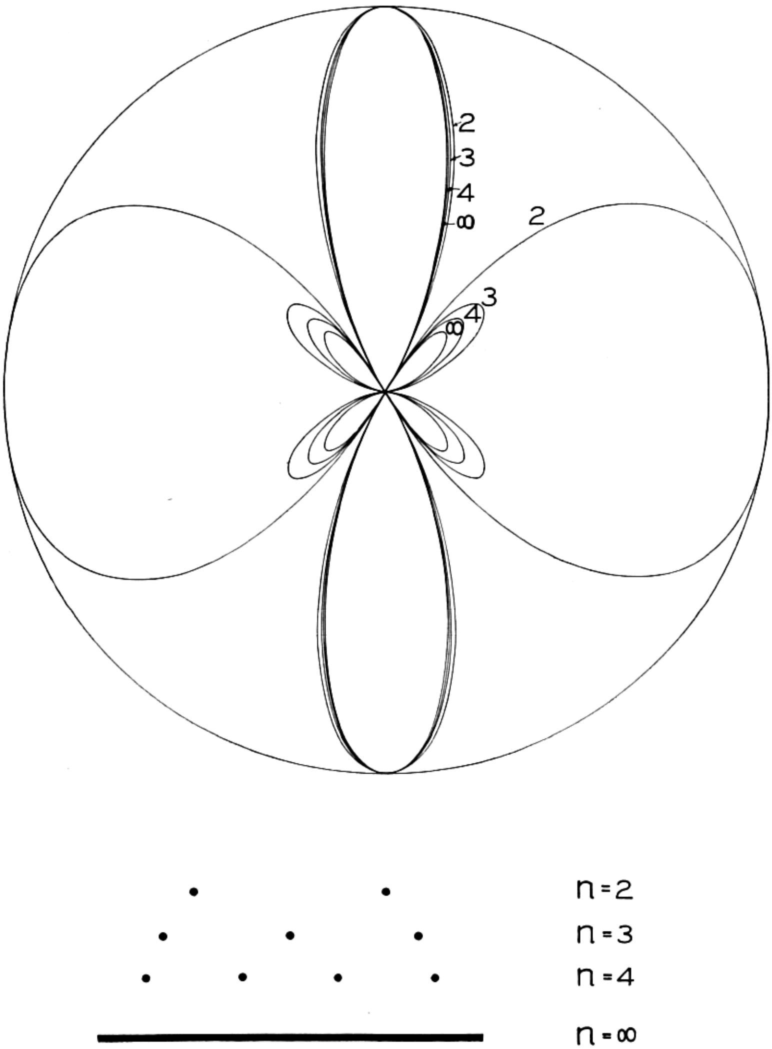

Fig. 9—Comparison of finite and infinite arrays within a total distance of two wavelengths, with zero phase difference

tional to the amplitude of the field of the radiation at a great distance from the array in the direction of the radius vector is required. Two particular cases will be illustrated in order to give some idea of the surface which shows the group effect of the array; for actual antennae, the radius vector of this surface must be multiplied by the corresponding radius vector of the space characteristic of the individual antenna in order to obtain the actual characteristic of the array.

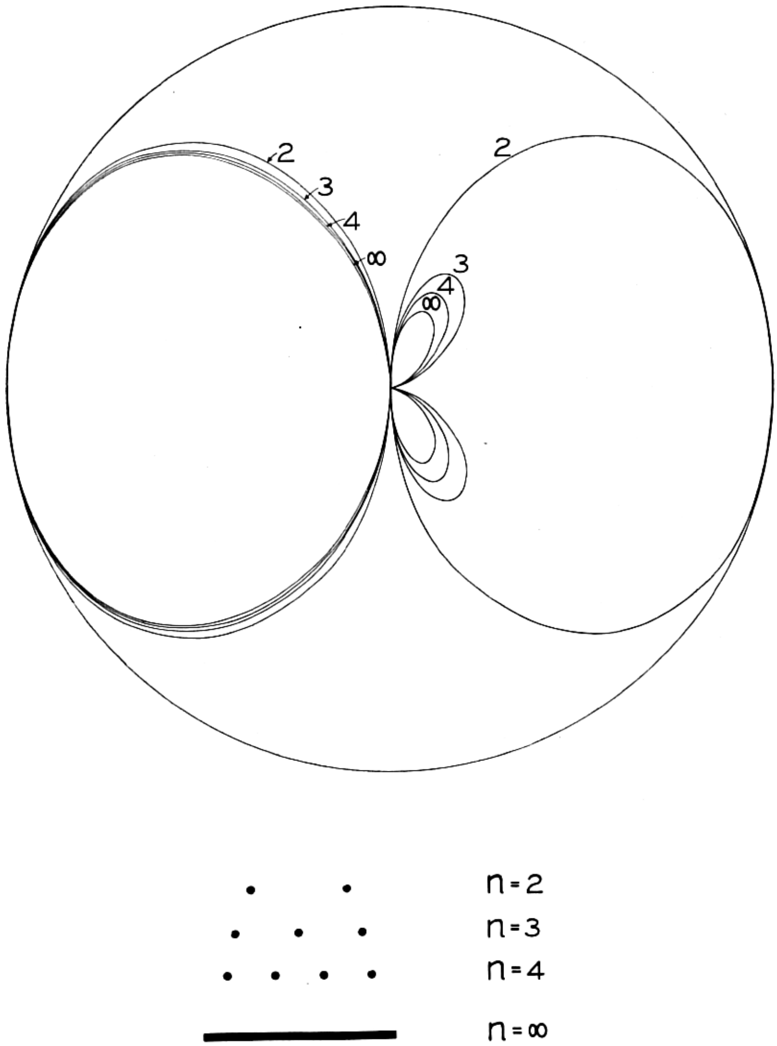

Fig. 10—Comparison of finite and infinite arrays within a total distance of one wave-length, and within a total phase interval of one period

A model of the upper half of the space characteristic of an array of four antennae located at the corners of a square is shown by Fig. 11, each side of the square having the parameters ($\frac{1}{2}\lambda$, $0T$). Below the model is shown the directive diagram in the plane of the array, which

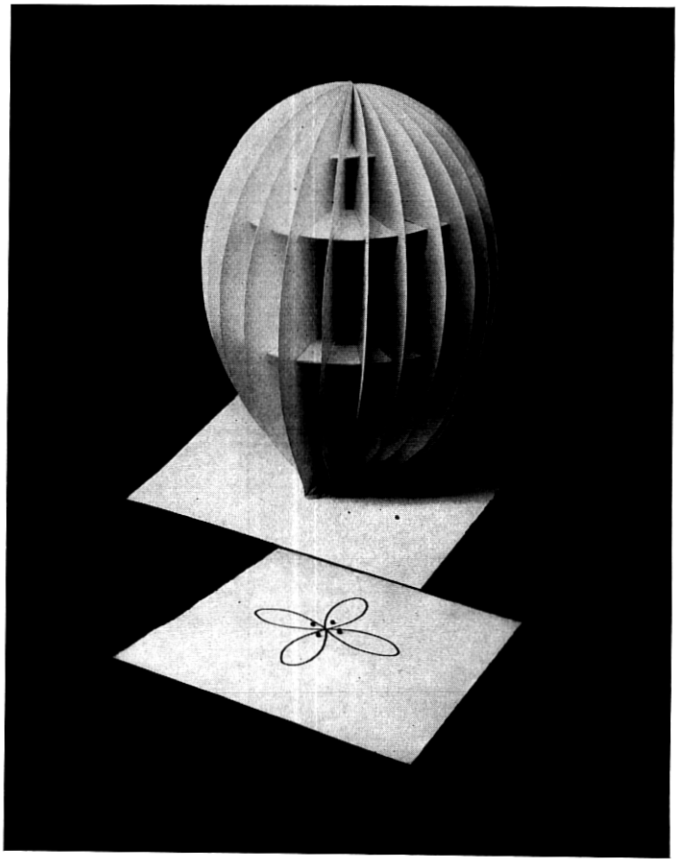

Fig. 11—Model of the space characteristic for an array of four antennae located at the corners of a square, with a separation of one-half wave-length between antennae on each side of the square, and with zero phase difference

is identical with the base of the model, together with a representation of the array itself. In the horizontal plane the maximum amplitude is slightly less than 1/5, occurring along the diagonals of the square; the amplitude reaches its absolute maximum of unity only in the vertical direction.

Fig. 12 shows a model of the upper half of the space characteristic of an array of 32 antennae located along the diagonals of a square, with the parameters ($\frac{1}{2}\lambda$, $0T$) in each diagonal. This space characteristic is a complicated surface, the main features of which are shown by the model, the smaller lobes not being shown clearly in detail. In

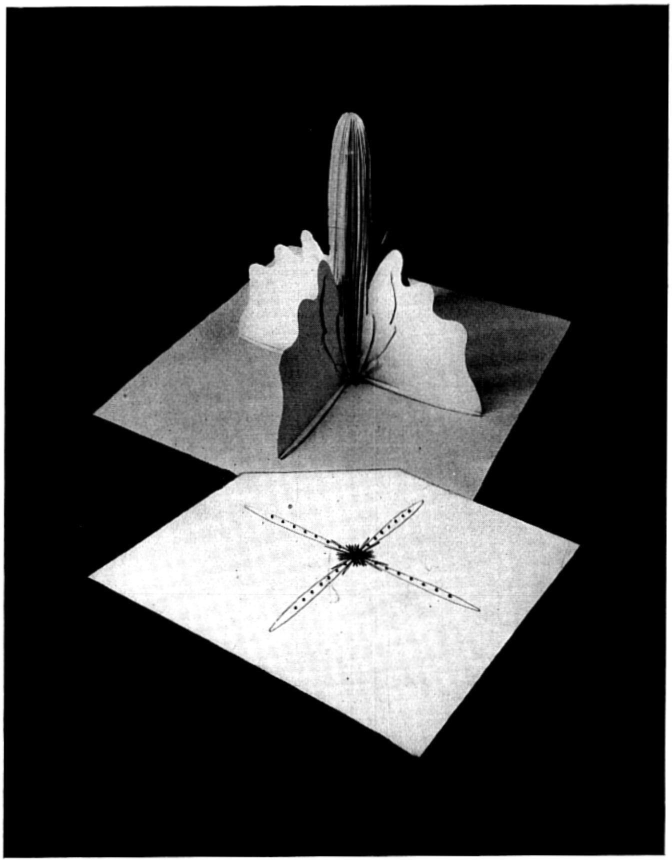

Fig. 12—Model of the space characteristic for an array of 32 antennae located along the diagonals of a square, with a separation of one-half wave-length between adjacent antennae in each diagonal, and with zero phase difference

the horizontal plane the maximum amplitude is 1/2, occurring along the diagonals of the square; the amplitude reaches a pronounced maximum of unity, however, in the vertical direction.

I am greatly indebted to Dr. Louisa E. Townshend for supervising the preparation of the drawings and models, and expecially for the accuracy attained in redrawing Fig. 1.

Appendix

Formulae for the directive diagrams of this paper are conveniently expressed in terms of polar coordinates, as follows: For a linear array of n antennae with the parameters $(a\lambda, bT)$,

$$r = \left| \frac{\sin n\,(\pi a \cos \theta + \pi b)}{n \sin\,(\pi a \cos \theta + \pi b)} \right|, \tag{1}$$

where θ is measured from the axis of the array. The area of this diagram, relative to the unit circle, is

$$S = \frac{2}{n^2}\left(\frac{n}{2} + \sum_{k=1}^{n-1} (n-k) J_0(2\pi k a) \cos\,(2\pi k b) \right). \tag{2}$$

For the special case $n=2$, Fig. 1, formula (1) reduces to

$$r = |\cos\,(\pi a \cos \theta + \pi b)|, \tag{3}$$

and formula (2) for the area to

$$S = \tfrac{1}{2}(1 + J_0(2\pi a) \cos\,(2\pi b)). \tag{4}$$

For the special case $n=\infty$ in a total interval $(A\lambda, BT)$, the limit of formula (1) for $a=A/n$ and $b=B/n$, as n becomes infinite, is

$$r = \left| \frac{\sin\,(\pi A \cos \theta + \pi B)}{\pi A \cos \theta + \pi B} \right|. \tag{5}$$

For the array of Fig. 11,

$$r = |\cos\,(\tfrac{1}{2}\pi \cos \theta \cos \phi) \cos\,(\tfrac{1}{2}\pi \sin \theta \cos \phi)|, \tag{6}$$

where ϕ is the angle which the radius vector makes with the plane of the array, and θ the angle which the projection of the radius vector in this plane makes with one side of the square. For the array of Fig. 12,

$$r = \left| \frac{\sin\,(8\pi \cos \theta \cos \phi)}{32 \sin\,(\tfrac{1}{2}\pi \cos \theta \cos \phi)} + \frac{\sin\,(8\pi \sin \theta \cos \phi)}{32 \sin\,(\tfrac{1}{2}\pi \sin \theta \cos \phi)} \right|, \tag{7}$$

where ϕ and θ are the same as for formula (6) except that the latter is measured from one diagonal of the square.