Irregularities in Loaded Telephone Circuits

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SYNOPSIS: The development of long distance telephone transmission has made the question of line irregularities a matter of great importance because of their harmful effect in producing echo currents and causing the repeaters to sing.

The structure of coil-loaded circuits permits the calculation of the probability of obtaining an assigned accuracy of balance between line and network when certain data are known or assumed regarding the accuracy of loading coil inductance and section capacity.

Formulae are given and the results of calculations compared with measurements made on circuits of known accuracy of loading.

INTRODUCTION

THE application of repeaters to telephone circuits in which the speech currents in the two directions of transmission pass through the same electrical path, has caused considerable emphasis to be placed on the matter of making the telephone circuits as free as possible from irregularities. This paper aims to present the theory of the relation between the irregularities in coil loaded lines and the effects resulting therefrom, which have an important bearing upon the operation of two-way telephone repeaters.

The idea of applying the theory of probability to the problem of summing up the effects of many small line irregularities was first suggested in 1912 by Mr. John Mills. The effect upon repeater operation of impedance unbalance had been mathematically analyzed by Dr. G. A. Campbell; and the effect upon impedance of a single irregularity of any type had been investigated by Mr. R. S. Hoyt. Using a probability relationship which was pointed out by Mr. E. C. Molina, Mr. Mills developed a formula which gives the average or probable impedance departure in terms of average or probable irregularities in inductance or capacity, which served at the time of the engineering of the transcontinental line (1913–14) and for some years after.

With the rapid growth of repeatered circuits in cable it became necessary to calculate what fraction of a large number of essentially similar lines would give a definite impedance unbalance at a given frequency. The necessary mathematical work to indicate the conditions for a large group of similar lines was recently carried out independently by Messrs. H. Nyquist and R. S. Hoyt.

The theory which has thus been evolved over a period of years is now presented in a manner which it is hoped will be found relatively simple and useful. Various charts are given which should be of material aid in the application of the theory. There are also given the results of some experiments made on cable circuits in which comparison is made between the impedance departures of the circuits as obtained by direct measurement with the departures as computed from data covering the individual irregularities. These impedance

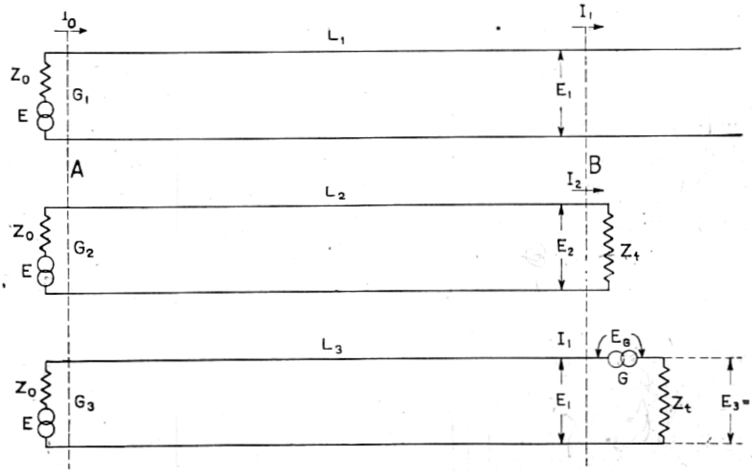


Fig. 1

departures are expressed as "return losses," the meaning of which is explained below. The agreement is shown to be close enough to constitute a good check as to the correctness of the underlying theory.

Magnitude of Reflected Current

In Fig. 1, are shown three regular[1] telephone lines of the same type beginning at a certain point A. The first line L_1 passes through another point B and continues on to infinity. The second line L_2 terminates at B where it is connected to an impedance Z_t which differs from the characteristic impedance Z_o of the three lines, thus constituting an irregular termination. The third line L_3 also terminates at B where it is connected to an impedance Z_t and a generator G of zero impedance whose purpose will be described later. At the sending end A each line is provided with one of three identical generators, G_1, G_2, G_3, having an impedance equal to Z_o the characteristic impedance of the line. The internal voltages of these generators are all equal and represented by E. The generator G_1 impresses a

[1] In this paper the term "regular" implies that a telephone line is free from electrical irregularities.

voltage $E_o = \frac{1}{2} E$ upon the sending end of the line L_1 and causes a current I_o to flow into it. The voltage and current waves are propagated regularly over the line to the point B where they set up a potential difference E_1 between the conductors and cause a current I_1 to flow. E_1 and I_1 are smaller in magnitude and later in phase than E_o and I_o because of the losses and finite velocity of transmission of the line L_1. These quantities have the relation

$$\frac{E_o}{I_o} = \frac{E_1}{I_1} = Z_o \tag{1}$$

since the line is regular.

In the second line L_2 a different set of conditions exists. In this case, the voltage E_2 and the current I_2 produced at B by the generator have the relation

$$\frac{E_2}{I_2} = Z_t. \tag{2}$$

When the e.m.f. of the generator G is zero, the conditions in the third line L_3 are the same as in L_2 but by adjusting the phase and magnitude of the e.m.f. of this generator the current in the terminal impedance Z_t can be made equal to I_1 and the drop across this impedance becomes

$$E_3 = I_1 Z_t. \tag{3}$$

Under these conditions the current I_1 flows at the end of the line L_3 and the potential difference E_1 exists between the conductors at this point. The line L_3 is then in the same condition as the line L_1 between the points A and B. When the waves arrive at B over the line L_3 the generator boosts or depresses the voltage at the terminus of the line by just the amount necessary to cause the terminal apparatus to take the desired current. Then the e.m.f. of the generator G is

$$E_G = E_3 - E_1. \tag{4}$$

Removing the e.m.f. of the generator G makes the conditions in line L_3 identical with the conditions in L_2, but removing this e.m.f. is the same thing as introducing another e.m.f. $-E_G$ in series with the generator which annuls its e.m.f. E_G. This e.m.f. $-E_G$ causes a current I_3 to flow back into the line

$$I_3 = -\frac{E_G}{Z_o + Z_t}. \tag{5}$$

Substituting from equations (1), (3) and (4) above

$$I_3 = \frac{Z_o - Z_t}{Z_o + Z_t} I_1. \tag{6}$$

That is, the effect of connecting an impedance Z_t to the end of a line of characteristic impedance Z_o is to return toward the source a current whose value is $\frac{Z_o-Z_t}{Z_o+Z_t}$ times the current that would exist at the terminus if the line were regularly terminated. The ratio between

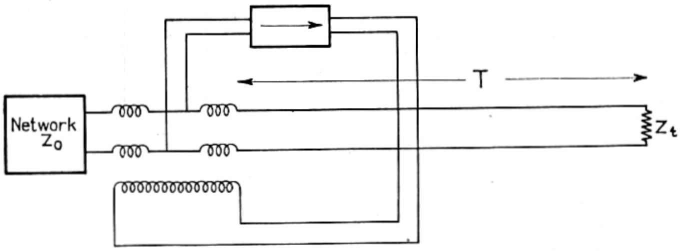


Fig. 2

the reflected and the incident current is known as the "reflection coefficient," the value of which is expressed as follows:

$$r=\frac{I_3}{I_1}=\frac{Z_o-Z_t}{Z_o+Z_t}. \tag{7}$$

This ratio can also be expressed in transmission units (TU). When expressed in TU this relation will be referred to in this paper as the "transmission loss of the returned current," or, briefly, as the "return loss."

If a condition occurs in a line which causes the impedance at any point to differ from the characteristic impedance it has the same effect as an irregular termination.

Return Loss at a Repeater Due to a Single Irregularity

Fig. 2 shows a No. 21-type repeater connected between a line and a network whose impedance is exactly equal to the characteristic impedance Z_o of the line. If the line is perfectly regular the repeater will be perfectly balanced and the gain can be increased indefinitely without causing the repeater to sing.

Assume now that the line is terminated by some apparatus having an impedance Z_t at a distance from the repeater such that the transmission loss of the intervening line is T TU. If a wave of current having a certain magnitude leaves the repeater, it is reduced in strength by T TU when it reaches the terminus. Of this current, a certain amount is transmitted back toward the repeater, suffering a further loss of T TU on the way; consequently, the relation expressed in TU between the strength of the currents leaving and returning to the repeater, that is, the return loss at the repeater, is given by the equation

$$S=20\log_{10}\frac{Z_o+Z_t}{Z_o-Z_t}+2T. \tag{8}$$

If the gain of the repeater, expressed in TU, is equal to or greater than S the repeater will sing provided the returning current has the correct phase relation to reinforce the original wave. For this reason the term "singing point" has frequently been applied to the quantity S, which is called returned loss in this paper.

If the line is shortened until the impedance Z_t is connected directly to the repeater terminals, the transmission loss T between the repeater and the irregularity is reduced to zero and the return loss becomes

$$S=20\log_{10}\frac{Z_o+Z_t}{Z_o-Z_t}. \tag{9}$$

Return Loss of Irregular Lines

In practice, lines are never perfectly regular. Not only is it impracticable to build apparatus which would form a perfectly regular termination for a line, but there are numerous causes of irregularity in the lines themselves, each one of which is capable of reflecting a portion of the waves which traverse the line. These irregularities can be kept smaller than any specified amount if sufficient care is used in building and maintaining the line but they cannot be entirely eliminated; consequently, if a length of actual line is terminated regularly by a network of impedance Z_o, the return loss will be high if the line is carefully built and low if it contains large irregularities. The return loss of such a line, when terminated regularly by a network is a measure of the quality of the line from the standpoint of repeater performance. In measuring the return loss of a line it is necessary that a rather long section of the line be available so as to include all irregularities near enough to have an appreciable effect upon the result. If the section measured is too short, the result will be too high because only a few irregularities will be included.

Calculation of the Return Loss of Coil Loaded Lines

Owing to the facts that the inductance of coil loaded lines is concentrated principally in the loading coils and the capacity is divided into elements of finite size by the loading coils and, further, that the electrical irregularities are due principally to the deviations of the inductance of the coils and the capacity of the sections from their average values for the line, it is possible to calculate by a fairly simple method the value of the return loss of a coil loaded line if the representative values of these deviations and the electrical properties of the line are known or assumed.

Since the return loss depends upon the accidental combination of a large number of unbalance currents there will not be one definite value applying to all circuits, but an application of the theory of probabilities makes it possible to compute what return loss will probably be surpassed by any assigned fraction of a large group of lines having the given deviations.

The method of calculating the return loss of coil loaded lines will now be described. The symbols used in this description and their meanings are given in the following table:

TABLE I

A = Attenuation Factor per Loading Section = Ratio of the Current Leaving a Loading Section to the Current Entering it.

C = Normal Capacity per Loading Section in Farads.

F = Fraction of a Large Group of Lines.

f = Any Frequency for which a Return Loss is to be Found.

f_c $= \frac{1}{\pi\sqrt{LC}}$ = Critical or Cutoff Frequency of the Line.

H_C = Representative[2] Deviation of the Capacity of Loading Sections.

h_C = Deviation of the Capacity of a Particular Loading Section.

H_L = Representative[2] Deviation of the Inductance of Loading Coils.

h_L = Deviation of the Inductance of a Particular Loading Coil.

H $= \sqrt{H_C^2+H_L^2}$ = Representative[2] Combined Deviation.

I_o = Current Entering the Line.

I' = Representative[2] Total In-Phase Returned Current at the Sending End.

I'' = Representative[2] Total Quadrature Returned Current at the Sending End.

I_F = Value of Returned Current which will be Exceeded in a Specified Fraction F of a Large Group of Lines.

i' = Total In-Phase Current at the Sending End of the Line.

i'' = Total Quadrature Current at the Sending End of the Line.

$i_1, i_2, i_3,$ - - - i_n = Currents Returned from the 1, 2, 3, - - - - - and nth Irregularities.

$i_1', i_2', i_3',$ - - - - - i_n' = In-Phase Components of $i_1, i_2, i_3,$ - - - i_n

$i_1'', i_2'', i_3'',$ - - - - - i_n'' = Quadrature Components of $i_1, i_2, i_3,$ - - - i_n

k $= \sqrt{\frac{L}{C}}$ = Nominal Characteristic Impedance of the Line.

L = Normal Inductance of a Loading Coil.

n = Number of Irregularities.

P = Probability Function for the Absolute Value of the Total Returned Current at the Sending End.

p' = Probability Function of the Total In-Phase Returned Current.

R_C = Representative[2] Reflection Coefficient at Capacity Irregularities.
R_L = Representative[2] Reflection Coefficient at Inductance Irregularities.
r_C = Reflection Coefficient at a Capacity Irregularity.
r_L = Reflection Coefficient at an Inductance Irregularity.
$r_1, r_2, r_3, \cdots r_n$ = Reflection Coefficient at the 1, 2, 3, - - - nth Irregularities.
S = Return Loss, Infinite Line.
S_n = Return Loss, Finite Line.
S_A = Attenuation Function.
S_F = Distribution Function.
S_H = Irregularity Function.
S_w = Frequency Function.
T = Transmission Loss in a Finite Line.
$\Theta_1, \Theta_2, \Theta_3, \cdots \Theta_n$ = Phase Angles of the Currents at the Sending End Returned by the 1, 2, 3, - - - nth Irregularities.
w = f/f_c.

Reflection at a Coil Irregularity

If a loading coil has too much or too little inductance, the effect is the same as if a small inductance $h_L L$ had been added to or taken away from the coil. The reactance of this increment is $2\pi f L h_L$. The additional reactance has the same effect wherever it may occur in the load but it is somewhat simpler to assume that the increment is introduced at mid-coil. Within the useful range of telephonic frequencies, the mid-coil impedance of a loaded line is given closely by the expression $k\sqrt{1-w^2}$.

In equation (7) $Z_o - Z_t$ corresponds to $2\pi f L h_L$ while $Z_o + Z_t$ is approximately equal to $2k\sqrt{1-w^2}$ when the irregularity is small, consequently:

$$r_L = \frac{\pi f L h_L}{k\sqrt{1-w^2}} \tag{10}$$

and, substituting for f and k their equivalents obtained from relations given in Table I,

$$r_L = h_L \frac{w}{\sqrt{1-w^2}}. \tag{11}$$

Reflection at a Spacing Irregularity

If a loading section has too much or too little capacity, the effect, neglecting conductor resistance, is the same as if a small bridged capacity $h_C C$ were added to or removed from the line. The effect

[2] The "representative" deviation or current is an index of the magnitude of the deviation or current that may be expected in accordance with the laws of the distribution of errors. It corresponds to the root-mean-square error. It must not be confused with the "effective" or r.m.s. value of a particular alternating current. The meaning of the term as used here is more completely explained in the paragraph following equation (24).

is the same for any point in the section, but it is somewhat simpler to assume that the additional capacity is applied at mid-section. The reactance of the added capacity is $\frac{1}{2\pi f h_C C}$ and the mid-section impedance is, closely, $\frac{k}{\sqrt{1-w^2}}$.

When the bridged reactance is large compared with the line impedance, the reflection coefficient r_C is given closely by the equation

$$r_C = \frac{1}{2}\frac{\dfrac{k}{\sqrt{1-w^2}}}{\dfrac{1}{2\pi f h_C C}} \tag{12}$$

from which, substituting the values of f and k as before

$$r_C = h_C \frac{w}{\sqrt{1-w^2}} \tag{13}$$

which is identical in form with equation (11) above.

Approximations Made in Deriving R_L and R_C

The expressions for the mid-coil and mid-section impedances used above in deriving equations (10) and (12) are simple approximations which take no account of the effects of the resistance of the line conductors and loading coils, leakage between conductors or distributed inductance. The errors due to these effects are negligible in the important parts of the frequency range involved in telephone transmission when the types of loading and sizes of conductors now commonly used are considered. The errors due to these causes tend to increase for frequencies which are very low or which approach the cutoff frequency. For accurate calculations relating to very light loading applied to high resistance conductors it would be desirable to take into account the effects of resistance. Because the use of the precise expressions would greatly complicate this discussion and would probably serve no very useful purpose at this time, the approximations given above are used.

Current Returned to the Sending End of the Line

Consider first a line having only one kind of irregularity as, for example, one in which only the loading coils are assumed to vary from their normal values. If a current I_o enters such a line, a current

i_1 is returned to the sending end from the first irregularity (assumed to be very near the sending end)

$$i_1 = r_1 I_o \tag{14}$$

a second current

$$i_2 = A^2 r_2 I_o \tag{15}$$

is returned from the irregularity located at a distance of one loading section away from the sending end, since the current is reduced by the factor A in going to the irregularity and again in returning.

Similarly, a current

$$i_n = A^{2(n-1)} r_n I_o \tag{16}$$

is returned from the nth irregularity.

The first current will return to the sending end with a certain phase angle Θ_1 with respect to the initial current, the second with a phase angle Θ_2, etc. Each returned current may be resolved into two components, one in phase with the initial current and one in quadrature.

The in-phase components of the currents are then:

$i_1' = I_o r_1 \cos \Theta_1$ from the first irregularity. (17)
$i_2' = I_o r_2 A^2 \cos \Theta_2$ from the second irregularity. (18)
$i_3' = I_o r_3 A^4 \cos \Theta_3$ from the third irregularity. (19)
$i_n' = I_o r_n A^{2(n-1)} \cos \Theta_n$ from the nth irregularity. (20)

and the quadrature components are:

$i_1'' = I_o r_1 \sin \Theta_1$ from the first irregularity. (21)
$i_2'' = I_o r_2 A^2 \sin \Theta_2$ from the second irregularity. (22)
$i_3'' = I_o r_3 A^4 \sin \Theta_3$ from the third irregularity. (23)
$i_n'' = I_o r_n A^{2(n-1)} \sin \Theta_n$ from the nth irregularity. (24)

Now the deviations of the inductance (and capacity) resemble the errors of measurement discussed in many text books dealing with the precision of measurement, consequently, they can be studied and their effects combined by the same mathematical law.

Examination of measurements of the inductance of large numbers of loading coils and the capacities of the pairs and phantoms in many reels of cable have shown that the most reasonable assumption is that the deviations of inductance and capacity follow the "normal" law of the distribution of errors.

The deviation at each irregularity is not known but it is possible to derive from the measurements of the inductance of large numbers of loading coils (and the capacity of many lengths of cable) representative values for these deviations similar to the "mean error." Because of the way in which the effects of irregularities combine, this *representative deviation* is taken as the square root of the mean of the squares of the deviations (r.m.s. deviation) of the individual coils. If the average deviation of a large group of coils is known, but the individual deviations are not, it may be multiplied by 1.2533 to obtain the representative deviation on the assumption that the deviations follow the normal law of errors.

If then the representative deviation H_L is substituted for the particular deviation h_L in equation (11), we obtain the representative reflection coefficient

$$R_L = H_L \frac{w}{\sqrt{1-w^2}}. \tag{25}$$

Now in the usual case where no effort is made to select the loading coils and so obtain a special distribution of the deviations the representative deviation and the representative reflection coefficient are the same for each coil. Substituting R_L for r_1, r_2, etc., in equations (17) to (24) each equation gives the representative value, at the sending end of the line, for the current reflected from the corresponding irregularity.

According to the laws for the combination of deviations which are demonstrated in treatises dealing with precision of measurements the representative value of the current due to all the irregularities would be the square root of the sum of the squares of the representative values of the different currents taken separately, consequently the representative in-phase current is

$$I' = I_o R_L \sqrt{(\cos^2\Theta_1 + A^4\cos^2\Theta_2 + A^8\cos^2\Theta_3 + \cdots A^{4(n-1)}\cos^2\Theta_n)} \tag{26}$$

and the representative quadrature current is

$$I'' = I_o R_L \sqrt{(\sin^2\Theta_1 + A^4\sin^2\Theta_2 + A^8\sin^2\Theta_3 + \cdots A^{4(n-1)}\sin^2\Theta_n)}. \tag{27}$$

By assuming that the representative in-phase and quadrature currents are equal the following steps can be greatly simplified. In view of the varying effects of frequency, distance from the sending end and nature of the irregularity upon the phase relations this appears to be a justifiable assumption, so combining I' and I'' in quadrature,

$$I' \dotplus I'' = \sqrt{\frac{I'^2 + I''^2}{2}} = \frac{I_o R_L}{\sqrt{2}} \sqrt{1 + A^4 + A^8 + \cdots A^{4(n-1)}} \tag{28}$$

For a finite number of irregularities, that is a finite line terminated by a perfect network just beyond the nth coil:

$$I' = I'' = \frac{I_0 R_L}{\sqrt{2}} \sqrt{\frac{1-A^{4n}}{1-A^4}} \tag{29}$$

which is obtained by summing up the series of terms under the radical in equation (28).

For an infinitely long line A^{4n} becomes zero since $A < 1$ and

$$I'_\infty = I''_\infty = \frac{I_O R_L}{\sqrt{2}} \sqrt{\frac{1}{1-A^4}}. \tag{30}$$

I' corresponds to the r.m.s. error in the ordinary theory of errors, consequently the probability function for the distribution of the in-phase currents is:

$$p' = \frac{1}{I'\sqrt{2\pi}} e^{-\frac{i'^2}{2I'^2}}. \tag{31}$$

Changing the accents, this equation also applies to the quadrature components.

The probability that the in-phase current lies between two near by values i' and $i'+di'$ is then equal to $p'\,di'$ and the probability that the quadrature component also lies between two values i'' and $i''+di''$ at the same time is $p'di' \times p''di''$. Transferring to polar coordinates,[3] the probability that the total returned current will be between a value $i = \sqrt{i'^2 + i''^2}$ and a slightly different value $i+di$ and also have a phase angle between Θ and $\Theta + d\Theta$ is

$$P = \frac{1}{2\pi I'^2} i e^{-\frac{i^2}{2I'^2}} \, di d\Theta. \tag{32}$$

Integrating with respect to the phase angle Θ between O and 2π to find the probability of obtaining a current between i and $i+di$ of any possible phase displacement

$$F = \frac{1}{I'^2} \int_{I_F}^{\infty} i e^{-\frac{i^2}{2I'^2}} \, di. \tag{33}$$

Integrating between I_F and infinity gives the probability that the total returned current will exceed the value I_F.

$$F = e^{-\frac{I^2_F}{2I'^2}}. \tag{34}$$

[3] For a more complete description of this operation, see "Advanced Calculus," by E. B. Wilson, page 390 et seq.

In a large number of lines, F is the fraction of the whole group which will have a return current in excess of I_F.

From the definition of the transmission unit the return loss of the line expressed in TU, is given by the expression

$$S=20\log_{10}\frac{I_o}{I_F}=-20\log_{10}\frac{I_F}{I_o} \tag{35}$$

from which

$$I_F^2=I_O^2 10^{-\frac{S}{10}}. \tag{36}$$

Substituting in (34)

$$F=e^{-\frac{I_o^2}{2I'^2}10^{-\frac{S}{10}}} \tag{37}$$

Taking logarithms to the base e and transposing

$$10^{-\frac{S}{10}}=-\frac{2I'^2}{I_o^2}\log_e F. \tag{38}$$

Taking logarithms to the base 10

$$S=10\log_{10}\left[\frac{1_o^2}{2I'^2\log_e\frac{1}{F}}\right]. \tag{39}$$

Substituting the value of I'_∞ from equation (30) for I'

$$S=10\log_{10}\left[\frac{1-A^4}{R_L^2}\times\frac{1}{\log_e\frac{1}{F}}\right] \tag{40}$$

and the value of R_L from equation (25)

$$S=10\log_{10}\left[\frac{1}{H_L^2}\times\frac{1-w^2}{w^2}\times(1-A^4)\times\frac{1}{\log_e\frac{1}{F}}\right]. \tag{41}$$

By a similar process of reasoning it is evident that if the line contains capacity deviations only, the return loss is given by this same expression with H_C substituted for H_L and if both types of irregularity occur the representative deviation is

$$H=\sqrt{H_L^2+H_C^2}$$

when H_C includes the effect of spacing irregularities as well as capacity deviations in the cable. The foregoing expression can, for convenience, be put in the form

$$S=S_H+S_w+S_F-S_A \tag{42}$$

in which each term depends upon only one independent variable and in which the symbols have the following meanings:

$$S_H = \text{Irregularity function} = 20 \log_{10} \frac{1}{H} \tag{43}$$

$$S_w = \text{Frequency function} = 20 \log_{10} \frac{\sqrt{1-w^2}}{w} \tag{44}$$

$$S_F = \text{Distribution function} = 10 \log_{10} \frac{1}{\log_e \frac{1}{F}} \tag{45}$$

$$S_A = \text{Attenuation function} = 10 \log_{10} \frac{1}{1-A^4} \tag{46}$$

Meaning of Equation (42)

To understand more clearly the meaning of equation (42) imagine that a large number of circuits of the same type and gauge are to be built in accordance with the same specifications so that the representative (r.m.s.) deviation including all causes has the same value H for each circuit. Further, imagine that the value of S has been calculated by formula (42) using a particular frequency f and a convenient fraction F. It is to be expected that when the circuits have been built and their return losses measured at the given frequency f the fraction F of the whole group will have return losses lower than S and the rest will have higher return losses.

In discussing expected results it is sometimes preferable to state the fraction $1-F$ of the circuits whose return losses will be greater than the assigned value rather than the fraction F whose return losses will be lower. This is done in Figs. 9 to 14 described below.

Location of the First Irregularity

In equations (14), (15) and (16) and all the equations which depend upon them it was assumed that the first irregularity occurs at the sending end of the line. Two other assumptions are equally plausible and might under some circumstances be preferable. These are that the first irregularity occurs (a) at one-half section from the end or (b) at a full section. In the first case (a) the current returned to the sending end from each irregularity will be reduced by the factor A and in the second (b) by the factor A^2, that is the return loss given by equation (42) should be increased by (a) the amount of the transmission loss in one loading section or (b) twice the amount of the transmission loss in one loading section respectively.

Return Losses of Short Lines

When a line is short and regularly terminated the returned current will be somewhat less than if it extends to infinity with irregularities and consequently the return loss will be higher. From equations (29) and (30), the returned current is lowered in the ratio $\frac{I'}{I'_\infty}=\sqrt{1-A^{4n}}$ by limiting the line to n sections; consequently

$$S_n=S+(S_n-S)=S+10\log_{10}\frac{1}{1-A^{4n}} \tag{47}$$

in which

$$S_n-S=10\log_{10}\frac{1}{1-A^{4n}} \tag{48}$$

is the increase in return loss.

Since the transmission loss in n sections of the line is

$$T=20\log_{10}\frac{1}{A^n} \tag{49}$$

it is easily seen that the increase of return loss can be expressed as a function of this loss. Transposing (49) and substituting in (48)

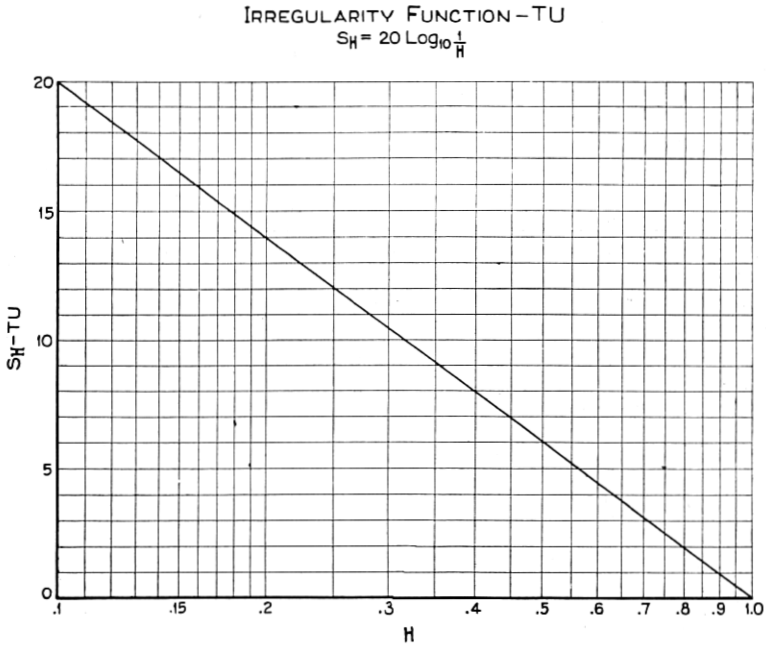
$$S_n-S=10\log_{10}\frac{1}{1-\left[\frac{1}{\log_{10}^{-1}\frac{T}{20}}\right]^4}. \tag{50}$$

Charts

The process of computing return losses can be greatly shortened by using the graphs of equations (43), (44), (45), (46), and (50) to obtain the values of the various functions. The accompanying Figs. 3 to 8, inclusive, have been prepared to illustrate these graphs and for use in rough calculations.

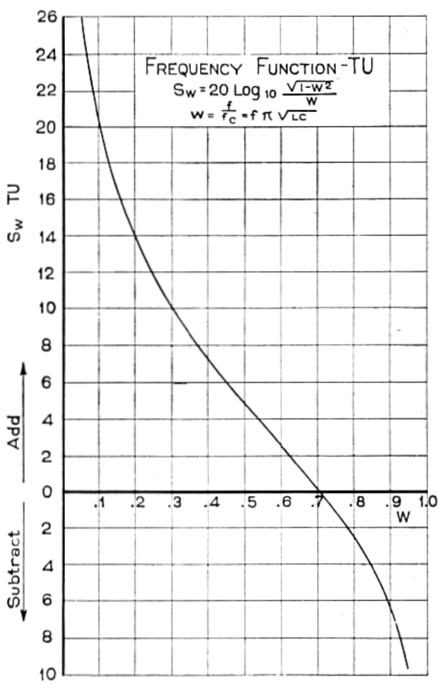
S_H may be obtained from any table or chart giving the relation between TU and current ratio by using H like a current ratio. Fig. 3 is a chart drawn especially for this purpose. For values of H lying between 0.1 and 0.01 look up a point on the curve corresponding to $10H$ and add 20 TU to the corresponding value of S_H, for values of H lying between 0.01 and 0.001 look up a point corresponding to $100H$ and add 40 TU to the value of S_H, and so forth.

Figs. 4, 5, 6, and 7 are curves giving the relations between the functions S_w, S_F and S_A, respectively, and the quantities upon which



IRREGULARITY FUNCTION – TU
SH = 20 Log10 1/H
SH – TU
20
15
10
5
0
.1
.15
.2
.3
.4
.5
.6
.7
.8
.9
1.0
H

Fig. 3



FREQUENCY FUNCTION - TU
Sw = 20 Log 10 √1-w² / w
w = f / fc = f π √LC
Sw TU
Add
Subtract
26
24
22
20
18
16
14
12
10
8
6
4
2
0
2
4
6
8
10
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
W

Fig. 4

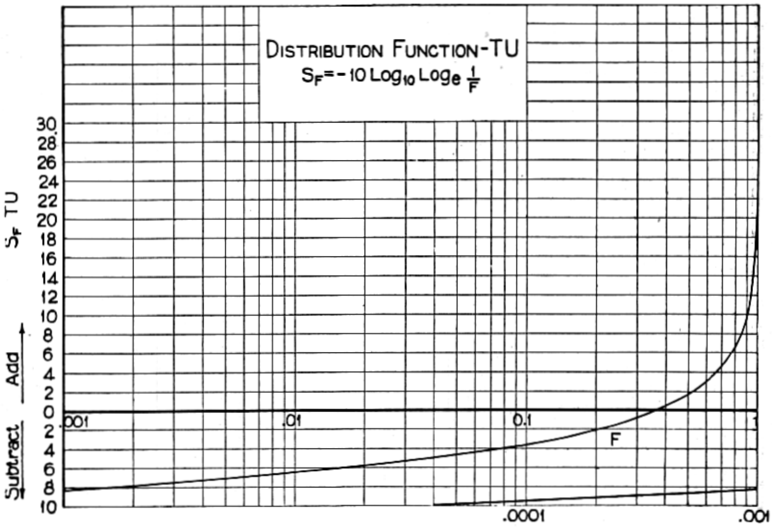


Fig. 5

ATTENUATION FUNCTION—TU

In terms of loss per loading section

$$S_A = 10 \log_{10} \frac{1}{1-A^4},$$

A = Attenuation factor per loading section,

$$L = 20 \log_{10} \frac{1}{A} = \text{loss per loading section in TU}$$

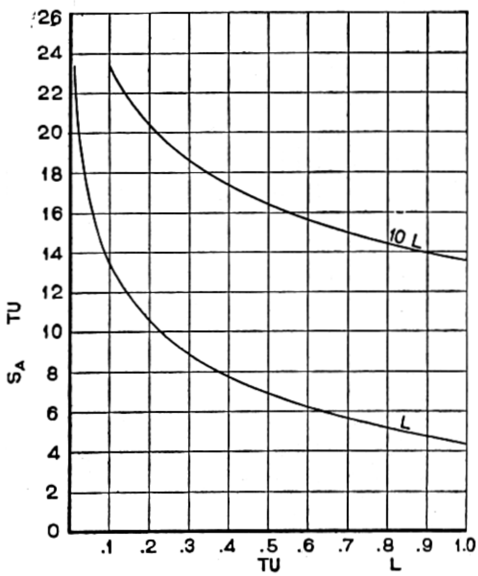


Fig. 6

each depends plotted from equations (44), (45) and (46). These are all positive except as indicated by the word "Subtract" on the diagrams.

A simple method for extending the curve of Fig. 5 is as follows: (a) choose a point on the curve within 3 TU of the lower end, (b) subtract about 3 TU (accurately, $10 \log_{10} 2$) from the value of S_F for this point, and (c) square the value of F for this point. The results obtained for (b) and (c) are the coordinates of another point on the extension of the curve.

Fig. 6 gives the relation between S_A and the transmission loss per loading section. On account of the wide use of 6,000 ft. spacing the curves of Fig. 7 are plotted to give the relation between S_A and the transmission loss per mile for 6,000 ft. spacing which is usually a more convenient arrangement.

Fig. 8 gives the amount, $S_n - S$, by which the return loss of a regularly terminated line of finite length (n sections) is greater than that of an infinite line as a function of the transmission loss of the finite line. This was calculated by formula (50).

Calculation of Return Loss

The process of finding the return loss by means of the curves is as follows:

(1) Determine the value of H_L, the representative deviation of the loading coils, and H_C, the representative deviation of the capacity of the loading sections. These depend upon the variations allowed in the specifications for loading coils and cable and upon the care with which the line is built. Calculate $H = \sqrt{H_L^2 + H_C^2}$, the representative combined deviation of the section. Look up the number of TU corresponding to H in any suitable table or chart, such as Fig. 3, to find S_H.

(2) Assume the frequency, f, to be considered. Calculate $w = \dfrac{f}{f_c}$ and look up the corresponding value of S_w on Fig. 4.

(3) Assume a value of F and look up the corresponding value of S_F on Fig. 5.

(4) Look up the value of S_A on Fig. 7, corresponding to the transmission loss per mile of the circuit at the frequency f if the coils are spaced 6,000 feet (1.136 miles) apart, or calculate the loss per section and look up S_A on Fig. 6, if some other spacing is used.

(5) Calculate $S = S_H + S_w + S_F - S_A$.

Attenuation Function—TU

In terms of loss per mile of the circuit length of loading section 6000 ft.

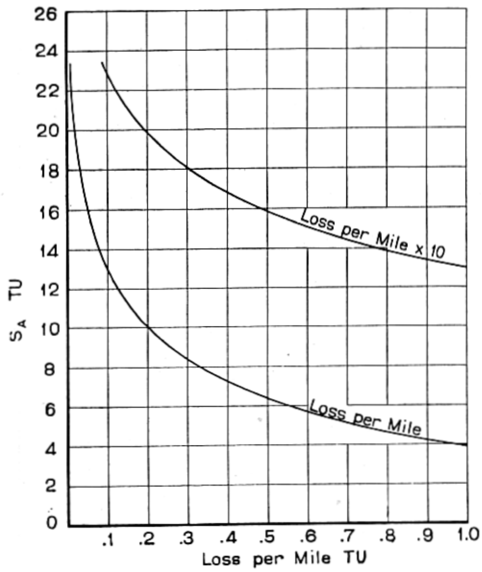


Fig. 7

Increase of the return loss when the line is limited to *n* sections

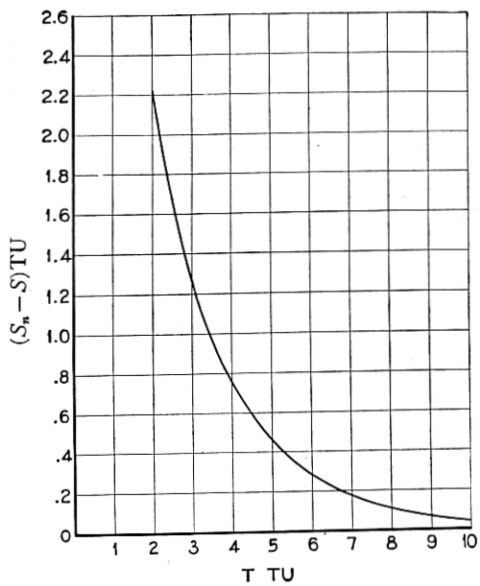


Fig. 8

(6) If the return loss of a finite length of line is desired determine the transmission loss of this length and look up the corresponding value of $S_n - S$ on Fig. 8. Add this amount to the value of S found in paragraph (5).

Example

As an example to illustrate the application of these methods let us calculate a return loss at 1,000 cycles for No. 19-H-174-63 [4] side circuits such that 90 per cent. of the circuits may be expected to have a higher value and only 10 per cent. to fall below it. The necessary data are given in Table II, below.

(1) $H = \sqrt{0.0062^2 + 0.0129^2 + 0.0045^2} = 0.0150.$
Fig. 3 gives 36.5 TU as the corresponding value of S_h.

(2) At 1,000 cycles $w = \dfrac{1000}{2810} = 0.356.$

Fig. 4 gives 8.4 TU as the corresponding value of S_w.

(3) Since 90 per cent. of the finished lines are to have return losses greater than S and 10 per cent. less $F = 0.1$ and Fig. 5 gives -3.7 TU as the corresponding value of S_F.

(4) The transmission loss per mile is 0.274. Since the coils are spaced 6,000 feet apart, Fig. 7 gives 8.7 TU as the value of S_A. This same value would be obtained less directly by calculating the loss per loading section, $0.274 \times \dfrac{6000}{5280} = 0.311$ and using Fig. 6. The latter method is used when the spacing is different from 6,000 feet.

(5) Using equation (42)

$$S = S_H + S_w + S_F - S_A = 36.5 + 8.4 - 3.7 - 8.7 = 32.5 \text{ TU}.$$

This will be found to agree with the 90 per cent. point on the smooth curve plotted in Fig. 10 which is described below.

(6) In case it is desired find the return loss of a length of this line having a transmission loss of, for example, 6 TU instead of the return loss of the infinite line. Fig. 8 gives $S_n - S = 0.3$ from which

$$S_n = 32.5 + 0.3 = 32.8 \text{ TU}.$$

Determination of Tolerable Deviations

To determine the deviations which correspond to an assigned value of the return loss find values of S_w, S_F and S_A as in paragraphs (2),

[4] In accordance with the practices of the Bell System, this notation indicates a phantom group of No. 19 B. & S. conductors in a cable with loading coils spaced 6,000 feet apart, the side circuit coils having 174 millihenrys inductance and the phantom coils 63 millihenrys.

(3) and (4) above and substitute in formula (42) to find the value of S_H. This with a table or chart of TU and current ratio gives the value of H. Limits can then be imposed on the loading coil inductances and section capacities that will insure that the representative deviation will not exceed the value H so found.

Comparison of Different Types of Circuits

These formulae are useful in comparing the return losses to be expected in various types of circuits which are built with the same accuracy in the matters of coil inductance and section capacity. In such cases it is merely necessary to calculate the quantity $S_w - S_A$ for each circuit and take the difference.

Example

As an example compare the No. 19-H-174-63 side circuits worked out above with No. 16-H-44-S [5] circuits at 1,000 cycles. Since the deviations and the fraction F are the same only S_w and S_A need be considered. For the No. 16-gauge circuit $f_c = 5560$ and the loss in TU per mile is 0.236. From these figures:

Gauge of Line	No. 19	No. 16
$w = \frac{1000}{f_c}$	0.356	0.18
S_w TU	8.4	14.8
S_A TU	8.7	9.4
$S_w - S_A$ TU	−0.3	5.4

These figures show that the return loss of the No. 16-H-44-S circuits should be higher than that of the No. 19-H-174-63 side circuits and the difference to be expected is $5.4 - (-0.3) = 5.7$ TU.

When the circuits to be compared have the same cutoff frequency the process of comparison is even simpler since the quantity S_w is then the same in each case. S_A is determined for each circuit as in paragraph (4) above. The difference between the two values of S_A is the difference between the return losses.

Example

As an example compare the No. 19-H-174-63 side circuits with No. 16-H-174-63 side circuits. In this case the cutoff frequencies are the same so w and S_w are the same. It is then only necessary to compare S_A. The loss per mile of the No. 16-gauge circuit is 0.161

[5] This notation indicates a side circuit of No. 16 B. & S. conductors in a cable loaded with 44 millihenry coils spaced 6,000 feet apart.

TU at 1,000 cycles from which $S_A = 11$ TU. In equation (42) S_A is negative hence the No. 19-gauge will have a higher return loss than the No. 16-gauge circuits and the expected difference is $11 - 8.7 = 2.3$ TU.

Comparison of Calculated and Measured Return Losses

In order to test the methods of calculation described above a series of measurements of return loss at 500, 1000 and 2000 cycles were made on a group of loaded side and phantom circuits in a cable using a No. 2-A unbalance measuring set.

The representative inductance deviations were found by analyzing the inductance measurements on a large group of loading coils similar to those used in the cable. The representative capacity deviations, not including the spacing irregularity were found by analyzing the shop measurements on a number of reels of the cable. This gave representative figures for reel lengths which were divided by $\sqrt{12}$ (in accordance with the laws of probability since this cable had 12 reel lengths in a loading section) to obtain the representative capacity deviations due to the cable for the loading sections. The spacing deviations were separately determined from the measured distances between the loading points.

The data used in the calculation were as follows:

TABLE II

	Sides	Phantoms
Representative inductance deviation	0.0062*	0.0061*
Representative capacity deviation	0.0129*	0.0138*
Representative spacing deviation	0.0045*	0.0045*
Combined representative deviation, H	0.0150*	0.0158*
Cutoff frequency fc (cycles sec.)	2810	3727
Transmission loss TU per mile — 500 cycles	0.265	0.271
Transmission loss TU per mile — 1000 cycles	0.274	0.279
Transmission loss TU per mile — 2000 cycles	0.317	0.296

The smooth curves of Figs. 9 to 14, inclusive, were calculated from the data in Table II using the methods described above. The abscissas are the percentages of a large group of circuits which may be expected to have return losses greater than the values given by the ordinates. This percentage is equal to 100 (1−F). The points plotted on the

* The figures are "fractional" deviations. Percentage deviations which are sometmes used are 100 times as large. Care should be taken to avoid errors caused by failure to divide percentage deviations by 100 before finding the value of F_H.

Return loss of No. 19–H–174–63 sides exceeded by various percentages of circuits at 500 cycles

Smooth curve—theoretical

⊙ 46–H–174–63 sides Pittsburgh to Ligonier

⊡ 12–H–174–63 sides Ligonier to Pittsburgh

△ 52–H–174–106 sides Pittsburgh to Ligonier

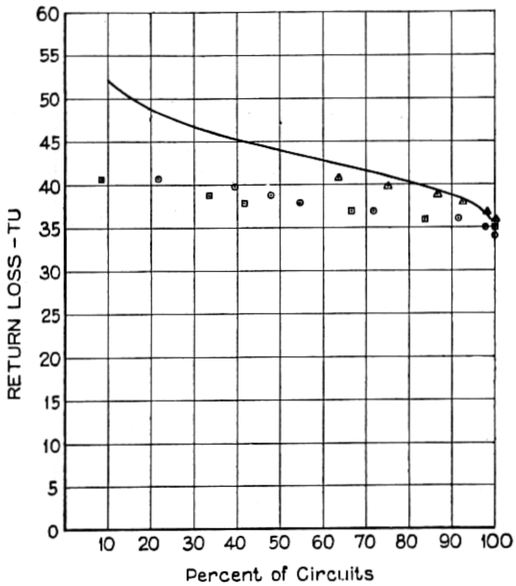


Fig. 9

Return loss of No. 19–H–174–63 sides exceeded by various percentages of circuits at 1000 Cycles

Smooth curve—theoretical

⊙ 46–H–174–63 sides Pittsburgh to Ligonier

⊡ 12–H–174–63 sides Ligonier to Pittsburgh

△ 52–H–174–106 sides Pittsburgh to Ligonier

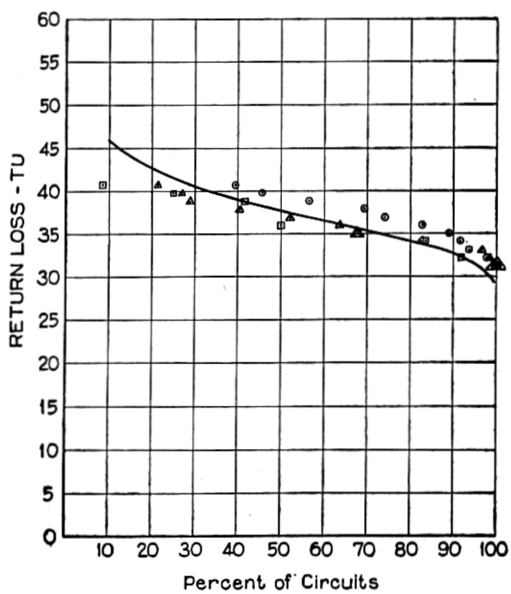


Fig. 10

Return loss of No. 19–H–174–63 sides exceeded by various percentages of circuits at 2000 cycles

Smooth curve—theoretical
⊙ 46-H-174–63 sides Pittsburgh to Ligonier
⊡ 12–H–174–63 sides Ligonier to Pittsburgh
△ 52–H–174–106 sides Pittsburgh to Ligonier

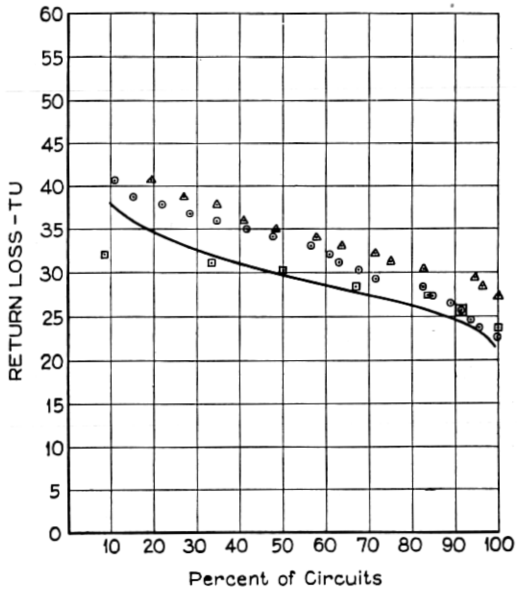


Fig. 11

Return loss of No. 19–H–174–63 phantoms exceeded by various percentages of circuits at 500 cycles

Smooth curve—theoretical
⊙ 25–H–174–63 phantoms Pittsburgh to Ligonier
⊡ 21–H–174–63 phantoms Ligonier to Pittsburgh

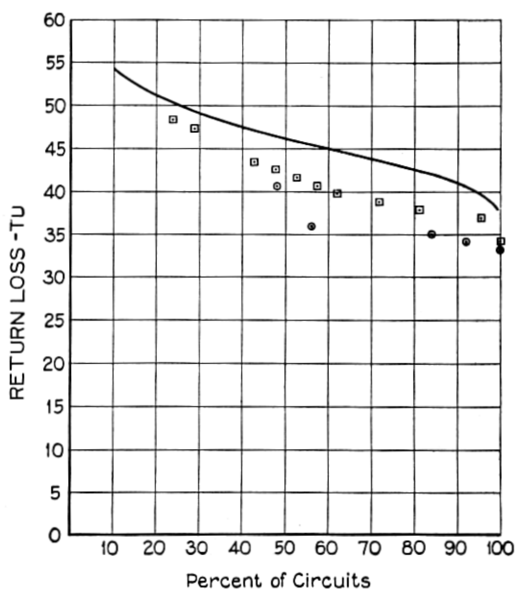


Fig. 12

Return loss of No. 19–H–174–63 phantoms exceeded by various percentages of circuits at 1000 cycles

Smooth curve—theoretical
⊙ 25–H–174–63 phantoms Pittsburgh to Ligonier
⊡ 21–H–174–63 phantoms Ligonier to Pittsburgh

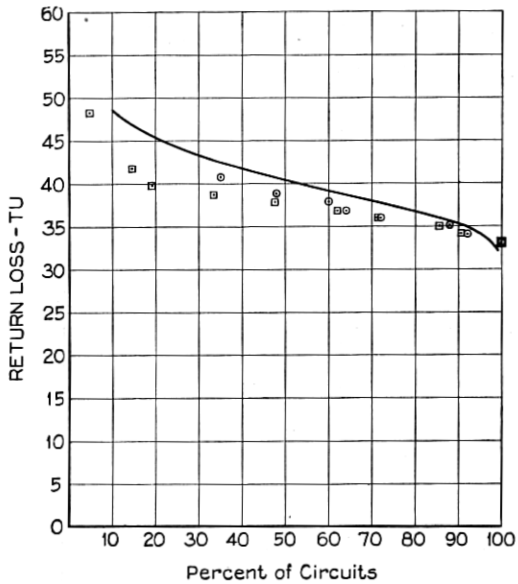


Fig. 13

Return loss of No. 19–H–174–63 phantoms exceeded by various percentages of circuits at 2000 cycles

Smooth curve—theoretical
⊙ 25–H–174–63 phantoms Pittsburgh to Ligonier
⊡ 21–H–174–63 phantoms Ligonier to Pittsburgh

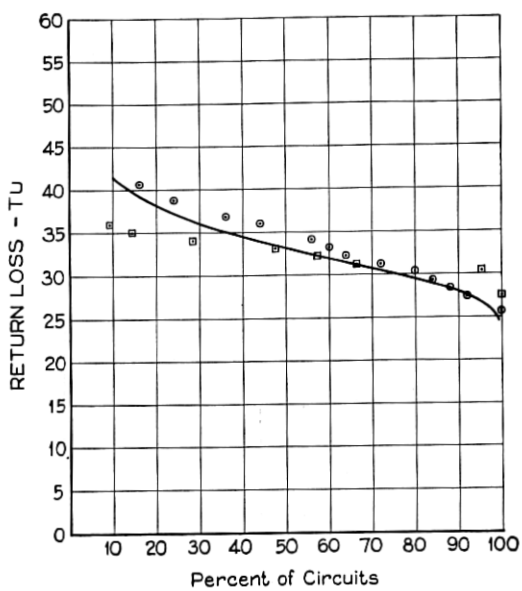


Fig. 14

curve sheets give the measured values of return loss found in the groups of circuits listed in the explanatory notes on the drawings.

In general, it will be observed that there is a fair agreement between the theoretical curves and the measured return losses especially at 1000 and 2000 cycles.

Due to the limited range of the measuring apparatus, readings of return losses greater than 40.7 TU were not made except in the case of the Ligonier to Pittsburgh phantoms shown on Figs. 12, 13 and 14, when a special arrangement was available to extend the range to 47.3 TU. For this reason points representing observed return losses above these limits are not available which causes the observed values for 500 cycles in Figs. 9 and 12 to appear somewhat low at first sight.

Where the highest point in a given set of data represents many circuits as in the cases represented by the small triangles and circles in Fig. 9 this point probably gives closely the return loss corresponding to the percentage of circuits it indicates but the points for higher return losses are not available. When the highest point represents only one or two circuits as in the case represented by the square in Fig. 9, it is likely that the actual return loss is higher than the point indicates.

It should also be noted that above 40 TU the actual impedance of the line and its characteristic impedance differ by less than 2 per cent. so that very small departures of the network from the true characteristic impedance of the line would tend to make the observed return loss low.

Conclusion

It is believed that the procedure described in this paper offers a reliable method for determining the probability of attaining a particular value of return loss at any assigned frequency when a circuit is built with definite limitations on inductance and capacity deviations so that the representative deviations are known.