Selective Circuits and Static Interference*

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SYNOPSIS: The present paper has its inception in the need of a correct understanding of the behavior of selective circuits when subjected to irregular and random interference, and of devising a practically useful figure of merit for comparing circuits designed to reduce the effects of this type of interference. The problem is essentially a statistical one and the results must be expressed in terms of mean values. The mathematical theory is developed from the idea of the spectrum of the interference and the response of the selective circuit is expressed in terms of the mean square current and mean power absorbed. The application of the formulas deduced to the case of static interference is discussed and it is shown that deductions of practical value are possible in spite of meagre information regarding the precise nature and origin of static interference.

The outstanding deductions of practical value may be summarized as follows:

1. Even with absolutely ideal selective circuits, an irreducible minimum of interference will be absorbed, and this minimum increases linearly with the frequency range necessary for signaling.
2. The wave-filter, when properly designed, approximates quite closely to the ideal selective circuit, and little, if any, improvement over its present form may be expected as regards static interference.
3. As regards static or random interference, it is quite useless to employ extremely high selectivity. The gain, as compared with circuits of only moderate selectivity, is very small, and is inevitably accompanied by disadvantages such as sluggishness of response with consequent slowing down of the possible speed of signaling.
4. A formula is developed, which, together with relatively simple experimental data, provides for the accurate determination of the spectrum of static interference.
5. An application of the theory and formulas of the paper to representative circuit arrangements and schemes designed to reduce static interference, shows that they are incapable of reducing, in any substantial degree, the mean interference, as compared with what can be done with simple filters and tuned circuits. The underlying reason lies in the nature of the interference itself.

I

THE selective circuit is an extremely important element of every radio receiving set, and on its efficient design and operation depends the economical use of the available frequency range. The theory and design of selective circuits, particularly of their most conspicuous and important type, the electric wave filter, have been highly developed, and it is now possible to communicate simultaneously without undue interference on neighboring channels with a quite small frequency separation. On the other hand too much has been expected of the selective circuit in the way of eliminating types of interference which inherently do not admit of elimination by any form of selective circuit. I refer to the large amount of inventive thought devoted to devising ingenious and complicated circuit ar-

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rangements designed to eliminate *static interference.* Work on this problem has been for the most part futile, on account of the lack of a clear analysis of the problem and a failure to perceive inherent limitations on its solutions by means of selective circuits.

The object of this paper is twofold: (1) To develop the mathematical theory of the behavior of selective circuits when subjected to random, irregular disturbances, hereinafter defined and designated as *random interference.* This will include a formula which is proposed as a measure of the *figure of merit of selective circuits with respect to random interference.* (2) On the basis of this theory to examine the problem of *static interference* with particular reference to the question of its elimination by means of selective circuits. The mathematical theory shows, as might be expected, that the complete solution of this problem requires experimental data regarding the frequency distribution of static interference which is now lacking. On the other hand, it throws a great deal of light on the whole problem and supplies a formula which furnishes the theoretical basis for an actual determination of the spectrum of static. Furthermore, on the basis of a certain mild and physically reasonable assumption, it makes possible general deductions of practical value which are certainly qualitatively correct and are believed to involve no quantitatively serious error. These conclusions, it may be stated, are in general agreement with the large, though unsystematized, body of information regarding the behavior of selective circuits to static interference, and with the meagre data available regarding the wave form of elementary static disturbances.

The outstanding conclusions of practical value of the present study may be summarized as follows:

(1) Even with absolutely ideal selective circuits, an irreducible minimum of interference will be absorbed, and this minimum increases linearly with the frequency range necessary for signaling.

(2) The wave-filter, when properly designed, approximates quite closely to the ideal selective circuit, and little, if any, improvement over its present form may be expected as regards static interference.

(3) As regards static or random interference, it is quite useless to employ extremely high selectivity. The gain, as compared with circuits of only moderate selectivity, is very small, and is inevitably accompanied by disadvantages such as sluggishness of response with consequent slowing down of the possible speed of signaling.

(4) By aid of a simple, easily computed formula, it should be possible to determine experimentally the frequency spectrum of static.

(5) Formulas given below for comparing the relative efficiencies of selective circuits on the basis of signal-to-interference energy ratio are believed to have considerable practical value in estimating the relative utility of selective circuits as regards static interference.

II

Discrimination between signal and interference by means of selective circuits depends on taking advantage of differences in their wave forms, and hence on differences in their *frequency spectra.* It is therefore the function of the selective circuit to respond effectively to the range of frequencies essential to the signal while discriminating against all other frequencies.

Interference in radio and wire communication may be broadly classified as *systematic* and *random,* although no absolutely hard and fast distinctions are possible. *Systematic interference* includes those disturbances which are predominantly steady-state or those whose energy is almost all contained in a relatively narrow band of the frequency range. For example, interference from individual radiotelephone and slow-speed radio telegraph stations is to be classified as systematic. *Random interference,* which is discussed in detail later, may be provisionally defined as the aggregate of a large number of elementary disturbances which originate in a large number of unrelated sources, vary in an irregular, arbitrary manner, and are characterized statistically by no sharply predominate frequency. An intermediate type of interference, which may be termed either *quasi-systematic* or *quasi-random,* depending on the point of view, is the aggregate of a large number of individual disturbances, all of the same wave form, but having an irregular or random time distribution.

In the present paper we shall be largely concerned with random interference, as defined above, because it is believed that it represents more or less closely the general character of *static* interference. This question may be left for the present, however, with the remark that the subsequent analysis shows that, as regards important practical applications and deductions, a knowledge of the exact nature and frequency distribution of static interference is not necessary.

Now when dealing with random disturbance, as defined above, no information whatsoever is furnished as regards instantaneous values. In its essence, therefore, the problem is a statistical one and the conclusions must be expressed in terms of mean values. In the present paper formulas will be derived for the *mean energy* and *mean square current* absorbed by selective circuits from random interference, and their applications to the static problem and the protection afforded by selective networks against static will be discussed.

The analysis takes its start with certain general formulas given by the writer in a recent paper[1], which may be stated as follows:

Suppose that a selective network is subjected to an impressed force $\phi(t)$. We shall suppose that this force exists only in the time interval, or epoch, $o \leqq t \leqq T$, during which it is everywhere finite and has only a finite number of discontinuities and a finite number of maxima and minima. It is then representable by the Fourier Integral

$$\phi(t) = 1/\pi \int_0^\infty |f(\omega)| \cdot \cos[\omega t + \theta(\omega)]\, d\omega \tag{1}$$

where

$$|f(\omega)|^2 = \left[\int_0^\infty \phi(t) \cos \omega t dt\right]^2 + \left[\int_0^\infty \phi(t) \sin \omega t dt\right]^2. \tag{2}$$

Now let this force $\phi(t)$ be applied to the network in the *driving* branch and let the resulting current in the *receiving* branch be denoted by $I(t)$. Let $Z(i\omega)$ denote the steady-state *transfer* impedance of the network at frequency $\omega/2\pi$: that is the ratio of e.m.f. in *driving* branch to current in *receiving* branch. Further let $z(i\omega)$ and cos $\alpha(\omega)$ denote the corresponding impedance and power factor of the receiving branch. It may then be shown that

$$\int_0^\infty [I(t)]^2 dt = 1/\pi \int_0^\infty \frac{|f(\omega)|^2}{|Z(i\omega)|^2} d\omega \tag{3}$$

and that the total energy W absorbed by the receiving branch is given by

$$W = 1/\pi \int_0^\infty \frac{|f(\omega)|^2}{|Z(i\omega)|^2} |z(i\omega)| \cos \alpha(\omega) \cdot d\omega. \tag{4}$$

To apply the formulas given above to the problem of random interference, consider a time interval, or epoch, say from $t=o$ to $t=T$, during which the network is subjected to a disturbance made up of a large number of unrelated elementary disturbances or forces, $\phi_1(t)$, $\phi_2(t) \ldots \phi_n(t)$.

If we write

$$\Phi(t) = \phi_1(t) + \phi_2(t) + \ldots + \phi_n(t),$$

then by (1), $\Phi(t)$ can be represented as

$$\Phi(t) = 1/\pi \int_0^\infty |F(\omega)| \cdot \cos[\omega t + \theta(\omega)]\, d\omega$$

[1] Transient Oscillations in Electric Wave Filters, Carson and Zobel, *Bell System Technical Journal*, July, 1923.

and

$$\int_0^\infty [I(t)]^2\,dt = 1/\pi \int_0^\infty \frac{|F(\omega)|^2}{|Z(i\omega)|^2}\,d\omega. \tag{3}$$

We now introduce the function $R(\omega)$, which will be termed the *energy spectrum* of the random interference, and which is analytically defined by the equation

$$R(\omega) = \frac{1}{T}|F(\omega)|^2 \tag{5}$$

Dividing both sides of (3) and (4) by T we get

$$\overline{I^2} = 1/\pi \int_0^\infty \frac{R(\omega)}{Z|(i\omega)|^2}\,d\omega, \tag{6}$$

$$\overline{P} = 1/\pi \int_0^\infty \frac{R(\omega)}{|Z(i\omega)|^2}|z(i\omega)| \cdot \cos\alpha(\omega)\,.\,d\omega. \tag{7}$$

$\overline{I^2}$, $\overline{P}$ and $R(\omega)$ become independent of the T provided the epoch is made sufficiently great. *$\overline{I^2}$ is the mean square current and $\overline{P}$ the mean power absorbed by the receiving branch from the random interference.*

In the applications of the foregoing formulas to the problem under discussion, the mean square current $\overline{I^2}$ of the formula (6) will be taken as the relative measure of interference instead of the mean power $\overline{P}$ of formula (7). The reason for this is the superior simplicity, both as regards interpretation and computation, of formula (6). The adoption of $\overline{I^2}$ as the criterion of interference may be justified as follows:

(1) In a great many important cases, including in particular experimental arrangements for the measurement of the static energy spectrum, the receiving device is substantially a pure resistance. In such cases multiplication of $\overline{I^2}$ by a constant gives the actual mean power $\overline{P}$.

(2) It is often convenient and desirable in comparing selective networks to have a standard termination and receiving device. A three-element vacuum tube with a pure resistance output impedance suggests itself, and for this arrangement formulas (6) and (7) are equal within a constant.

(3) We are usually concerned with relative amounts of energy absorbed from static as compared with that absorbed from signal. Variation of the receiver impedance from a pure constant resistance would only in the extreme cases affect this ratio to any great extent. In other words, the ratio calculated from formula (6) would not differ greatly from the ratio calculated from (7).

(4) While the interference actually apperceived either visually or by ear will certainly depend upon and increase with the energy absorbed from static, it is not at all certain that it increases linearly therewith. Consequently, it is believed that the additional refinement of formula (7) as compared with formula (6) is not justified by our present knowledge and that the representation of the receiving device as a pure constant resistance is sufficiently accurate for present purposes. It will be understood, however, that throughout the following argument and formulas, $\overline{P}$ of formula (7) may be substituted for $\overline{I}^2$ of (6), when the additional refinement seems justified. The theory is in no sense limited to the idea of a pure constant resistance receiver, although the simplicity of the formulas and their ease of computation is considerably enhanced thereby.

The problem of random interference, as formulated by equations (6) and (7) was briefly discussed by the writer in "Transient Oscillations in Electric Wave Filters" [1] and a number of general conclusions arrived at. That discussion will be briefly summarized, after which a more detailed analysis of the problem will be given.

Referring to formula (6), since both numerator and denominator of the integrand are everywhere $\geqq o$, it follows from the mean value theorem that a value ω of ω exists such that

$$\overline{I}^2 = \frac{R(\overline{\omega})}{\pi} \int_0^\infty \frac{d\omega}{|Z(i\omega)|^2}. \tag{8}$$

The approximate location of $\overline{\omega}$ on the frequency scale is based on the following considerations:

(a) In the case of efficient selective circuits designed to select a continuous finite range of frequencies in the interval $\omega_1 \leqq \omega \leqq \omega_2$, the important contributions to the integral (6) are confined to a finite continuous range of frequencies which includes, but is not greatly in excess of, the range which the circuit is designed to select. This fact is a consequence of the impedance characteristics of selective circuits, and the following properties of the spectrum $R(\omega)$ of random interference, which are discussed in detail subsequently.

(b) $R(\omega)$ is a continuous finite function of ω which converges to zero at infinity and is everywhere positive. It possesses no sharp maxima or minima, and its variation with respect to ω, where it exists, is relatively slow.

On the basis of these considerations it will be assumed that $\overline{\omega}$ lies within the band $\omega_1 \leqq \omega \leqq \omega_2$ and that without serious error it may be taken as the mid-frequency ω_m of the band which may be defined either as $(\omega_1+\omega_2)/2$ or as $\sqrt{\omega_1\omega_2}$. Consequently

$$\bar{I}^2=\frac{R(\omega_m)}{\pi}\int_0^\infty \frac{d\omega}{|Z(i\omega)|^2}. \tag{9}$$

From (9) it follows that the mean square current $\bar{I}^2$, due to random interference, is made up of two factors: one $R\ (\omega_m)$ which is proportional to the energy level of the interference spectrum at mid-frequency $\omega_m/2\,\pi$: and, second, the integral

$$\rho=1/\pi\int_0^\infty \frac{d\omega}{|Z\ (i\omega)\ |^2} \tag{10}$$

which is independent of the character and intensity of the interference. Thus

$$\bar{I}^2=\rho R(\omega_m). \tag{11}$$

Formula (11) is of considerable practical importance, because by its aid the spectral energy level $R\ (\omega)$ can be determined, once $\bar{I}^2$ is experimentally measured and the frequency characteristics of the receiving network specified or measured. It is approximate, as discussed above, but can be made as accurate as desired by employing a sufficiently sharply selective network.

The formula for the *figure of merit of a selective circuit with respect to random interference* is constructed as follows:

Let the signaling energy be supposed to be spread continuously and uniformly over the frequency interval corresponding to $\omega_1 \leqq \omega \leqq \omega_2$. Then the mean square signal current is given by

$$\frac{E^2}{\pi}\int_{\omega_1}^{\omega_2} \frac{d\ \omega}{|Z(i\omega)|^2}$$

or, rather, on the basis of the same transmitted energy to

$$\frac{E^2}{\pi(\omega_2-\omega_1)}\int_{\omega_1}^{\omega_2} \frac{d\omega}{|Z(i\omega)|^2}=E^2\frac{\sigma}{\omega_2-\omega_1}. \tag{12}$$

The ratio of the mean square currents, due to signal and to interference, is

$$\frac{E^2}{R(\omega_m)}\cdot\frac{1}{\omega_2-\omega_1}\frac{\sigma}{\rho}. \tag{13}$$

The first factor $\dfrac{E^2}{R(\omega_m)}$ depends only on the signal and interference energy levels, and does not involve the properties of the network. The second factor depends only on the network and measures the efficiency with which it excludes energy outside the signaling range. It will therefore be termed *the figure of merit of the selective circuit* and denoted by S, thus

$$S=\frac{1}{\omega_2-\omega_1}\frac{\sigma}{\rho}=\frac{1}{\omega_2-\omega_1}\int_{\omega_1}^{\omega_2}\frac{d\omega}{Z(i\omega)^2}\div\int_0^\infty\frac{d\omega}{|Z(i\omega)|^2}. \tag{14}$$

Stated in words, *the figure of merit of a selective circuit with respect to random interference is equal to the ratio of the mean square signal and interference currents in the receiver, divided by the corresponding ratio in an ideal band filter which transmits without loss all currents in a "unit" band* ($\omega_2-\omega_1=1$) *and absolutely extinguishes currents outside this band.*

III

Before taking up practical applications of the foregoing formulas further consideration will be given to the hypothesis, fundamental to the argument, that over the frequency range which includes the important contributions to the integral $\int_0^\infty \frac{d\omega}{|Z(i\omega)|^2}$ the spectrum $R(\omega)$ has negligible fluctuations so that the integral

$$\int_0^\infty \frac{R(\omega)}{|Z(i\omega)|^2}d\omega$$

may, without appreciable error, be replaced by

$$R(\omega_m)\int_0^\infty \frac{d\omega}{|Z(i\omega)|^2}$$

where $\omega_m/2\pi$ is the "mid-frequency" of the selective circuit.

The original argument in support of this hypothesis was to the effect that, since the interference is made up of a large number of unrelated elementary disturbances distributed at random in time, any sharp maxima or minima in the spectrum of the individual diturbances would be smoothed out in the spectrum of the aggregate disturbance. This argument is still believed to be quite sound: the importance of the question, however, certainly calls for the more detailed analysis which follows:

Let
$$\Phi(t)=\sum_1^N \phi_r(t-t_r) \tag{15}$$

where t_r denotes the time of incidence of the r^{th} disturbance $\phi_r(t)$. The elementary disturbances $\phi_1, \phi_2 \ldots \phi_N$ are all perfectly arbitrary, so that $\Phi(t)$ as defined by (15) is the most general type of disturbance possible. The only assumption made as yet is that the instants of incidence $t_1 \ldots t_N$ are distributed at random over the epoch $o \leqq t \leqq T$; an assumption which is clearly in accordance with the facts in the case of static interference. If we write

$$C_r(\omega) = \int_0^\infty \phi_r(t) \cos \omega t \, dt,$$

$$S_r(\omega) = \int_0^\infty \phi_r(t) \sin \omega t \, dt, \tag{16}$$

it follows from (2) and (15), after some easy rearrangements that

$$| F(\omega) |^2 = \sum_{r=1}^{N} \sum_{s=1}^{N} \cos \omega(t_r - t_s)[C_r(\omega)C_s(\omega) + S_r(\omega)S_s(\omega)] = \sum C_r^2(\omega) + S_r^2(\omega) \tag{17}$$
$$+ \sum \sum \cos \omega(t_r - t_s) \, [C_r(\omega)C_s(\omega) + S_r(\omega) \, S_s(\omega)], \; r \neq s.$$

The first summation is simply $\sum |f_r(\omega)|^2$. The double summation involves the factor $\cos \omega \, (t_r - t_s)$. Now by virtue of the assumption of random time distribution of the elementary disturbances, it follows that t_r and t_s, which are independent, may each lie anywhere in the epoch $o \leqq t \leqq T$ with all values equally likely. The mean value of $| F(\omega) |^2$ is therefore gotten by averaging[2] with respect to t_r and t_s over all possible values, whence

$$| F(\omega) |^2 = \sum | f_r(\omega) |^2 + 2/T^2 \frac{1 - \cos \omega T}{\omega^2} \times \sum \sum [C_r(\omega)C_s(\omega) + S_r(\omega)S_s(\omega)] \tag{18}$$

and

$$\bar{I}^2 = \frac{1}{\pi T} \sum \int_0^\infty \frac{| f_r(\omega) |^2}{| Z(i\omega) |^2} d\omega + \frac{2}{\pi T^2} \sum \sum \int_0^\infty \frac{1 - \cos \omega T}{\omega^2 T} [C_r(\omega)C_s(\omega) + S_r(\omega)S_s(\omega)] \frac{d\omega}{| Z(i\omega) |^2}.$$

[2] The averaging process with respect to the parameters t_r and t_s employed above logically applies to the average result in a very large number of epochs during which the system is exposed to the same set of disturbances with different but random time distributions. Otherwise stated, the averaging process gives the mean value corresponding to all possible equally likely times of incidence of the elementary disturbances. The assumption is, therefore, that if the epoch is made sufficiently large, the actual effect of the unrelated elementary disturbances will in the long run be the same as the average effect of all possible and equally likely distributions of the elementary disturbances.

Now in the double summation if the epoch T is made sufficiently great, the factor $\frac{(1-\cos\omega T)}{\omega^2 T}$ vanishes everywhere except in the neighborhood of $\omega = o$. Consequently, the double summation can be written as

$$\frac{2}{\pi T^2}\int_0^\infty \frac{1-\cos\omega T}{\omega^2 T^2}d\omega T\cdot\sum\sum\frac{C_r(o)C_s(o)}{|Z(o)|^2}=\frac{1}{T^2}\sum\sum\frac{C_r(o)C_s(o)}{|Z(o)|^2}.$$

Finally if we write $N/T = n$ = average number of disturbances per unit time, and make use of formula (2), we get

$$\bar{I}^2=\frac{n}{N}\sum 1/\pi\int_0^\infty\frac{|f_r(\omega)|^2}{|Z(i\omega)|^2}d\omega$$
$$+\frac{n^2}{N^2}\cdot\frac{1}{|Z(o)|^2}\cdot\sum\sum\int_0^\infty\phi_r(t)dt\cdot\int_0^\infty\phi_s(t)dt, \qquad (19)$$

which can also be written as

$$\bar{I}^2=\frac{n}{N}\sum\int_0^\infty i_r^2dt+\frac{n^2}{N^2}\sum\sum\int_0^\infty i_rdt\cdot\int_0^\infty i_sdt\cdot \qquad (20)$$

when $i_r = i_r\ (t)$ is the current due to the r^{th} disturbance $\phi_r\ (t)$.

Now the double summation vanishes when, due to the presence of a condense or transformer, the circuit does not transmit direct current to the receiving branch. Furthermore, if the disturbances are oscillatory or alternate in sign at random, it will be negligibly small compared with the single summation. Consequently, it is of negligible significance in the practical applications contemplated, and will be omitted except in special cases. Therefore, disregarding the double summation, the foregoing analysis may be summarized as follows:

$$R(\omega)=\frac{n}{N}\sum|f_r(\omega)|^2=n\cdot r(\omega), \qquad (21)$$

$$\bar{I}^2=\frac{n}{N}\sum 1/\pi\int_0^\infty\frac{|f_r(\omega)|^2}{|Z(i\omega)|^2}d\omega \qquad (22)$$

$$=\frac{n}{N}\sum\int_0^\infty i_r^2dt=n\int_0^\infty\bar{i}^2dt, \qquad (23)$$

$$\bar{P}=\frac{n}{N}\int_0^\infty\frac{r(\omega)}{|Z(i\omega)|^2}|z(i\omega)|\cdot\cos\alpha\ (\omega)\cdot d\omega \qquad (24)$$

$$=\frac{n}{N}\sum w_r=n\cdot\bar{w}. \qquad (25)$$

In these formulas n denotes the average number of elementary disturbances per unit time, w_m the energy absorbed from the r^{th} disturbance $\phi_r(t)$, and $\bar{P}$ the mean power absorbed from the aggregate disturbance. $r(\omega)$ is defined by formula (20) and is the mean spectrum of the aggregate disturbance, thus

$$r(\omega)=1/N\sum|f_r(\omega)|^2=R(\omega)/N. \tag{26}$$

We are now in a position to discuss more precisely the approximations, fundamental to formulas (9)–(14),

$$\int_0^\infty \frac{R(\omega)}{|Z(i\omega)|^2}d\omega=R(\omega_m)\int_0^\infty \frac{d\omega}{|Z(i\omega)^2|}. \tag{27}$$

The approximation involved in this formula consists in identifying $\omega_m/2\pi$ with the "mid-frequency" of the selective circuit, and is based on the hypothesis that over the range of frequencies, which includes the important contribution to the integral (22), the fluctuation of $R(\omega)$ may be ignored.

Now it is evident from formulas (21)–(22) that the theoretically complete solution of the problem requires that $R(\omega)$ be specified over the entire frequency range from $\omega=o$ to $\omega=\infty$. Obviously, the required information cannot be deduced without making some additional hypothesis regarding the character of the interference or the mechanism in which it originates. On the other hand, the mere assumption that the individual elementary disturbances $\phi_1 \ldots \phi_N$ differ among themselves substantially in wave form and duration, or that the maxima of the corresponding spectra $|f_r(\omega)|$ are distributed over a considerable frequency range, is sufficient to establish the conclusion that the individual fluctuations are smoothed out in the aggregate and that consequently $r(\omega)$ and hence $R(\omega)$ would have negligible fluctuations, or curvature with respect to ω, over any limited range of frequencies comparable to a signaling range.

It is admitted, of course, that the foregoing statements are purely qualitative, as they must be in the absence of any precise information regarding the wave forms of the elementary disturbances constituting random interference. On the other hand, the fact that static is encountered at all frequencies without any sharp changes in its intensity as the frequency is varied, and that the assumption of a systematic wave form for the elementary disturbances would be physically unreasonable, constitute strong inferential support of the hypothesis underlying equation (27). Watt and Appleton (*Proc. Roy. Soc.*, April 3, 1923) supply the only experimental data regarding the wave forms of the elementary disturbances which they found to be classifiable under general types with rather widely variable amplitudes and durations. Rough calculations of $r(\omega)$, based on their results, are in support of the hypothesis made in this paper, at least in the radio frequency range. In addition, the writer has made calculations based on a number of reasonable assumptions regarding variations of wave form among the individual disturbances, all of which resulted in a spectrum $R(\omega)$ of negligible fluctuations over a frequency range necessary to justify equation (27) for efficient selective circuits. However the problem is not theoretically solvable by pure mathematical analysis, so that the rigorous verification of the theory of selectivity developed in this paper must be based on experimental evidence. On the other hand, it is submitted that the hypothesis introduced regarding static interference is not such as to vitiate the conclusions, qualitatively considered, or in general to introduce serious quantitative errors. Furthermore, even if it were admitted for the sake of argument that the figure of merit S was not an accurate measure of the ratio of mean square signal to interference current, nevertheless, it is a true measure of the excellence of the circuit in excluding interference energy outside the necessary frequency range.

IV

The practical applications of the foregoing analysis depend upon the formulas

$$\bar{I}^2 = \frac{R(\omega_m)}{\pi} \int_0^\infty \frac{d\omega}{|Z(i\omega)|^2} = \rho \cdot R(\omega_m) \tag{11}$$

and

$$S = \frac{1}{\omega_2 - \omega_1} \cdot \int_{\omega_1}^{\omega_2} \frac{d\omega}{|Z(i\omega)|^2} \div \int_0^\infty \frac{d\omega}{|Z(i\omega)|^2} = \frac{1}{\omega_2 - \omega_1} \cdot \frac{\sigma}{\rho} \tag{14}$$

which contain all the information which it is possible to deduce in the case of purely random interference. They are based on the principle that the effect of the interference on the signaling system is measured by the mean square interference current in the receiving branch, and that the efficiency of the selective circuit is measured by the ratio of the mean square signal and interference currents. As stated above, in the case of random interference results must be expressed in terms of mean values, and it is clear that either the mean square current or the mean energy is a fundamental and logical criterion.

Referring to formula (11), the following important proposition is deducible.

If the signaling system requires the transmissions of a band of frequencies corresponding to the interval $\omega_2-\omega_1$, and if the selective circuit is efficiently designed to this end, then the mean square interference current is proportional to the frequency band width $\frac{(\omega_2-\omega_1)}{2\pi}$.

This follows from the fact that, in the case of efficiently designed band-filters, designed to select the frequency range $\frac{(\omega_2-\omega_1)}{2\pi}$ and exclude other frequencies, the integral $\int_0^\infty \frac{d\omega}{|Z(i\omega)|^2}$ is proportional to $\omega_2-\omega_1$ to a high degree of approximation.

The practical consequences of these propositions are important and immediate. It follows that as the signaling speed is increased, the amount of interference inevitably increases practically linearly and that this increase is inherent. Again it shows the advantage of single vs. double side-band transmission in carrier telephony, as pointed out by the writer in a recent paper.[3] It should be noted that the increased interference with increased signaling band width is not due to any failure of the selective circuit to exclude energy outside the signaling range, but to the inherent necessity of absorbing the interference energy lying inside this range. The only way in which the interference can be reduced, assuming an efficiently designed band filter and a prescribed frequency range $\frac{(\omega_2-\omega_1)}{2\pi}$, is to select a carrier frequency, at which the energy spectrum $R(\omega)$ of the interference is low.

Formula (11) *provides the theoretical basis for an actual determination of the static spectrum.* Measurement of $\bar{I}^2$ over a sufficiently long interval, together with the measured or calculated data for evaluating the integral $\int_0^\infty \frac{d\omega}{|Z(i\omega)|^2}$, determines $R(\omega_m)$ and this determination can be made as accurate as desired by employing a sufficiently sharply tuned circuit or a sufficiently narrow band filter. It is suggested that the experimental data could be gotten without great difficulty, and that the resulting information regarding the statistical frequency distribution of static would be of large practical value.

The selective figure of merit S as defined by (14) is made up of two factors, $\frac{1}{(\omega_2-\omega_1)}$ which is inversely proportional to the required signaling frequency range; and the ratio of the integrals σ/ρ. This

[3] Signal-to-Static-Interference Ratio in Radio Telephony, *Proc. I. R. E. E.*, June, 1923.

ratio is unity for an ideally designed selective circuit, and can actually be made to approximate closely to unity with correctly designed band-filters. Formula (14) is believed to have very considerable value in comparing various circuits designed to eliminate interference, and is easily computed graphically when the frequency characteristics of the selective circuit are specified.

The general propositions deducible from it may be briefly listed and discussed as follows:

With a signaling frequency range $\frac{(\omega_2-\omega_1)}{2\pi}$ *specified, the upper limiting value of S with a theoretically ideal selective circuit is* $\frac{1}{(\omega_2-\omega_1)}$, *and the excellence of the actual circuit is measured by the closeness with which its figure of merit approaches this limiting value.*

Formula (14) for the figure of merit S has been applied to the study of the optimum design of selective circuits and to an analysis of a large number of arrangements designed to eliminate or reduce static interference. The outstanding conclusions from this study may be briefly reviewed and summarized as follows:

The form of the integrals σ and ρ, taking into account the signaling requirements, shows that the optimum selective circuit, as measured by S, is one which has a constant transfer impedance over the signaling frequency range $\frac{(\omega_2-\omega_1)}{2\pi}$, and attenuates as sharply as possible currents of all frequencies outside this range. Now this is precisely the ideal to which the band filter, when properly designed and terminated, closely approximates, and leads to the inference that *the wave filter is the best possible form of selective circuit, as regards random interference.* Its superiority from the steady-state viewpoint has, of course, long been known.

An investigation of the effect of securing extremely high selectivity by means of filters of a large number of sections was made, and led to the following conclusion:

In the case of an efficiently designed band-filter, terminated in the proper resistance to substantially eliminate reflection losses, the figure of merit is given to a good approximation by the equation

$$S=\frac{1}{\omega_2-\omega_1}\,\frac{1}{1+1/16\,n^2}$$

where n is the number of filter sections and $\frac{(\omega_2-\omega_1)}{2\pi}$ the transmission band. It follows that *the selective figure of merit increases inappreciably with an increase in the number of filter sections beyond* 2, *and that the* *band filter of a few sections can be designed to have a figure of merit closely approximating the ideal limiting value,* $\frac{1}{(\omega_2-\omega_1)}$,

This proposition is merely a special case of the general principle that, as regards static interference, it is useless to employ extremely high selectivity. The gain obtainable, as compared with only a moderate amount of selectivity is slight and is inherently accompanied by an increased sluggishness of the circuit. That is to say, as the selectivity is increased, the time required for the signals to build up is increased, with a reduction in quality and possible signaling speed.

Another circuit of practical interest, which has been proposed as a solution of the "static" problem in radio-communication consists of a series of sharply tuned oscillation circuits, unilaterally coupled through amplifiers.[4] This circuit is designed to receive only a single frequency to which all the individual oscillation circuits are tuned. The figure of merit of this circuit is approximately

$$S=L/R\frac{2^{2n-2}\,(n-1)\,!^2}{(2n-2)\,!}$$

where n denotes the number of sections or stages, and L and R are the inductance and resistance of the individual oscillation circuits. The outstanding fact in this formula is the slow rate of increase of S with the number of stages. For example, if the number of stages is increased from 1 to 5, the figure of merit increases only by the factor 3.66, while for a further increase in n the gain is very slow.[5] This gain, furthermore, is accompanied by a serious increase in the "sluggishness" of the circuit: That is, in the partciular example cited, by an increase of 5 to 1 in the time required for signals to build up to their steady state.

The analysis of a number of representative schemes, such as the introduction of resistance to damp out disturbances, balancing schemes designed to neutralize static without affecting the signal, detuning to change the natural oscillation frequency of the circuit, demodulation through several frequency stages, etc., has shown that they are one and all without value in increasing the ratio of mean square signal to interference current. In the light of the general theory, the reason for this is clear and the limitation imposed on the solution of the static problem by means of selective circuits is seen to be inherent in the nature of the interference itself.

[4] See U. S. Patent No. 1173079 to Alexanderson.

[5] When the number of stages n is fairly large, the selective figure of merit becomes proportional to $\sqrt{n}$ and the building-up time to n.